# Pedagogically Effective Technology

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Technology is now almost universally available and its use in our classrooms is increasing rapidly. In some quarters it has become almost the badge of reform to be known as a technology user. Our students are inundated with graphing calculators here, CAS numbers 1, 2, and 3 there, and specialized software packages x, y, and z in their respective niches. How is it going, is our plan pedagogically sound, and are we heading in a direction that best benefits our students? Below we address the questions raised by the conference organizers about technology tradeoffs, how we should be using technology, and where it is leading us.

**How should technology affect what and how we teach?**

*Technology should change the set of skills and knowledge that we have always thought of as fundamental.*

Some skills are no longer important to most potential users of mathematics (e.g., most of our students) because technology is almost universally available that can execute them sufficiently well for us. These include techniques of integration, drawing and graphing, and root finding. Solutions to systems of equations, both linear and nonlinear, continue to be taught by hand but are better done with technology. The ability to make nearly effortless numerical evaluations has greatly reduced the importance of trigonometric identities, yet we still employ them just because they are available and we (teachers) know them. We often bemoan the fact that our students don't know the trig functions at special angles, but why do we value that so much? 30° and 60° angles are largely artificial; they show up so rarely in real applications that maybe we should satisfy ourselves with numerical evaluation as needed, just as we do for 31° and 59° angles. Cross products and curls are easily done with technology and there is no mathematical insight gained through calculating them by hand, yet we always teach and test them.

As we accept the fact that some skills such as those discussed above are no longer important, there is a corresponding set of technology skills that we need to be teaching. These include skills in numerical evaluation, computer algebra, linear algebra, numerical computation (of roots, eigenvalues and eigenvectors, combinations and permutations, and statistical measures), and graphing and visualization. We have taken a very selective approach here; most of us teach a few random and personally convenient skills, but few of us have a comprehensive inventory of what students need or a plan for covering them all systematically. Reference [1] contains a sample plan of such skills, with representative realizations.

Some skills and knowledge remain important, and deserve added attention because of technology-induced atrophy. These include basic geometry, the algebra of polynomials, exponentials, and logarithms, the derivatives, integrals, graphs, and behaviors of the elementary functions, and the domains and ranges of the vector differential operators. Reference [2] contains a sample list of such skills.

*Technology should allow students to see that, at least on one facet, mathematics is an experimental science.*

In today's reform-minded environment we talk a lot about learning through student discovery, but don't always do a lot about it. We need to get serious by targeting places where deductive and analytic approaches have historically failed the students and see if technology

offers a better approach. For example, deductive approaches to convergence of series are traditionally disasters for students, both in understanding and in execution. Instead we could replace these with a week of experiments designed to lead students toward a visceral understanding of convergence tests. For example: Guess at a test for convergence ($a_n > a_{n+1}$ ?). Given an intelligently selected collection of instructor-provided series, graph "enough" partial sums to tell if each is converging or diverging; can you use these to give evidence for or against your conjecture? Plot the ratio $a_{k+1}/a_k$; is there any relationship between convergence and the graph of this ratio over k? Does the harmonic series look convergent or divergent? Group its terms into packets such that the $n^{th}$ packet contains $2^n$-$2^{n-1}$ terms and plot the sum of each packet; can you draw any conclusions? This approach is applicable to other traditional student problem areas, such as limits and continuity of multivariable functions.

Technology allows us to disprove conjectures in a straightforward manner, but we need to use it more to do "proof by example". Of course, what we really mean here is to do enough positive examples to convince the student that the conjecture is very feasible, and that (coupled with our assurance) he or she really should believe it. What this step also does is motivate the need for proof; with so many examples under our belt and the willingness to believe it's true, we're now motivated to pursue the proof (if we have an audience that has either the need or an inclination in that direction).

*We should move beyond using technology to demonstrate toy problems and really start doing more involved problems*

As technology proponents we have often touted the ability to do more realistic problems, but even with its almost universal presence, we usually don't do this. What we actually do is teach the traditional skills and techniques on the canonically easy problems, then repeat those problems using technology to show how it is done and how technology makes it is easier. We shouldn't introduce nonlinear equations, uglier integrands, and more complicated functions just to show it is possible. Rather, we should actually take advantage of the power available by refining our models to more closely reflect reality and show how solving refined models leads to refined solutions.

