# How Do We Describe an Interdisciplinary Curriculum?

Thomas R. Berger
Department of Mathematics and Computer Science
Colby College
Waterville, ME, 04901
trberger@colby.edu

The Committee on the Undergraduate Program in Mathematics (CUPM) is addressing the various majors in the mathematical sciences. About 1970 [i] CUPM released documents describing the major in terms of some courses and their mathematical content. By the early 80's [ii] this task was more difficult so the resulting document is more ambiguous. A more recent effort in the early 1990's [iii] was not conclusive since the scene was rapidly changing and the situation very diverse. In 1995 [iv] CUPM took another approach. They looked at successful programs and presented these as models of good curricula. There is a growing feeling that these case studies do not go far enough. It is time again to try to describe a curriculum.

During this same thirty-year period attitudes changed about curriculum based upon broad experience with massive numbers of mathematics students and with research on teaching and learning. There is a general feeling that we should describe education in terms of expected outcomes. No one has exactly figured out how to do this yet. In other words, the major should have goals for the students and the program. If those goals can be articulated, then they can be combined with information about a school's students, the faculty teaching those students, the types of careers for graduates, and the environment of the college or university. From these factors a program can be planned that fulfills the goals and serves the students, institution, and faculty.

Probably even greater have been the outside forces that act upon curriculum decisions. Just to list a few:

First and foremost, technology has affected the tools we have to teach. It has become a demand from employers. We ignore it at the peril of our students.

Second, a wide range of teaching styles have been tried and discussed. Much research has been done on student learning. The results indicate that these different teaching styles may have much to recommend them. Third, our majors are no longer prepared primarily for entry into graduate study in mathematics. Many graduates go on to graduate and professional schools, but mainly in disciplines other than mathematics. The vast majority of graduates enter the workforce directly. We can single out a few places (for example, teaching, finance, and engineering) where it might be possible to delineate the mathematics students should know for areas that attract moderate numbers of graduates. [v] Except possibly teaching, these few areas still attract less than ½ of the majors. Diversity is the most salient feature of careers of recent graduates in the mathematical sciences.

Fourth, employers seek mathematics connected with other areas of knowledge. Students should be able to translate the mathematics they know into the situations they will encounter on the job. This exerts pressure on faculties to develop courses in "relevant mathematics" when sometimes we don't even know what the word "relevant" should mean. Fifth, because so many students enter the job market, additional skills are expected. The students may need technical writing skills, practice in interpersonal relationships in a technical working group, experience studying with students in a variety of disciplines, and other very non-traditional skills.

Sixth, a discovery that the success of students in a major has a great deal to do with the context in which learning takes place. The respect students receive, the care taken for their welfare, and the pleasure they take in their studies are important factors. The excitement and pleasure faculty take in their courses is another important factor.

With all this in mind, how can we specify our expectations of a major in the mathematical sciences in such a way that a department might plan a program? This paper presents some preliminary ideas that have come to CUPM. First, how might a description of a curriculum be given if it describes what students should know at the end of two years? Second, if process skills are important, which ones do we describe and how do we do that? The paper mostly just raises questions and offers a few tentative examples.

## I. Goals for Content

How might a description of expectations look? No one really knows, but many are trying. This paper makes a modest start by using calculus as an example. For the curriculum in the mathematical sciences majors, calculus is probably the most described and taught of all courses. Describing our expectations ought to be easy. It isn't.

In the past it was expected that students should have completed a course in the calculus at the end of the first two years for a mathematical sciences major. A traditional curriculum would discuss the usual or minimum coverage of topics. Turning this around, we ask ourselves the question:

**"What knowledge does the average student carry out of this course that is durable or will be heavily exercised in future coursework?"**

At the most basic level of curriculum description, what skills do we expect of students? Do we expect students to know differentiation and integration formulas and techniques? Experience and research shows we rarely attain the traditional goals we set for skills even with very good students. Now computer algebra systems (like Maple, Mathematica, and Derive) provide powerful tools to simplify traditional differentiation and integration problems. If so, do we even want the same level of competence that traditional curricula prescribe? A perusal of textbooks shows that these matters are not resolved. Essentially all texts claim to be fully "calculus reformed." Some texts place heavy emphasis on traditional skill development (including some of the most widely used texts). Others downplay this role while increasing emphasis on conceptual understanding.

It is in this context that we must describe our expectations for students of the calculus. (Others have discussed calculus [vi,vii] and the content of the first two years. [viii]) Here then are some possible expectations of our students for calculus.

1. Every mathematical sciences major should have an intense calculus experience lasting at least one year.

   A sample attempting to describe a student by level of competence is given next. This standard should be applied one or more years after the course is completed. For curricular purposes, it provides a basis for discussion of the calculus course and how it fits into the mathematical program. Let's try to describe computational levels of achievement.

