Technology and Interdisciplinary Calculus:

Three Challenges for Educators

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The move toward more interdisciplinary work in mathematics has been fueled by -- at times even motivated by -- the ready availability of inexpensive, portable computing technology. From the earliest versions of scientific calculators that transformed the teaching of trigonometry and logarithms to the CAS-equipped graphing calculators and laptop computers that are changing calculus instruction today, handheld technology has affected what we teach and how we teach it.

Although technology may not be the *primary* reason for cross-disciplinary collaboration, it makes the sharing of common goals easier. Advances in numerical and graphical methods let calculus students tackle more realistic applications for the sciences; computing packages give physics students access to mathematical techniques formerly out of reach (or out of sequence); multiple representations in the form of tables and graphs lead to new connections for engineering students. Technology facilitates explorations and discovery-based activities that blend a variety of mathematical and scientific concepts.

With calculators and computers, students across the disciplines can investigate interesting problems despite gaps in their mathematical knowledge. Using technology as a panacea for mathematical weaknesses, however, can be dangerous and misleading. In fact, calculators and computers can introduce a new set of problems to mathematics instruction and client disciplines. I shall describe some of those problems as I address one of the questions posed at this conference: "How should technology affect what and how we teach?" The answer depends upon our responses to three challenges that face mathematics educators: 1) keeping up with technology advances; 2) dealing with the classroom effects of technology; and 3) preparing students to use technology effectively.

Challenge #1: Keeping up with technology advances. It is tempting to dismiss this challenge quickly: We can't keep up with advances, so why try? As soon as we adapt the curriculum to reflect technology currently available, there will be something better, faster, more comprehensive, more amazing. While a generation in curriculum development is at least a decade (witness the pace of the calculus reform movement or production time for a text from inception to wide adoption), a generation for handheld technology may be as short as a year. Consider, for example, how many versions graphing calculators passed through from the first Casio 7000 in 1986 (without numerical integration or a "zoom-box" feature) to the TI-92 in 1995 (with CAS, 3-D graphing, and geometry software).

Changing the calculus curriculum every year to keep pace with technology is not feasible. How then should we react to new machines and software? Some innovations are so useful (e.g., numerical integration) or contribute so substantially to understanding (3-D graphing) that

instructors will bring them into the classroom as soon as possible; but fundamental changes to the core calculus should not be determined by available functionalities on this year's calculator. We should not toss a new application into calculus just because the technology now exists to solve that problem; nor should we abandon a by-hand procedure just because a machine does it faster. Rather, there is a serious need for more venues -- like this workshop -- to discuss what remains "core" in calculus and what procedures, problems, and applications *further* students' understanding of calculus concepts or make them better at modeling problems. That discussion must include the so-called "client" disciplines, for calculus *is*, after all, a tool important to other courses.

Utility, however, cannot be the sole criterion for inclusion. Calculus instruction must also preserve some ideas that reflect the historical development and the beauty of the subject -- even if a machine would make the topic moot. One of my personal favorites from differential calculus is Newton's method. Numerical or CAS "solve" keys may produce numerical solutions faster, but Newton's algorithm is a simple application of the derivative that effectively demonstrates the power of linear approximation. Calculus is one of the great achievements of the human intellect; Newton's method reminds us of that in a way technology can't.

To decide when and how to use technology, we need a clear understanding of the goals of calculus instruction. Kutzler (1997) suggests that one teaching goal is to make students capable of solving real world problems through modeling by translating such problems into model problems, applying algorithms to get a model solution, and interpreting the model solution for the real world (p. 30). It is at the stage of "applying algorithms" that technology becomes most useful -- and perhaps most controversial. If the end goal is the answer, then a black-box use of technology (in which the method of solution is not open to the student) may suffice. On the other hand, if the end goal is understanding the modeling and solution processes, one might want to apply a step-by-step algorithm a few times. Performing the separate steps may take longer but may lead to a better understanding of subsequent problems.

Technology can still play a role, albeit a more transparent one, such as solving a differential equation by using Derive to manipulate both sides to separate variables and then integrate the results. In this "white-box" use of technology, students benefit from the power of CAS while still directing the steps in the solution process (Kutzler, 1997). Even when state-of-the-art technology is available, there are times when black-box methods (emphasis on the answer) should be limited in favor of white-box methods (emphasis on the mathematics) that reveal more of the calculus and lead to better understanding.

