CONSISTENCY OF TEACHING APPROACHES IN MULTI-SECTION LARGE ENROLLMENT COURSES

Bill Haver
Virginia Commonwealth University

In the not-so-distant past, most mathematics departments offered a consistent set of courses at the lower division level. All sections of a given course used the same text and, in many large universities, common hour exams and finals were set department wide. In addition, the mode of instruction and the assignments made to students remained constant across all sections.

I can remember, as a graduate student at Rutgers University in the late 1960s, participating in group grading sessions in which more that 1,000 hour exams were graded in one setting. Each graduate assistant graded the same problem on all tests to assure consistency. The department also had a syllabus which included exercises to be assigned at the end of each section of the text. Everyone knew that the exams would consist of problems that were identical (except for the numbers) to some of these exercises.

To the best of my knowledge, the structure of classes nationwide was virtually identical. At some institutions, calculus was taught in sections of 20-40 students, while at others a lecture/recitation format was used. In either case, a large portion of class time was dedicated to the instructor: answering student questions; working problems that had been assigned the period before; discussing the theory behind the new mathematics to be studied, including proofs of some theorems; working problems similar to those to be assigned for the student to complete before the next period. In the 1960's, most students were studying calculus for the first time.

Of course, within this basic structure, there was a great deal of qualitative difference in instruction provided. Faculty had the opportunity to introduce students to calculus and inject motivational examples, history, applications, and heuristic arguments that contained at least the germ of an actual proof. Some instructors were much more gifted than others in exposition and in helping students to understand. Some instructors were more capable than others of conveying the power and excitement of mathematics. The best instructors managed to elicit the best performance from their students. Generally students responded to the ability and caring of their instructors. This was particularly apparent in the lecture recitation format; some faculty maintained over 90% student attendance in their lecture halls, and in other lectures, student attendance quickly fell below 10%.

In my experience, this pattern was the norm from at least 1960, when I studied calculus, linear algebra and differential equations at Bates College, through the mid 1980s. During this time I taught mathematics at Rutgers University, Bates College, SUNY Binghamton, the University of Tennessee, Knoxville College, J. Sargeant Reynolds Community College and Virginia Commonwealth University. In retrospect, it is surprising how similar the course content and perceived notions of student learning and student expectations were across this broad spectrum of types of institutions. The texts used were virtually isomorphic and the examinations remarkably similar. In the classes that I taught over this period of time, proof and formal logic were much more prevalent in the 1960s than in the late 1970s. I think that this change reflected my perception that the students were not gaining anything from theoretical lectures, and also reflected

a national trend. The difference seemed to be more in how class time was spent; I don't think many theoretical questions were asked nationwide on examinations, even at the beginning of the period in question.

Of course, not all courses followed this mode. At many institutions honors sections were offered that varied considerably from the norm. A small number of innovators experimented with using technology and with student writing.

But these variations were the exception. For example, a person teaching any course (offered by the physics, engineering or mathematics departments) that had a prerequisite of the two semester calculus sequence knew, or could have found out, which exercises students were assigned as homework and what students needed to be able to do to pass the final exam. Of course, there was great dissatisfaction in the amount of mathematics studied and in the level of understanding reached by the typical student. Faculty at all levels and in all fields lamented the fact that students did not know what they should be expected to know and were "not able to think." It should also be noted that most science and engineering faculty never did make the effort to find out what topics actually were taught, but this predictability did exist.

Recent Changes

During the 1990s, many mathematicians have decided that a large percentage of students can gain a much greater degree of understanding through participation in courses that expect and demand more active student involvement than that expected by the end of the previous decade. Some mathematicians reached this conclusion through studying the limited amount of educational research that could be applied to college level mathematics classes, some through attending seminars and workshops, and others by hearing informal testimonials from their colleagues. However, most mathematicians who reached this conclusion did so after observing increased student learning while trying student-centered approaches in their own courses.

Another motivation for the changes that have occurred is the fact that in recent decades a large percentage of students studying introductory calculus at many colleges have already been introduced to calculus in high school. These students either did not take an AP course or did not do sufficiently well to receive advanced placement. They have developed a preconception of the nature of calculus and often are expecting a technique-oriented class that does not require any more effort than the students put forth in high school. A desire to move students away from this view of mathematics has been a motivating factor for many faculty as they have developed new instructional approaches.

A variety of different changes in the mode of instruction has occurred in the calculus sequence, differential equations, linear algebra and other lower division courses typically required of students majoring in engineering and scientific fields. Among these changes are a much greater use of computers, graphing calculators and other technology; assignment of longer term projects; emphasis on written and oral communication by the students; and group or cooperative learning assignments. In varying degrees, student activities have replaced time formally devoted to lectures; longer assignments requiring substantial in-depth thinking have replaced short answer daily exercises; complicated problems requiring substantive use of technology have replaced paper and pencil assignments. In addition, many faculty have supplemented or replaced traditional tests

with other forms of assessment. The nature of this change has been described in detail by this author [2] and others [3,4].

Impact of the variety of teaching approaches

By their very nature, these various reform approaches result in a greater inconsistency in what students know when they have completed their lower level mathematics program. If our mathematics programs no longer resemble assembly lines, the products of these programs no longer resemble the products of an assembly line. There are at least three reasons for the inconsistency of what students know.

