The Old Time Religion: College Calculus

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Introduction

Since this paper is somewhat critical of the changes currently taking place to "reform" the college mathematics curriculum, it is appropriate to provide a very brief sketch of my background. After more than 40 years in industry and the academic, I have retire to start a small company that publishes engineering textbooks intended for first year engineering students. During my active tenure, I taught at four different colleges, a military academy, and worked as a practicing engineer for three different companies.

Most of my experience, upon which this paper is based, is with the University of Maryland (U of MD) at College Park. The U of MD has a large engineering program with more than 600 students entering the freshman class each year. The students are relatively bright with an average SAT in excess of 1250, although about a third of this class requires remedial mathematics. The required mathematics courses (15 credit hours) are taught over the first four semesters with differential equations as the final offering. The book adopted for the first three semesters is Calculus with Analytic Geometry, 5th Edition, by Robert Ellis and Denny Gulick.

To prepare for this paper, I purchased the book (a 1024-page tome not counting the appendices or the 488-page solution manual). I also contacted the Chair of the Mathematics Department to obtain copies of the syllabi for the four courses and representative copies of the final examinations for each course for the past two semesters. Finally, I dug out my old college calculus book[1] to make a comparison between the coverage in the old and the new textbooks. The new book(s) is printed using three colors, covers more material, and costs about 50 times more than the old book, yet there are remarkable similarities in content. The old book covered 105 topics in 12 Chapters while the new book covered 117 topics in 15 chapters. Most of the additional content in the new book deals vectors and partial derivatives.

Interdisciplinary Efforts

There have been significant efforts to integrate the freshman curriculum at Drexel University, Rose-Hulman Institute of Technology and by others in the Foundation Coalition. These efforts which integrated 12 credit hours of technical courses (mathematics, science and engineering) have provided an improved pedagogy for presenting basic content in an interdisciplinary fashion with documented improvements in student performance. However, these efforts seem to have stalled because of inertia in the system. System inertia is defined here as the tendency of both students and faculty to cling tenaciously to the status quo. The system works—don't change it. Of course, it is clear that the system is seriously flawed. Mathematics is a killer subject that causes many good students to lose interest in a technical education long before they complete

[1] Elementary Calculus, by Woods and Bailey, a book with a 1928 copyright was still in use at Carnegie Institute of Technology in 1948. This book contains 385 pages including solutions and appendices.

high school. Many other engineering students with entry level skills in mathematics encounter significant difficulties with the abstract content of the Calculus courses and transfer to other colleges where the mathematics requirements are much less demanding.

At the U of MD, the College of Engineering sends it students to a Mathematics Department that is in the College of Computer, Mathematical and Physical Sciences. Four organizational walls exist between the students and the curriculum of a typical engineering program. Cooperation is limited, communication is rare, and change is extremely difficult to implement. However, recently cooperation has occurred between the Electrical Engineering and Mathematics Departments in offering EE 214 an Electrical Engineering course on numerical methods that is taken concurrently with, and in addition to, the required course in differential equations. Unfortunately, instead of being team-taught, the course is offered by the faculty of one department or the other on alternating semesters.

It is the author's opinion that very little is being accomplished to foster inter departmental cooperation in spite of the fact that mathematics requirements represent over 12% of the credit hours for College of Engineering programs leading to the B. S. degree. There is an overwhelming sense of indifference on the part of faculties from the Engineering and Mathematics Departments. There are several reasons for the indifference.

Mathematics Faculty

- It is a service course.
- The students are not interested.
- Expectations are low.
- It generates student credit hours.
- The material is so old and worn.
- No one cares.

Engineering Faculty

- It is out of College and Dept.
- Impossible to make changes.
- Dean handles the negotiations.
- It is a not a priority item.
- We will teach a departmental mathematics/computer course to provide the tools our students need.

Much can be accomplished to markedly improve the student's experience in this massive exposure to calculus and differential equations. We will describe some apparent difficulties and make suggestions for improvements in the following sections.

Content

There is little or no effort to integrate the content in the mathematics courses with the engineering courses. The syllabi prepared by the Mathematics Department largely ignore the needs of engineering students. The first problems arise in the second semester when most of our engineering students take courses in statics and physics (classical mechanics). Several mathematics skills are required in both courses. Vectors are used to represent forces and moments and the students lack previous exposure to vector analysis. The material on vectors (dot and cross products) is taught in the mathematics department in the third semester—two semesters later than the student's need. Because of this disconnect, it is necessary for the engineering faculty to introduce vector mechanics as they simultaneously try to cover equilibrium principles, free body modeling, friction, etc.

