Technology in the First Two Years of Collegiate Mathematics

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Technology affects the teaching and learning of mathematics in the first two years of collegiate mathematics in many ways as:

- a platform for presenting ideas,
- an engine for restructuring mathematical course content,
- a tool for tutorial review,
- an Internet communication and data collection device,
- a way of performing mathematical computations with numbers, symbols, and graphs.

A Platform for Presenting Ideas

Historically, professors have presented material at a blackboard with chalk. In more recent times, we have begun to use color chalk and, in some cases, transparencies on overhead projectors. We have often used these tools poorly, especially overhead projector which blind the instructor and obstructs the view of students directly in front of the projector. With the new, optically sharp, lightweight computer projectors that can be mounted in the ceiling and connected to a variety of devices that are controlled from a computer, an instructor can display marvelous class notes, simulations, animations, and computer-driven computations quickly and easily. Preparing presentations that take full advantage of this technology is time consuming but can be very effective.

An Engine for Restructuring Mathematical Course Content

Computer software that performs mathematical computations can be a driving force for reexamining the content and emphasis of courses. We have seen how even low level technology has in the past changed physics courses. Students no longer spend long hours computing answers using paper and pencil or slide rules, but rather are engaged in working many more problems because they perform the computations with a scientific calculator or computer. The content of the courses may not have changed much, but the time students spend on critical thinking about physical principles has increased. In mathematics, we have not yet fully addressed the difference between mathematics and mathematical computations. This can be seen most vividly in the importance that has historically been given to the differential equations course of computational gymnastics that was frequently the capstone course in mathematics for engineers. The wonderful applications that become accessible with an understanding of differential equations were frequently overlooked in the quest for increasingly elaborate computations to solve more and more specialized differential equations. That most differential equations could best be approached from a numerical, qualitative, and graphical standpoint is often lost in the effort to make sure students can compute quickly and accurately, something a computer is better suited to do.

Some mathematicians do not value the applications of mathematics and as a consequence do not give emphasis to them. Others like even less the inroads that computer-driven computations and

graphics have made in the use of mathematics. Theory is an essential (if not the essential) ingredient of our discipline. However, most students do not major in mathematics and need to know the limitations that theoretical mathematics place upon the investigations of applied situations in the real world. If we seldom treat applied problems, then our students will not develop the ability to understand and use the lively interaction between theory and the real world. For example, students unfamiliar with the need for existence theorems in differential equations may begin to use numerical techniques to solve equations that have no solution or no unique solution.

The content of courses from calculus through linear algebra and differential equations could stand a complete reexamination based on the kinds of computational tools that are available to all our students in inexpensive, portable, and powerful devices. Some professors have begun to revamp the differential equations and linear algebra courses that can most benefit from extensive numerical computations. Calculus, the centerpiece of the undergraduate mathematics curriculum at many institutions, also could benefit from such review, in part because of the changes in the differential equations and linear algebra courses. The calculus reform movement has made many changes in the pedagogy of such courses and some changes have been made in the emphases of the course. In addition, most calculus textbooks these days have down played theory and rigor because of the perceived decrease in student ability. However, the notion that some topics should be de-emphasized or eliminated and the order of the topics rearranged has not been entertained. That differential equations should come earlier and be treated in more depth has been foreshadowed by some of the reform efforts but has not been fully implemented because (I think) computer symbolic manipulations have not yet been fully embraced. There is perhaps a need to introduce difference equations early in the first course in calculus and to use them as a springboard to differential equations (as has been done at the USMA). Perhaps the interaction between differential equations and difference equations in investigating real world situations could be exploited in such courses. Also, the requirement that power series be treated in a first course rather than multivariate calculus concepts should perhaps be revisited.