## What are the strengths and drawbacks of different technology choices?

In general, it seems that the universality and general public familiarity of a given technology choice is inversely proportional to its power. One way to group the different technology choices is by function. Most of our technology choices fall into the area of technology that helps us *do mathematics*, a few choices help us *demonstrate mathematics*, and a few help us to *communicate* as part of the learning process.

### *Doing Mathematics*

Graphing calculators have proven to be economical and popular with the rising "Game Boy" generation. Their reasonable costs have made them ubiquitous in high schools and as graduation presents. Since many or possibly most students enter college possessing both a calculator and some familiarity of use, this can provide a free and instant jump-start into college technology use. Their portability and the availability of sensor-peripherals makes them perfect for turning a classroom into an instant laboratory. A major disadvantage is the rapid pace of development; there are so many manufacturers and models present with more appearing annually that compromises must be made in what will be used or required or in who will have to switch and learn a new calculator. Another disadvantage is in the power available and the ease of use to access it. Visualization is often mediocre and navigating the layers of menus required to execute some operations is arcane and tedious.

Spreadsheets are universally available and familiar to many. They are ideally suited to many applications involving sequences, series, difference equations, discrete approximations to

continuous operations, iteration, or visualization of the same. It is both an advantage and a disadvantage that they require the student to understand the mathematics well enough to be able to mentally flow chart what they are trying to accomplish and to construct and implement the algorithm themselves. Being constrained to numerical, discrete applications, they are often bypassed in favor of packages that are somewhat more general, but are often favored by faculty in our partner departments.

Computer algebra systems (CAS's) have often proven to be a more popular choice than others because of their ability to handle a wide range of discrete, continuous, and visualization tasks. Cheaper CAS's are accessible to students, but make tradeoffs in power and generality. More powerful CAS's are more expensive, which often limit availability to labs and servers and keep them out of the hands of students. Nonintuitive syntax is a problem with almost all CAS's; students find the learning curve steep. And because of the presence of several different popular CAS's, compromises must again be made about which to use and who must learn the new package.

A fourth choice is one of the increasing numbers of specialized mathematics/engineering packages. Packages for linear algebra, ODE's, statistics, complex analysis, and other specialized areas are available for use in the appropriate courses. Advantages include a programming structure and command set that are tailored for the field of interest. This is also their chief disadvantage: the packages are seldom used or useful outside of one or two courses in the students' programs.

A choice that involves choice of platforms is the use of laptop technology. Several schools have experimented with issuing laptops to students, either as a required purchase for all incoming students or as one-semester or one year loans in order to conduct classroom studies of effectiveness [3]. The ability to turn any given lesson of any given class into a laboratory session is a great benefit, as is the ability to have students actively involved with examples in class and the ability to transport work and programs between class, dorms, and trips. Disadvantages include weight, security, durability, backup power in the classroom, and boot time in the classroom.

***Demonstrating Mathematics***

Web applets are an increasingly popular way to demonstrate to students such mathematical concepts as Riemann sums, secant and tangent lines at a point, the area interpretation and direction of a cross product, etc. These can be made available to large audiences at the individual's convenience via web access and offer an active and visual way for students to play with a concept. However, they require some amount of faculty creativity and programming expertise.

In another mode, CAS's also provide a way of demonstrating mathematical concepts. This is through the creation and posting or mailing to students of prepared materials that require the student to only execute them, and allows the student to change no more than a few parameters or functions to solve very similar problems. This is via quicksheets in Mathcad, notebooks in Mathematica or Maple, etc. These are popular with students because they require little actual competence in the packages to actually use them for common mathematical tasks. Disadvantages include the instructor time and ingenuity required in creating these, and the faculty worry that making too many of these publicly available to students will provide too many crutches that will empower them without helping them learn the mathematics involved.

***Communicating***

Email over campus networks is an increasingly common way to enhance learning for students. We increasingly send out to students administrative notices and requirements (which saves more classroom time for teaching and learning), night-before tips for what to key on and what to de-emphasize in a given night's reading assignment, immediate corrections or clarifications to something that happened in the class that just adjourned, etc. As this medium

matures, it seems that we move from the initial stage where we contact students, to an intermediate stage where we augment what we are saying to students, and finally to an advanced stage where most of a student's professors are inundating him or her with daily odds and ends.

A web presence, such as a personal or course web page with pertinent course materials and important notices and updates posted, makes materials available to students without pushing (or inundating in an unwelcome fashion). Effectiveness depends on individual student web habits and compulsiveness.