2. A student completing a calculus course should achieve computational proficiency in the course. This might be measured as follows.
   a. Basic Proficiency

The student is presented with a problem typical of the course and its solution. The student can read and understand both the problem and its solution. The student can describe each step leading to the solution.

c. Proficient
The student has basic proficiency and can describe the solution by explaining the methods used, why they were used, and how they were used. The student exhibits some understanding of the connection of this problem with general knowledge in the subject.

d. Skilled
The student is proficient and can frequently supply solutions to such problems without being presented with them, especially if given a little warning and time.

e. Highly Skilled
The student is skilled and can solve some complex problems requiring significant symbolic computation (either manually or with the help of a computer algebra system or both). The student can take a page of such computation and discuss and modify it in interaction with another highly skilled person.

This approach to a curriculum is sensitive to the kinds of problems posed for students. A traditional curriculum would focus on problems now familiar to us. An interdisciplinary curriculum would emphasize problems rooted in a variety of disciplinary contexts. For today's students, problems should probably contain a mix of applied and pure. Understanding how the calculus tells us about the economic, social, living, and physical world around us broadens students' understanding of the mathematics itself and provides a basis for making mathematics useful.

The levels and descriptions above are similar to graduation standards being written around the country for high school students. For a school student the description might refer to an algebra, rather than a calculus course. Recently I spent a day looking at statewide test results in order to obtain a community consensus for performance levels described as above. The goal for skill level for the course would be that all students achieve, at least, Basic Proficiency with some students becoming Highly Skilled. This rather elaborate style can be continued through each of the following types of knowledge. I'll be a bit briefer.

Skill development is only part of understanding calculus. A curriculum should provide a foundation for a conceptual understanding. Stating measures of achievement in conceptual understanding is more difficult since our tests tend to only indirectly measure this quality.

3. The student should be proficient in the basic principles of the calculus.
   a. A student should be able to describe the derivative by means of examples with tables, graphs, and formulae, including interpretations as both slope and instantaneous rate of change. The student should be able to move comfortably among the various representations. The student should have some notion how the derivative is approximated by the difference quotient. Depending upon the level of the course and expectations of the program, students may be able to relate these concepts with their mathematical explication including connections with the difference quotient and limits. If a course has a theory component, students should be able to explain connections with definitions and simple theorems. They should be aware that the subject depends upon fundamental properties of the real number system.

b. A student should be able to describe the integral by means of examples with tables, graphs, and formulas, including interpretations as both area and total change. The student should be able to move comfortably among the various representations. The student should have some notion how the integral is approximated by sums of areas of rectangles. Depending upon the level of the course and expectations of the program, students may be able to relate these concepts with their mathematical explication including connections with Riemann sums and limits. If a course has a theory component, students should be able to explain connections with definitions and simple theorems. They should be aware that the subject depends upon fundamental properties of the real number system.

**Are standards like these for content appropriate?**

A curriculum should provide a basis for discussion within a department for a curriculum appropriate to the students, department, and institution. Because these factors can vary so much, it isn't possible to list every topic. Enough information should be provided to guide a department to make wise decisions compatible with those being made at other institutions.

**What other content expectations should be placed upon students for the first two years of a mathematical science major?** (In all programs and majors in the mathematical sciences.)

A. If the student is specializing in an area like statistics or computer science, should we place an expectation of completing an intensive yearlong course in the specialty?
B. Do we expect every major to have experience with linear algebra in the first two years?
C. Do we expect every major to have experience with differential equations in the first two years?
D. Do we expect every major to have experience with infinite sequences and series in the first two years?
E. Do we expect every major to have experience in discrete mathematics in the first two years? (Topics from computer science, combinatorics, optimization, mathematical modeling, etc.)
F. Do we expect statistical learning for all mathematical science majors in the first two years?
G. For pure mathematics majors, do we expect introductory experience with proofs at an appropriate level?
H. Do we expect that students will also develop interdisciplinary knowledge in appropriate courses? If so, how should this be described so as to serve a diverse student body?

Students will be exposed to this variety of mathematics in a sequence of courses. Should there be summative demands on the curriculum so that students pull together the knowledge they are acquiring from various courses? Issues of content are ones we claim to understand. Can we find sufficient consensus to write content standards for the first two years?

## II. Goals for Process.

The world expects our students to have skills usually not explicitly taught in school. Might it not be time to make some of this explicit? Let me state some particular skills so that you can react to them. These types of skills involve general competencies applied particularly to the mathematical enterprise. Frequently in English courses students learn how to use the campus library. But when asked to do so in a mathematics course, the students are often unable to perform the simplest tasks. That is, our assumption that "English can take care of this issue."

is not valid. Knowledge does not transfer quite so easily as we hope. So we may want to help our students learn process skills in the context they will use them within our discipline.