Applications used to illustrate the utility of integration are not well matched to the sequence of engineering mechanics courses (statics, mechanics of materials, and dynamics). Indeed, a review of the final examinations for the four required mathematics courses over two semesters did not reveal a single problem on integration that could be related to an engineering application. Yet engineering applications such as finding centroids, first and second moments of areas, products of inertia, mass moments of inertia, moments, etc. are much more appropriate than the abstract examples used for assignments and examinations. Furthermore, if these application problems do appear in some calculus texts, they are usually relegated to "optional" problems or motivational activities, rather than routine problems.

The capabilities of computers and software have made remarkably little impact on the first three of the four required mathematics courses at the U of MD. Some calculators are allowed on examinations but others (TI-89 and TI-92), capable of differentiation and integration, are not allowed at the U of MD. These rules imply that the emphasis is on how to memorize and execute recipes for differentiation and integration. High-speed calculation and graphing capabilities of low-cost computers and software programs have made many of the algorithms (e. g. integration) taught in these courses irrelevant. The focus on closed form solutions is as extinct today in engineering as the dinosaur. The value of a course outcome of having students integrate functions like $\int x^2 \sec^2 (x^3)dx$ (a typical final examination problem) is not clear to the author considering existing computational capabilities that will continue to improve dramatically in the future.

Emphasis

The content of the three calculus courses and the textbook used at the U of MD ignores the developments of the past two decades in numerical computation methods. While it is useful to understand **how** to differentiate and integrate some simple functions, we need not become consumed in the process. More important considerations are **why** we are differentiating, and **what** is the function. Sure we know that if a function is differentiated, its maximum, minimum and inflection points may be located. That was a very important procedure before we could compute at 500 MHz and plot graphs with a laser printer. A casual glance at the graphical output identifies all of these characteristics. Today, it still nice to know about the properties of differentiated functions, but it is much less important.

Integration is performed to determine a large number of properties of areas and masses of critical interest to engineers. However, today we can determine these properties with ease by summation (rather than formal anti-differentiation resulting from the integration as currently taught) using even the simplest spreadsheet. As with differentiation, it may be useful to understand **how** to integrate some simple functions, but it is more important for the student to understand **what** integration implies and **why** we must add all of those little slices or squares together

The determination of limits, which appears to be almost sacred, is still another example of the disregard of computational capabilities that exist today. Consider the well-worn example of f(x) = x/sin (x) as x goes to zero. Ah! Apply L'Hospital's rule, differentiate numerator and denominator, and let x in each expression go to zero. Or forget the rule, open a spreadsheet and compute the value of x/sin (x) as x goes to zero as shown below. Actually, the limit could be determined with only a few lines; we showed six lines here to demonstrate the approach to the limit.

x(rad)	sin x	x/sin x
1.57	0.9999996829	1.57000049779718
0.157	0.1563558123	1.00412001137264
0.00157	0.0015699994	1.00000041081678
0.000157	0.0001570000	1.00000000410817
0.0000157	0.0000157000	1.00000000004108
0.00000157	0.0000015700	1.00000000000041

If the **how** part of calculus and differential equations was de-emphasized and modern computational methods employed appropriately, time would be available to increase the scope of the coverage to much more relevant topics that are not included in the 15 credit hours currently devoted to mathematics. Or the time saved could be used to develop a deeper understanding of mathematics and the need for analysis.

The f(x) Fixation

Calculus textbooks are filled with f(x) or perhaps g(t). These functions are usually defined as a close form relation, and the student is required to perform some type of operation with f(x) to arrive at another closed form relation. Unfortunately, f(x) has little to do with real life and/or engineering. Engineers design and produce products or processes. In this endeavor, engineers make many measurements and accumulate data that defines a process or the performance of a product. The CEO or VP of Engineering does not hand down f(x).