**How should we match technology choices to the intended audience?**

We have not done very well in this area. We tend to sub-optimize, both as individual instructors and as one department in an academic institution. The truth is that we usually choose the technology to be used (if any) in a particular course based first upon what we as faculty are comfortable with. We may secondarily consider what the course has historically used and if our predecessors have left any worksheets or materials that we can learn from and reuse. The result is a technological experience for students that has them learning (or attempting to learn) several new technology choices in several different courses in an often disjoint way, and then forgetting them as they are no longer required or used.

In our service courses, we need to give more weight to the students' overall technology needs in their academic programs. We need to determine what technology is used in their majors, and talk to those departments and see what we can do to prepare them for their majors' experience and to reinforce the technological skills they have already seen in their major and in other institutional courses. As departmental faculty, we need to commit to one principal choice or to a few common choices that we agree to jointly and consecutively use and build on so that students can develop long term familiarity, faith in their own competency, and willingness to use technology on problems that arise in other courses or in open-ended contexts.

**What are the possible effects of future technology?**

One trend that will probably continue is the development of effective demonstration software. Applets, templates and notebooks, and simulation software will increasingly allow students to see mathematical ideas in action in an active manner without having to think about the underlying software. Specialized simulation packages will increasingly allow intelligent system interaction with students. Web access to this demonstration software will support the growth of distance learning, enabling teachers to put learning materials before students in other locations, with time to explore according to their schedule.

As a second possible trend, some believe that we are seeing the emergence of a new learning style among our students. Growing up with technology may be making our current and future students more comfortable learning from electronic sources than from printed or hard copy sources. When syllabus and lesson assignments are available both in hard copy and on the web, some students will ignore the hard copy in the notebook on the side of their desk and look up the web version. Many students will ignore the texts on the shelf in front of them and instead surf to research and find information. The traditional feeling has often been that "electronic is nice, but I need hard copy to study from"; this may soon no longer be the case for our students. A lifetime of exposure to technology could be changing attitudes about which medium is preferable.

## Conclusion

There are many individual success stories concerning technology in the classroom. Graphing calculator and CAS use have exploded in the last ten years. There are many factors working in our favor: increasingly powerful technology choices at reasonable cost, the slow trend toward syntax that may actually be intuitive to the journeyman user, the infusion of new faculty who grew up with technology, and a sincere belief on the part of many faculty that technology can be a useful and illuminating vehicle for learning. An increasing number of faculty talk, write, and give and attend workshops about how to use technology in the classroom. Institutional support for classroom technology initiatives is almost surprisingly strong. In all, it appears that technology in the classroom is past the fad stage, and is a recent part of our pedagogy that is here to stay.

## References

[1] Department of Mathematical Sciences, *Core Mathematics at USMA*, West Point, NY, 1999, pp. 28-35.
[2] Department of Mathematical Sciences, *Core Mathematics at USMA*, West Point, NY, 1999, pp. 24-27.
[3] J. Loy, J. Myers, and C. Tappert, "Notebook Versus Desktop Computers for Cadets at West Point", *IEEE Transactions on Education*, Vol 39, No 4, Nov 1996, pp. 497-504.

## Appendix A: Sample List of Required Skills in Computing (from US Military Academy)

The following list constitutes the basic, required scientific computing skills which you will learn and use during your four-semester core mathematics experience and in your future math/science/engineering courses. Once each of the listed topics is covered in class, you must be able to execute this skill upon demand using the specified scientific tool(s) listed below (with its reference book as an authorized reference) in any future lesson or course. Reference books, for the respective computing system, are the chief source of information for learning these skills. Annotated beside each item are the course and lesson number which denotes the exact point in time you are responsible for that particular item ( e.g. 103/1 means you are responsible for that skill following the first lesson of MA103).