First, like it or not, the monolithic instructional approach that existed in the 1960s and persisted through much of the 1980s no longer exists. Common hour exams and final exams have not existed for many years. Those faculty who have learned the power of graphing calculators or, in other cases, computer algebra systems will never again teach without using these tools. Faculty who have experienced the power of written essay assignments to focus student thinking will never forsake this tool. On the other hand, except for a small number of special cases, at most institutions significant number of faculty are not incorporating large scale changes into their classrooms. A few are vocally and actively "anti-reform", but most of these are individuals of all ages and ranks who do not believe that they are capable of increasing the performance of their students through using, what are to them, radically different approaches. Most have, however, significantly altered their methods of instruction and use of technology. In short, within most departments, there exists a wide spectrum of approaches.

There are counter examples to the lack of uniformity. For example, at Virginia Commonwealth University, my home institution, all 2,000 students enrolled in all sections of a contemporary mathematics general education course write two mathematics papers, conduct a long-term study of a mathematical topic, complete a learning log with weekly entries, and make oral presentations to other students in their class in addition to taking tradition tests. This uniformity is possible because very few regular faculty teach this course and the graduate assistants and part-time instructors can be assigned this course based upon their commitment to this type of instruction. Similarly, calculus at the University of Michigan is taught primarily by graduate assistants and individuals with short-term appointments who make a commitment to a certain teaching approach at the time they are hired. Small collegial departments can make a commitment to specific teaching and learning approaches (Ithaca College comes to mind). In other departments, including West Point, there is a long tradition of a department approach to teaching. The mathematics department at West Point has impressively merged elements of mathematics reform with a tradition of providing a common experience to all students carefully related to the mathematics that will be employed in other courses, See for example [1]. But these relatively standardized courses and programs are the exception rather than the rule.

Second, those who are committed to student-centered approaches are not agreed upon a monolithic approach. In fact, if there is any area of agreement, it is that different instructional strategies are most effective for different instructors and different students. It may be unlikely that those who have taken a position as being opposed to reform will change their courses. It is even more unlikely that those committed to reform will agree to a common definition of this reform and a common experience to be provided to all students. The unlikelihood of reaching such an agreement is perhaps most apparent in the area of technology. As the cost of technology decreases and the capability of hand held calculators continues to increase, the level of disagreement increases. For example, some believe that "computers" offer much more than "calculators", but year to year the differences decrease. Some faculty value the graphing

capabilities and dislike the computer algebra capabilities. When consensus is reached within a department or university, the technological advances of the next year inevitably challenge this consensus. Similar variations in thinking occur in the areas of assessment of student learning, of long term projects (should the entire course consist of a long-term project or should such a project be one component of the course), of collaborative learning, and of student written and oral communication.

Finally, and most importantly, is the belief by many that the goal of instruction is not to produce students who can perform specific tasks or who have been trained to follow specific algorithms. Neither is it the goal of any course or curriculum to produce a group of students with identical skills or even identical levels of understanding. Those faculty who assign long-term projects (group or individual projects) most clearly take this view. The ultimate purpose of a long-term project is not to answer the particular questions raised by the project. Rather, the ultimate purpose is to encourage students to think broadly and clearly, to acquire skills as needed, and to think in careful and sustained manners about specific issues and questions. Similar goals are behind collaborative learning, oral and written communication, and other instructional approaches. The purpose of instruction in mathematics, according to this view, is to teach students how to think about quantitative matters and to bring a wide array of skills and approaches to bear on real situations.

Implications of Non-Monolithic Programs

Indeed, mathematics departments no longer have as their primary goal the production of students with a specific set of skills and experiences. Given the current situation, it would take a major change within the discipline to make the development of common skills become the major goal. Rather, in my view, the goal of mathematics instruction has now become to produce students with a strong conceptual understanding of mathematical ideas, with the ability to seek and use whatever mathematical or technological tools are available, and with the ability to think rationally and creatively about a wide range of situations. I believe that most mathematics faculty and departments have, at least tacitly, reached this conclusion.

Mathematics departments need to respond to this new reality in at least two different ways:

- First, mathematics programs do need to learn what *specific* skills students who complete our courses actually need. If students are to subsequently take a course that immediately requires them to be able to invert a matrix, then why not ensure that they do know how to invert matrices. If we know what specific skills are expected, we can assure that students learn these skills without subverting the entire course to skill development;

- Most importantly, mathematics as a discipline globally, and mathematics departments locally, need to be clear concerning what goals we do wish to achieve. We need to refine these goals based upon broad dialogue outside of the mathematics community; we need to enunciate these goals to others in our universities and to future employers. I believe that employers and faculty in other disciplines share our view of what are the most desirable outcomes of mathematics instruction. I am not sure that they understand what mathematics courses and programs are trying to accomplish.

As has always been the case, mathematics and mathematics instruction is influenced by the broader social and political realities. The world is no longer divided into western and communist blocks, industry is no longer based upon the assembly line model, and large numbers of engineers that can perform routine procedures are no longer needed. Rather, what our society and our economy need are graduates of our programs with a wide variety of different talents and abilities who can marshal these talents in multiple situations.

The changes in our instructional program of the past decade are indeed headed toward preparing students with these broad-range of abilities. As we move in this direction, we do need to minimize the down side of not offering a consistent product to employers and user departments, but we also need to recognize that such consistency is no longer the most important outcome of our courses.

References

1. Dossey, J. A. (Ed). *Confronting the Core Curriculum; MAA Notes 45, Washington D. C. 1998*.

2. Haver, W. E. (ed). *Calculus: Catalyzing a National Community for Reform.* MAA, Washington, D. C. 1998.

3. Roberts, A. W. ed. *Calculus the Dynamics of Change*, MAA Notes No. 39, 1996.

4. Tucker, A. C. and Leitzel, J.R.C.(ed), *Assessing Calculus Reform Efforts*, MAA Reports No. 6, 1995.