A Tool for Tutorial Review

There are now available many assessment and tutorial programs for various mathematics courses given at the college level. Infrequently, these programs are combined into a package that both assesses the student in some content area and then provides the appropriate tutorial topics to master the content area. ALEKS (Assessment and LEarning in Knowledge Spaces, www.aleks.uci.edu) is a NSF funded cognitive sciences project that is now reaching the commercial stage that does just this. At the moment, it is only available on the Internet for beginning algebra, intermediate algebra, and geometry. Nevertheless, even such a package of services for the student could improve the retention rate in calculus courses by providing students with the opportunity to determine their deficiencies in low level algebra skills and correct them before or during the course using the Internet. The program can discover that a student is having problems with the difference quotient because the student is not comfortable with complex fractions in arithmetic and then fix that deficiency. This benefits the individual student who is potentially more successful in the course, the instructor who can be more comfortable with students' current algebra skills, and the class as a whole because they can move through the material with more attention to ideas than to arithmetic and algebraic details. In future, such packages will include precalculus and calculus and will become an alternative to our current courses. We may also use them as a supplementary part of traditional courses to make sure that computational skills are maintained while we devote more attention to critical thinking skills and problem-solving. Finally, using the assessment and tutorial features of such packages of

instructional services, professors in other disciplines can be assured of the computational skills of students who have taken our mathematics courses as prerequisites to their courses.

An Internet Data Collection and Communication Device

As more and more applications of mathematics arise, the collection of data related to other fields for mathematical and statistical analysis will become more and more important. The current availability of sites where such data is available is growing at an enormous rate. The ability to use this information and for institutions to provide opportunities for students to participate in joint projects with students at other institutions is enticing. The availability of large scale computing facilities for students to access over the Internet will increase with the development of the Digital Libraries projects that are being funded by the NSF. The time for professors to avail themselves of these new opportunities seem to be decreasing, however.

Although the capabilities that the Internet provides and promises to provide are exciting, the human resources needed to make these accessible to students in an efficient, timely, and educationally appropriate manner are often difficult to orchestrate. On a smaller scale, the development of Internet courses that for the first two years of collegiate mathematics but specialize the topics of, say, calculus for specific disciplines becomes possible.

At the moment many disciplines require two semesters of college level calculus (usually called calculus for science and engineering). These courses all seem to have applications, but not enough specific applications to satisfy any particular discipline. Biology, physics, mechanical engineering, electrical engineering, systems engineering, biochemistry, ecology, environmental science, decision science, economics, and psychology all have slightly different or vastly different requirements for what they need and want from calculus. The Internet provides an opportunity to tie such specialized laboratory courses to the traditional calculus courses taught at each school. Faculty from each of these different disciplines could then be satisfied that their students had seen the calculus in the particular light that they need while the mathematics faculty would not be forced to teach material that they felt uncomfortable with. For example, a consortium of colleges could agree that each college, following its strengths, would offer a laboratory course geared to one discipline and provide this course over the web. Students from each of the consortium schools could take the appropriate laboratory course for their discipline while taking a traditional course on campus. Mathematics students from each school could take several of these courses to fulfill the applied component of the mathematics major. In this way, the mathematics departments could freshen their offerings while providing mathematics majors with the applied background needed to obtain employment.

A Way of Performing Mathematical Computations With Numbers, Symbols, and Graphs

The second David Report of the NRC included many new breakthroughs in mathematics, a majority of which involved the use of computers. Although there are still mathematicians that do not believe that the four color problem has been solved, much mathematics is done today using computers as computational tools for exploration, for completing and verifying complex numerical and symbolic computations, and for graphical presentation. Almost all major mathematical journals now accept papers in electronic form, some only in electronic form. Much of the mathematical literature is being converted to electronic form to improve its use and usefulness. The use of computers by mathematicians has become a commonplace activity, but such use in

classrooms has not. That there was a time when mathematics was done in the sand and not with paper and pencil is not a surprise. That most mathematics in the next century will be done in silicon again would be a surprise to almost every one in mathematics, but perhaps not to those in other disciplines who are already using the tools.

A challenge for us in the mathematics community charged with teaching the young is to create sets of activities that will provide our students with the understanding needed to use these powerful tools effectively to solve problems and develop theories that we have not envisioned ourselves. What and how much mental and paper and pencil activity do students need to understand the mathematics well enough to use a particular computer tool? Can we teach the mathematical concepts with the computer computational tools? What are the essential ideas that must be taught for the student to learn new ideas in a technology-rich environment?

In the future, a combination of the uses of technology suggested here will eventually be developed. Professors will create new courses with substantially different content with in depth projects from a variety of disciplines that rely on student computer computational capabilities that will be presented in classrooms (or not) with video projectors to students that are Internet-connected at their e. The student and the professor will rely on student mastery of prerequisites prior to the class meeting using ALEKS-like instructional services, How do we ensure that this new educational experience that will come to pass will be of high quality and provide the understanding and skill needed to further the mathematical enterprise?