**CALCULATOR (HP48G / TI - 89 / or equivalent)**

1. Program a first or second order discrete dynamical system (DDS):

   E.g. Given the DDS $a(n+1) = 3\,a(n) + n^2$, find $a(15)$. (103/09)

2. Perform matrix operations in two and three variables:

   E.g. Given the linear system of equations,

$$\begin{bmatrix} a(n+1) \\ b(n+1) \\ c(n+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} a(n) \\ b(n) \\ c(n) \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

   a. Find the determinant of the coefficient matrix. (103/23)
   b. Find the inverse of the coefficient matrix. (103/23)
   c. Find the eigenvalues and eigenvectors of the coefficient matrix. (103/23)

   E.g. Given the linear system of equations,

$$\begin{bmatrix} 2 & 5 & 6 \\ 7 & 8 & 4 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$$

   a. Find the determinant of the coefficient matrix. (103/23) (364/15)
   c. Perform row reduction operations on the augmented matrix. (103/23)
   b. Solve the system of equations. (103/23) (364/15)

3. Find the real roots of a polynomial or transcendental function:

   E.g. Find the roots of the function $r^3 - 7r^2 - 14r + 8 = 0$. (103/30)

   E.g. Find the roots of the function $1 - \sin(2x) = 0$, $0 < x < p$ . (104/02)

4. Graph algebraic and elementary transcendental functions:

   E.g. Graph the particular solution (103/11)

$$a(k) = 2.3(4)^k + 5k^2 + 2, 0 \le k \le 25 \ .$$

   E.g. Graph the function $y = \sin(3x) + x^3$. (104/02)

E.g. Graph the following functions individually and together: (104/08)

$$y_1 = 2x^2 + 3x^{1.5} - 24e^{2x}$$
$$y_2 = \sin(3x) + e^x - 2\ln(4x)$$ ,

5. Evaluate derivatives:

E.g. Given the function $y = \frac{1}{x} + \sin(2x^3 - x)$ ,

a. Find the derivative. (104/02)
b. Plot the derivative. (104/02)

E.g. Given the function $f(x, y) = \frac{1}{y} + \sin(2x^3 - y)$ , find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ . (205/30)

6. Evaluate definite integrals:

E.g. Evaluate $\int_0^p \left(x^2 + \cos(2x)\right)dx$ . (104/20)

E.g. Evaluate $\int_0^3 \int_0^2 \left(x^2 y\right) dy\, dx$ (205/46)

7. Taylor Polynomials:

E.g. Find the 5th order Taylor approximation to the function cos(2$x$) about the origin. (205/63)

8. Differential Equations (DE):

E.g. Given the DE $\frac{dy}{dx} = \cos(px) + e^{-x} - 2$ ,

a. Plot the direction (slope) field of the DE. (104/35)
b. Graphically approximate a solution curve to the DE for $y(0) = 2$. (104/35)
c. Find a numerical approximation for $y(3)$ given $y(0) = 2$. (104/37)

9. Probability and Statistics:

a. Compute combinations and permutations: (206/5)

E.g. A small company will start a night shift. There are 20 employees. If the night crew consists of three employees, how many different night crews are possible?

b. Compute the sample and population mean, median, variance, and standard deviation: (206/3)

E.g. Given the following data: 3.0 2.5 3.7 2.7 3.3, compute the mean, median, variance, and standard deviation.

c. Use calculator programs to find probabilities and percentiles for binomial, hypergeometric, Poisson, uniform, exponential, normal, $t$, and $c^2$ distributions. (206/35).

E.g. Given that $X$ follows a binomial distribution with parameters $n = 14$ and $p = .72$, find $P(X \geq 11)$ .

E.g. Find the 85th percentile of a normal distribution with parameters $\mu = 9$ and $\sigma^2 = 25$.

10. Convert complex numbers between polar and rectangular forms: (364/12)

E.g. Express $7 + 3i$ in polar form.

E.g. Express $6e^{i\frac{p}{3}}$ in rectangular form.

## EXCEL

1. Perform iterative operations:

E.g. Given the DDS $a(n) = 3\,a(n\text{-}1) + n^2$, find $a(15)$. (103/09)

E.g. Solve the following system DDS's for $F_o = 2/3$: (364/15)

$$\begin{bmatrix} u_1(n+1) \\ u_2(n+1) \\ u_3(n+1) \\ u_4(n+1) \end{bmatrix} = \begin{bmatrix} 1-2F_o & F_o & 0 & 0 \\ F_o & 1-2F_o & F_o & 0 \\ 0 & F_o & 1-2F_o & F_o \\ 0 & 0 & F_o & 1-2F_o \end{bmatrix} \begin{bmatrix} u_1(n) \\ u_2(n) \\ u_3(n) \\ u_4(n) \end{bmatrix} + F_o \begin{bmatrix} 3 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

2. Create single and multiple graphs:

E.g. Graph the particular solution (103/20)

$$a(k) = 2.3(4)^k + 5k^2 + 2\,, 0 \le k \le 25 \ .$$

E.g. Graph the function $y = \sin(3x) + x^3$. (103/35)