Engineers in practice work with data and from this data they **may** infer f(x) or f(x,t) or even more complex functions. It would be refreshing if the mathematics faculty would recognize this fact, and start with data and arrive at relations for f(x) that represent useful functions. These are the functions that should be used in teaching integration methods. Of course, if this were the case we would never encounter $\int x^2 \sec^2 (x^3)dx$ or similar abstract functions commonly found on final examinations.

When dealing with real data, engineers find the content in a Statistics course to be more valuable than much of the content in the typical coverage of Calculus. With Statistics, engineers learn about characterizing a population (a process) and the importance of minimizing its variability. They learn about probability, designing experiments, and determining the importance of process parameters. These are all concepts that could be included in an exposure to mathematics that eliminated some of the coverage in Calculus and Differential Equations made obsolete by modern computational methods

One Size Fits All

A typical Mathematics Department services students from all the Colleges in the University. This population has widely different skills and interests, and Mathematics Departments have introduced courses for students from different colleges and their own mathematics majors. On first examination it appears that the offerings are sufficiently broad to cover this wide range of skills and interests. Yet the number of offerings is small compared to the number of students involved. At the University of Maryland, the Mathematics Department offers 50 different courses for an undergraduate population that approaches 30,000. That is about one course for

each 600 students. There appears to be room for more courses specifically tailored to the skills and student's interests.

Twenty years ago the curriculum for almost all of the engineering disciplines was common for the first two years. Today, only the first semester is common at the U of MD. The Electrical and Chemical Engineering curriculum is significant different from the other engineering disciplines. The mathematics requirements are also different, particularly when we consider the sequence in which the material should be presented and the applications, which maintain the student's interest. It is clearly time to consider a sequence of mathematics courses for electrical engineers (200 students) and a modified sequence for mechanical and aerospace engineers (200 students). The numbers of students are sufficient to justify the benefits of presenting relevant content, sequenced so as to support required courses in science and engineering, with applications that are matched to the curriculum.

Many engineering departments find it necessary to teach one or more courses of applied mathematics as part of their undergraduate engineering program. For example, in Mechanical Engineering a course on linear algebra and MATLAB is offered in the fourth semester to prepare the students for the computational requirements of the upper division ME courses. Mechanical engineering faculty also teach two other courses—statistical methods and finite element methods—both with significant mathematics content. Clearly, the mathematical content in a typical engineering program has grown to 20 or more credit hours if one counts the in-house efforts of a typical department. It would appear that more content could be covered in less time if the courses in the mathematics and engineering departments were synchronized and coordinated much more closely than they are today. We would gain efficiency by "designed coherent redundancy" as noted by Brian Winkel and Jeff Froyd at Rose-Hulman.

Future Directions

The changes underway for the past twenty years in computational methods will continue. Computers will be faster (xxGHz), be equipped with more memory, and cost less. Software programs capable of quickly solving the problems encountered in any mathematics course will become even more user-friendly with improved graphical interfaces. What changes should be made in the mathematics offering to recognize these developments

Clearly, we will not abandon Calculus or Differential Equations; however, the content should be taught in conjunction with software that removes the need to teach recipes and how to do this or that. The author has interviewed students exposed to pilot programs where a symbolic program was introduced together with calculus. They complained bitterly because the calculus content was not changed, and they had to learn how to manipulate a new software package without a matching reduction in the mathematics load. In these early pilot efforts, the two approaches were simply superimposed and not appropriately integrated.

The author does not believe that significant reform will occur unless small groups of faculty join together to offer interdisciplinary courses with markedly different content. In most universities, college and department structures seriously inhibit the possibility of twelve credit hour courses, but integration of six or perhaps even nine credit hours may be possible. The key to success of any reforms with integration of content from two or three courses is to:

1. Eliminate irrelevant material.
2. Recognize and utilize the capability of computation.

3. Achieve nearly seamless integration.
4. Sequence content providing information sequenced to student's needs (Just In Time).

Mathematics is currently loaded into the front end of the curriculum with 15 credit hours required in the first two years. Of course, the rational for this approach is that mathematics is supposed to be a foundation for the engineering courses that come later in the curriculum. Unfortunately, knowledge is perishable (particularly when students memorize rules and recipes) and the foundation crumbles. Consideration should be given to presenting the mathematics offerings over three years (with 18 credit hours). If some of the courses were offered jointly between the engineering and mathematics faculties and tightly coupled to the discipline content in the third year, much of the more applied mathematics taught by engineering faculty would not be necessary.