Here are some possible process skills for the first two years. Are there ones on the list that are optional? Are there missing ones that are crucial?

1. Learning skills.
   a. Students should be able to join a team and learn the basic principles of a new topic from a given body of text or reference material.
   b. Students should, on their own, be able to learn the basic principles of a new topic from a given body of text or reference material.
   c. Students should be able to acquire the basic principles of a new topic from a lecture at the appropriate level.
2. Resource Skills
   a. Given a specific mathematical topic, students should be able to
      i. find resources at the appropriate level on the given topic in a library,
      ii. find resources on the Web and know how check for validity and accuracy, and
      iii. find resources within the community.
   b. Students should be able to organize the results of research appropriately for the task at hand.
   c. Students should know about standards for plagiarism, bibliographic style, and presentation style within their area of concentration.
3. Communications Skills
   a. Students should be able to write solutions to problems in a way that communicates to a general mathematical audience.
   b. Students should have experience in writing extended reports on mathematics.
   c. Students should be able to orally present mathematics of the appropriate level to a group of peers.
4. Working Skills
   a. Students should be able to learn to use computer tools and have some knowledge of their own learning curve for such tools (i.e. how long will it take to learn and what level of effort must be invested?).
   b. Students should work effectively as a member of a team on an extended mathematical problem.
   c. Students should have experience working on an interdisciplinary team on an interdisciplinary problem.
5. Problem Solving Skills
   a. Students should have experience in working on extended mathematical problems?
   b. Students should understand problem solving processes and be able to articulate and apply these processes?

For many years, in several engineering and science disciplines, students have been given ill-posed, large-scale problems and asked to form teams to attack these problems. Sometimes courses on mathematical problem solving address issues of this kind. These experiences are valuable for students seeking jobs right after graduation. Many students who continue to graduate study also testify to the value of such experience. Might there then be process standards that are summative in this sense?

6. Summative skills

Students should have experience working on a team and bringing together most of their process skills and much of their knowledge. The experience should involve problem clarification,

resource gathering, problem solving, application of mathematics, use of appropriate technological tools, report generation, and written and oral communication of results.

In no sense are the above skills being proposed as essential to a curriculum. However, they are given in order to pose the question:

Should curriculum standards for mathematics describe process skills?

In other words, should we look at our entire curriculum to see that a major does acquire a certain minimum of process skills?

## III. Other Goals

I have only suggested content descriptions for some of calculus and for a few process skills. There are many other aspects of a curriculum. For example:

1. How serious are we about an interdisciplinary culture in our curriculum?
   Are we serious enough to ask that applications of a subject be more than peripheral?
   Should this topic be addressed in a curriculum description?
2. Students learn in an environment. In other words, the mode of learning is itself an integral part of the curriculum. How should a curriculum description address this issue?
3. Technology has been implicit in many of the suggestions above. Does the issue need explicit mention in a curriculum?

## IV. Summary Comments

The book "Models that Work" [Note IV] makes interesting reading. Frequently courses are mentioned but never quite described in terms of content. A successful curriculum depends very heavily upon the context for learning and its appropriateness to the faculty, students, and university. In trying to understand why the models are effective, one wants to read studies like: [ix] "Talking About Leaving," a study trying to understand why bright students leave mathematics, science, and engineering majors. [x] "What Matters in College?," a summary of many years of research by Astin's team isolating factors that influence student success in college, and [xi ], "How College Affects Students," a summary of twenty years of research on all facets of the college experience. Since the success of a curriculum includes the context in which courses are delivered, how should this aspect of the curriculum be discussed in a curriculum document?

In "Commentary on a General Curriculum in Mathematics For Colleges" (1972) [See Note I, Volume I, page 33], CUPM describes majors in the mathematical sciences by listing courses and their content. At the entry level, topics are described in some detail including suggested time for presentation. These types of descriptions focus attention on coverage rather than understanding. A shift to descriptions of expected student outcomes might refocus attention on what students actually learn. Since experience with outcome descriptions is limited, the task is difficult. I've attempted to give some feeling for the kinds of descriptions currently being used.

Now that a vast majority of mathematical sciences majors enter the workforce immediately upon graduation, the process skills we address in the curriculum become more important. They provide the context in which the students do mathematics. Experience with process skills enables students to use their mathematics. I've attempted to mention a variety of skills that have been discussed or mentioned in CUPM meetings.

In order for a curriculum description to be useful to the larger mathematical community, we need a great deal of discussion and reaction. Your suggestions and reactions are welcome. As you write about these issues you might try to spend some time focussing on goals and expected outcomes for students. You can address email comments to CUPM at "cupm-curric@ams.org. <mailto:cupm-curric@ams.org.>"

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