E.g. Graph the following functions individually and together: (104/08)

$$y_1 = 2x^2 + 3x^{1.5} - 24e^{2x}$$

$$y_2 = \sin(3x) + e^x - 2\ln(4x)$$

E.g. Graph the solution $\boldsymbol{u_s(n)}$ as a function of the subscript $\boldsymbol{s}$ ($s$ = 1,2,3,4) (364/6)
and $\boldsymbol{n}$ for the linear system of equations:

$$\begin{bmatrix} u_1(n+1) \\ u_2(n+1) \\ u_3(n+1) \\ u_4(n+1) \end{bmatrix} = \begin{bmatrix} 1-2F_o & F_o & 0 & 0 \\ F_o & 1-2F_o & F_o & 0 \\ 0 & F_o & 1-2F_o & F_o \\ 0 & 0 & F_o & 1-2F_o \end{bmatrix} \begin{bmatrix} u_1(n) \\ u_2(n) \\ u_3(n) \\ u_4(n) \end{bmatrix} + F_o \begin{bmatrix} 3 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

3. Approximate definite integrals numerically: (104/30)

E.g. Given the definite integral $\int_2^6 \left[(6x-1)\cos\left(2x^3\right)\right] dx$ ,

evaluate using the Trapezoidal Rule and $n$ = 16.

4. Approximate numerical solutions to differential equations: (104/35)

E.g. Given $y' = 1 - \sin(y)$ , $y(0) = 2$ , find $y(3)$ using Euler's method and $h = 0.2$ .

5. Export a table or a graph to a Microsoft Word document. (103/15)

## MATHCAD

1. Iterate a DDS and graph the result:

E.g. Given the DDS $a(n+1) = 3\,a(n) + n^2$, find $a(15)$ (103/10).

2. Perform matrix operations in two and three variables:

E.g. Given the linear system of equations,

$$\begin{bmatrix} a(n+1) \\ b(n+1) \\ c(n+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} a(n) \\ b(n) \\ c(n) \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

a. Find the determinant of the coefficient matrix. (103/23)
b. Find the inverse of the coefficient matrix. (103/23)
c. Find the eigenvalues and eigenvectors of the coefficient matrix. (103/23)

E.g. Given the linear system of equations,

$$\begin{bmatrix} 2 & 5 & 6 \\ 7 & 8 & 4 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$$

a. Find the determinant of the coefficient matrix. (103/23) (364/15)
c. Perform row reduction operations on the augmented matrix. (103/23)
b. Solve the system of equations. (103/23) (364/15)

3. Find the real roots of a polynomial or transcendental function:

E.g. Find the roots of the function $r^3 - 7r^2 - 14r + 8 = 0$. (103/32)

E.g. Find the roots of the function $1 - \sin(2x) = 0$, $0 < x < p$ . (104/08)

4. Graph:

a. Algebraic and elementary transcendental functions:

E.g. Graph the following functions individually and together: (103/46)

$$y_1 = 2x^2 + 3x^{1.5} - 24e^{2x}$$
$$y_2 = \sin(3x) + e^x - 2\ln(4x)$$
,

b. Surfaces and space curves:

E.g. Graph the space curve (205/18)

$$\vec{r}(t) = t^2\,\vec{i} + 2t\,\vec{j} + \sin(t)\,\vec{k}\,, t \in [0, p]$$

E.g. Graph the surface $z = \dfrac{x^2}{4} + \dfrac{y^2}{16}$. (205/14)

c. Level curves:

E.g. Graph the level curves for $x^2 + y^2 = k$ (205/28)

5. Find limits: (103/40)

6. Derivatives:

E.g. Given the function $y = \dfrac{1}{x} + \sin(2x^3 - x)$ ,

a. Plot the derivative. (103/46)
b. Find the derivative. (103/46)

E.g. Given the function $f(x, y) = x^2 y + xy^2$ ,

c. Find the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial^2 f}{\partial x \partial y}, \dfrac{\partial^2 f}{\partial x^2}$. (205/31)

e. Find the directional derivative of $f(x,y)$ at the point (2,3) in the direction of $2\,\vec{i} - 4\,\vec{j}$ . (205/33)

7. Integrals:

a. Definite integrals.

