Mathematics for Use Across the Curriculum
A Tool, a Language or What?

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What metaphor shall we use to most adequately convey the goals and intentions of the mathematics across the curriculum movement?

The metaphor we adopt for describing the use of mathematics in other fields is an essential part of communicating understanding to faculty and students. The metaphor we select is rooted in our understanding of what it means to know and use mathematics. This understanding reflects our mathematical epistemology; i.e. what we believe it means to know mathematics. Our epistemology, properly formulated, will include our understanding of both mathematics and the thinking of the people who use mathematics.

Let us begin by examining three different ways to talk about mathematics in the classroom. Given that most students in introductory mathematics courses have no intention of becoming professional mathematicians, the way we approach mathematics can have a significant influence on their attitude and their future uses of mathematics.

"The rest is just mathematics…

What understanding of mathematics do you think is presented to the students by the phrase, "The rest is just mathematics…"?

This phrase is sometimes heard in science and engineering courses at some point in a problem solving exercise. For example, a heuristic that has be used in some physics courses is ***EDPIC***, an extension of the problem-solving strategies advocated by Fuller (1982) and Reif (1976). The letters stand for the phases of the problem solving process that is used.

Exploration:	Examine a physical system and collect some experimental data.
Description:	List explicitly the given and desired information. Draw a diagram of the situation. (The result of this step should be a clear formulation of the problem.)
Planning:	Select the basic relations pertinent for solving the problem and outline how they are to be used. (The result of this step should be a specific plan for finding the solution. Most physics problems involve mathematical reasoning and model building.)
Implementation:	Execute the preceding plan by doing all necessary calculations. (The result of this step should be a solution of the problem.)

Checking: Check that each of the preceding steps was valid and that the final answer makes sense. (The result of this step should be a trustworthy solution of the problem.)

Typically, the students do an interactive video application, hands-on activity, or microcomputer-based laboratory data gathering in the **exploration** activity. This gives them some feeling and ownership for the data, as well as experience with the actual physics principles. Then the **description** begins with a drawing or diagram, such as a vector force diagram, free-body diagram, or sketchy graph. The relevant physical principles such as Newton's law of motion, or the ideal gas law enter as the **planning**. At this point, typically, the students have a reasonably difficult mathematical problem to handle. Sometimes handling the equation involves graphing, sometimes solving a differential equation. In either case, a computer algebra system (CAS) is an excellent platform for **implementing** the mathematical analysis of the model. Finally, the students derive the answer or result with mathematical modeling. The result is **checked** against common sense, well-known physics results, the experiment, or the results obtained by other students or groups in the class.

When such a heuristic is used in a physics classroom, it is not unusual to hear the instructor work through the first three parts of the strategy, E-P-D, and then say, "the rest is just mathematics…". From there the instructor may, or may not, go through the rest of the problem solving strategy in detail.

Please take a few moments to examine the context of that phrase and what it may show about the mental model the instructor is conveying to the students about mathematics.

In this instance, the instructor seems to be showing that, from the point-of-view of the instructor, the difficult and challenging parts of the exercise are completed by the time the instructor claims to use mathematics. In this setting, mathematics is assumed to be a simple algorithm the problem-solver can use to obtain a correct answer. In this context, mathematics is seen as subservient to the larger and more interesting problem solving strategy being used in physics. On the other hand it points to the usefulness of mathematics in reaching a satisfactory answer to the problem.

Unfortunately this reference to "just mathematics" neglects the mathematical reasoning that is inherent in most of the laws and principles of physics. The predictive nature of physical laws is based on their mathematical structure.

Nevertheless, since the setting up of a problem is still a long, long way from a useful solution, maybe the mathematics across the curriculum movement should adopt as it slogan, "the rest is just mathematics…"?

Mathematics is a tool to…

What understanding of mathematics do you think is presented to the students by the expression, "Mathematics is a tool to …" ?

A common way of talking about mathematics in science and engineering courses is to view mathematics as a tool. In these contexts, the task at hand is typically a problem to solve to obtain a result. The result requires the doing a variety of operations for which particular tools are especially useful or needed. A difficult task is made much easier by having the proper tool. Professionals nearly always have the correct tool available for their use. Hence, in the context of science and engineering as professions, mathematics is presented as a tool to be used for the successful completion of the task.