Challenge #2: Dealing with the classroom effects of technology. The effects of technology usually assist, rather than challenge, instructors. Research supplies evidence of positive benefits in computation, problem solving, and function and graph concept development (Dunham & Dick, 1994). Technology permits effective resequencing of content to teach concepts before manipulation skills with no difference in achievement on manipulation skills (Heid, 1997). Moreover, graphing technology may have greater benefits for some special populations -- in effect, leveling the playing field for women, non-traditional college students, low-ability students, and students with less spatial visualization ability (Dunham, to appear). Studies also point to positive changes in classroom dynamics and pedagogy as well as student attitudes (Heid, 1997).

Computing technology, despite reducing the tedium of long calculations or making seemingly impossible problems accessible, does bring other difficulties to the fore. Several studies indicate a class of misconceptions that are induced by technology. Examples include: a) graphing asymptotic rational functions as if they were continuous (in the "connect" mode); b) accepting an initial view of a graph in a window that doesn't reveal important behavior (shape, intercepts, etc.); c) misreading slopes because of misconceptions about scale and aspect ratios; and d) treating all values as rational decimals (Dunham, to appear). Lauten and others (1994) note a loss of distinction between variables for graphing calculator users. They observed "equal" treatment of "x" and "y" -- wherein independent-dependent functional relationships of x and y and the height interpretation of y are lost -- resulting from treating graphs as objects and using TRACE commands to produce both coordinates simultaneously. Our science colleagues have long complained about students' lack of graphing skills and inability to read graphs; if technology promotes new misconceptions about graphs, we have cause for concern in mathematics, too.

Research also reveals that most students develop an unquestioning reliance on technology answers, even when those answers may be wrong. (Consider, for example, calculators that produce a numerical derivative at 0 for $f(x) = |x|$.) When Glasgow and Reys (1998) studied the issue of misplaced confidence or lack of skepticism about machine solutions by programming calculators to give wrong answers, they found that students usually discounted their own intuition about a problem and relied on the machine's answers -- even when that answer was very wrong.

Combating technology-induced errors and making students aware of machine limitations constitute a challenge for all calculus instructors. We must be careful to avoid replacing one set of errors (algebraic difficulties) with another (technology-based misconceptions).

Challenge #3: Preparing students to use technology effectively. Instructors can counteract technology-related errors with careful teaching. Placing greater emphasis on estimation, checking for reasonableness of answers, and encouraging students to question solutions might reduce blind acceptance of machine answers. Explaining how calculators plot graphs and represent numerical approximations, discussing machine limitations (e.g., round-off and truncation error, discrete representations of continuous functions), and teaching explicitly about scale issues can alleviate some misconceptions as well as provide an opportunity to talk about some interesting mathematics. Intelligent technology use demands more attention to the mathematics of numerical methods, especially as we increase our focus on applied science problems.

Greater reliance on graphical and numerical representations in calculus also means a greater need for instruction in how to read and interpret graphs and tables. This is a meeting place for interdisciplinary interests, where mathematics, science and engineering instructors can help and support one another by giving students more experience with graph production. For example, both calculus and workshop physics classes can focus on understanding kinetic graphs with motion activities using CBL data collection devices.

Finally, effective instruction with technology requires time to develop a classroom culture in which technology is thoroughly integrated and students are comfortable with the machinery and its capabilities (Dunham, to appear). Novices use technology in a different way and get less benefit from it; they need time to develop expertise and confidence with the tools before they can employ them effectively. One solution to this problem is pairing inexperienced users with more expert

users to speed up the learning process. Another possibility is coordination of tools or software packages across courses; if students see the same computing technology in both math and science classes, they have more opportunity to become familiar with its features and become efficient and effective users. Students cannot mine the potential of technology without thoughtful instruction in its use.

Conclusion

Computing technology has an important place in the future of calculus instruction. To ensure that place, we must face the three challenges. Instructors and curriculum developers, informed by research, must consider what kinds of technology uses aid student understanding in calculus, how technology can affect student learning, and how to teach with technology to get the greatest benefit from it.

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