Please take a few moments to examine the concept of a "tool" and what it may show about the mental model the scientist, or engineer, is conveying to others about mathematics. Perhaps, in this instance, the professional is showing that from this point-of-view the task can best be accomplished by the use of mathematics. In this setting, mathematics is a device, or technique, the problem-solver can use to obtain a correct answer. Mathematics is seen as serving the larger and more interesting problem solving strategy being used in this course. While at the same time it shows the importance of mathematics is reaching a satisfactory answer to the problem.

It can be argued that the "tool" metaphor for mathematics is rooted in a behaviorist view of knowledge, a view that sees problem solving as a process to obtain a correct answer. This view of knowledge sees the correct answer as predetermined by the problem. The process of figuring out the answer will have no influence of what the correct answer really is. In this view of knowledge the tool influences neither the process nor the answer. This view of knowledge sees that the tool and its properties are completely independent of the task

To what degree does the tool metaphor reflect the understanding of mathematics by scientists? Or engineers? Or mathematicians? In what ways is it a useful metaphor for the mathematics across the curriculum movement?

"Mathematics is a language for…"

What understanding of mathematics do you think is presented to the students by the expression, "Mathematics is a language for …" ?

Physics is sometimes described as the building of mental models to explain and predict the behavior of natural systems. Such models are always linked to the measurement of some physical observables of the system. The models may have exotic properties like eleven dimensions of a string, but they are finally based on physical observables. The models of physics are most productive when they use the language of mathematics to convey their symmetry and structure. In fact, there are examples in the history of physics where the exploration of a physical system led to the need for new mathematics. On the other hand, mathematical systems of beauty have been developed without regard to any requirement to be an expression of the physical universe, only to be discovered, perhaps years later, to be just exactly what the physicists needed to help them complete an analysis of some natural phenomena.

The mathematics-as-a-language metaphor seems to have its roots in a constructivist view of knowledge. In this view, as a problem is being solved the scientist, or engineer, is constructing a framework for describing and explaining the problem. This constructivist epistemology sees the interaction between the problem-solver and the problem as an essential feature of the process. In such a dynamic process the communication of the activity is carried out between human beings

via language. If the results of the problem solving process are going to be given a numerical, or mathematical, result, then a mode of communication based on the principles of mathematics will be required. The grammar and vocabulary of mathematics will become essential aspects of the problem solving process. These attributes of mathematics will interact with the mental models being developed by the problem solver to shape and transform the mental activity of the problem solver and the nature of the answer being sought to the problem. In this view of mathematics the dynamic interaction between the problem solver and the problem occurs through the intermediary of a mathematical language which helps to provide structure to the problem solving process and the mental models being used. In addition, mathematics becomes a lively part of problem solving and is altered itself by the process. Such a dynamic view of problem solving requires that the subjects and objects of the process are dynamic and open to transformation during the process. Clearly, real-life problem solving experiences have such characteristics. As people seek to solve problems in their lives, the information and the context of the problems get changed as the person interacts more and more with the problems at hand. The dialogue between the problem-solver and the problem also changes the language used to communicate the problem solving process.

In some science and engineering courses, students will hear mathematics discussed as the language used in the problem solving process of the discipline. While this seems to be a particularly rich metaphor for the use of mathematics it is infrequently discussed in any detail in a science, or engineering, course. Perhaps, filling out the details of such a metaphor is a useful task for the people who are participating in mathematics across the curriculum activities.

Ultimately, the success, or failure, of the mathematics across the curriculum movement may depend upon the metaphors that we use to describe mathematics. It is an important task that demands our attention. What metaphors shall we choose?

Footnotes and References:

[1] Visiting from the Department of Physics and Astronomy, University of Nebraska - Lincoln

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Reif, F. J. H. Larkin, and G. C. Brackett, (1976) "Teaching general learning and problem-solving skills," **American Journal of Physics** 44, 212-217.