E.g. Evaluate $\int_1^2 \left(x\ln(x) - e^x\right)dx$ . (104/8)

E.g. Evaluate $\int_0^1 \int_0^{y^2} \left(3y^3 e^{xy}\right)dx\,dy$ . (205/47)

E.g. Evaluate $\int_0^3 \sin\left[\frac{(2n-1)p\ x}{6}\right] \sin\left[\frac{(2m-1)p\ x}{6}\right] dx$, (364/33)

for $m = n$ and $m \neq n$ .

b. Indefinite integrals (104/22)

c. Partial fraction decomposition: (104/39)

8. Differential Equations (DE):

E.g. Given $y(x) = e^x - e^{2x}$,

a. Verify that *y(x)* is a solution to the DE $y' - y = -e^{2x}$ , $y(0) = 0$ . (104/31)

9. Compute the gradient, divergence, and curl: (364/35)

E.g. Calculate the gradient of $f\ (x, y, z) = e^{xy} + xz^2$ .

E.g. Calculate the divergence and curl of $\vec{F} = 2xy\vec{i} + e^y\vec{j} + 2z\vec{k}$

10. Vectors:

a. Given 3 vectors, compute their magnitude, cross product, dot product, and triple scalar product. (205/8)

b. Extract from a vector an element to be used in another operation. (205/9)

11. Probability and Statistics: Evaluate probabilities and expectations for univariate and multivariate continuous probability distribution functions. (206/22)

12. Solve nonlinear systems of equations: (364/15)

a. Constrained application

b. Constrained optimization (Lagrange) (205/40)

c. Solving for relative extrema (205/37)

13. Create a formal written document that includes written, graphical, numerical, and analytical modeling and analysis. (103/11)

**MINITAB**

1. Analyze data with descriptive statistics and descriptive plots. (206/01)

2. Simulate data (generate random looking data) (206/09)

3. Perform and interpret tests for normality (206/19)

4. Construct and interpret confidence intervals (206/30)

5. Perform and interpret hypothesis tests (206/35)

## Appendix B:

**UNITED STATES MILITARY ACADEMY**
**MATHEMATICAL RECALL KNOWLEDGE**

The following constitutes a basic mathematical vocabulary that will be built upon during each cadet's four-semester core mathematics experience and in their future math/science/engineering courses. Once each of these basic ideas has been covered in class, each cadet can be required to reproduce, upon demand in any future lesson of any math/science/engineering course, that idea. Annotated beside each item are the course number and lesson number which denotes the exact point in time when the student is responsible for each item (e.g.: 103/1 means that each cadet is responsible for knowing that item the first lesson of MA103). These items are recall knowledge - cadets are also required to be proficient in the more conceptual, less-verbatim ideas and skills reflected in each core math course's Course Objectives.

### ALGEBRA

1. $ax^2 + bx + c = 0 \implies x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (103/1)
2. $a^b \cdot a^c = a^{b+c}$ (103/1)
3. $(a^b)^c = a^{bc}$ (103/1)
4. $\dfrac{a^b}{a^c} = a^{b-c}$ (103/1)
5. $y = \log_b x \implies x = b^y$ (103/1)
6. $\log_b b^x = b^{\log_b x} = x$ (103/1)
7. $\log_b x^a = a \log_b x$ (103/1)
8. $\log_b ac = \log_b a + \log_b c$ (103/1)
9. $\log_b \dfrac{a}{c} = \log_b a - \log_b c$ (103/1)
10. $\log_b a = \dfrac{\log_c a}{\log_c b}$ (103/1)

### ANALYTIC GEOMETRY

Rectangle: Area = $lw$ (103/1) Perimeter = $2l + 2w$ (103/1)

Circle: Area = $\pi r^2$ (103/1) Circumference = $2\pi r$ (103/1)

Rectangular Solid: Volume = $lwh$ (103/1) Surface Area = $2lw + 2lh + 2hw$ (103/1)

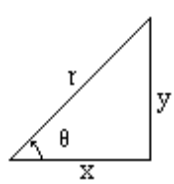
Cylinder: Volume = $\pi r^2 l$ (103/1) Surface Area = $2\pi r^2 + 2\pi rl$ (103/1)

Sphere: Volume = $\frac{4}{3}\pi r^3$ (103/1) Surface Area = $4\pi r^2$ (103/1)

Distance between $(x_1, y_1)$ and $(x_2, y_2)$ = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (103/1)

### TRIGONOMETRY

With reference to the right triangle:




$2\pi$ radians = 360 degrees (103/1)

$\sin q = \dfrac{y}{r}$ (103/1) $\cos q = \dfrac{x}{r}$ (103/1) $\tan q = \dfrac{y}{x}$ (103/1)

$\tan \theta = \dfrac{\sin q}{\cos q}$ (103/1) $x^2 + y^2 = r^2$ (103/1) $\sin^2 q + \cos^2 q = 1$ (103/1)

$\cot \theta = \dfrac{1}{\tan \theta}$ (103/1) $\sec \theta = \dfrac{1}{\cos \theta}$ (103/1) $\csc \theta = \dfrac{1}{\sin \theta}$ (103/1)

## RELATIONSHIPS

Distance = average rate × time

Corresponding sides of similar triangles are proportional

## DIFFERENTIATION (MA 103-104)

1. $\frac{d}{dx}(a) = 0$ (103/47)

2. $\frac{d}{dx}(x) = 1$ (103/47)

3. $\frac{d}{dx}(au) = a\frac{du}{dx}$ (103/47)

4 $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$ (103/47)

5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ Product Rule (103/50)

6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Quotient Rule (103/50)

7. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$ Power Rule (103/49)

8. $\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$ Chain Rule(103/54)

9. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$ (103/51)

10. $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$ (103/51)

11. $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$ (103/49)

12. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ (104/4)

## INTEGRATION-(MA 104)

13. $\int a\,dx = ax + C$ (104/24)

14. $\int (u+v)\,dx = \int u\,dx + \int v\,dx$ (104/22)

15. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ (104/24)

16. $\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$ (104/32)

17. Understand and be able to apply the Substitution Rule (104/32)

18. $\int \frac{du}{u} = \ln|u| + C$ (104/32)

19. $\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$ (104/30)

20. $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$ (104/30)

21. If $f$ is integrable on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$ where $\frac{dF}{dx} = f(x)$

(Fundamental Theorem of Calculus)

## VECTOR CALCULUS (MA-205)

22. $|\vec{A}| = \sqrt{a_i^2 + a_j^2 + a_k^2}$ (205/6)

23. $\vec{A} \bullet \vec{B} = a_i b_i + a_j b_j + a_k b_k = |\vec{A}||\vec{B}|\cos q$ (205/8)

24. $|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|\,|\sin\theta|$ (205/9)

26. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix}$ (205 / 9)

25. $\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$ (205/33)

**PROBABILITY & STATISTICS (MA 206)**

27. If an event $A$ is one of $n$ possible outcomes among the total number $N$ of all possible outcomes (all being mutually exclusive and equally likely), then the probability of event $A$ is defined to be $n/N$ and the range of $P(A)$ is [0,1], (i.e., $0 \le P(A) \le 1$). (206/3)

28. PDFs and CDFs: (206/16)

Discrete: PDF: $P(X = x) = p(x)$.

CDF: $P(X \le x) = \sum_{y \le x} p(y)$.

Continuous: PDF: $f(x)$ is used to find probabilities - $P(a \le X \le b) = \int_a^b f(x)dx$.

CDF: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy$.

29. The total accumulation of a probability distribution function is one. (206/15)

Discrete: $\sum_{\forall x} p(x) = 1$ Continuous: $\int_{-\infty}^{\infty} f(x)dx = 1$

30. Inclusion-Exclusion (simple form): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (206/3)

31. Intersection & Conditional Probability:

$$P(A \cap B) = \begin{cases} P(A \mid B) \cdot P(B) & \text{(In General)} \\ P(A) \cdot P(B) & (A \text{ and } B \text{ Independent}) \\ 0 & (A \text{ and } B \text{ Mutually Exclusive}) \end{cases} \qquad (206/5)$$

32. Calculate and interpret the expected value (mean) of a random variable.(206/16)

Discrete: $E(X) = \sum_{\forall x} x \cdot p(x)$ Continuous: $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$

33. Calculate and interpret the variance of a random variable. (206/16)
The variance is an expected value: $V(X) = E[(X - m)^2] = E(X^2) - m^2$.

34. Percentiles of random variables including the median (50th percentile). (206/17)

35. Central Limit Theorem: Let $X_1, X_2, \ldots, X_n$ be a random sample from *any* population with a finite mean and variance. Then, if $n$ is sufficiently large, $\overline{X} = \sum X_i / n$ and $T_0 = \sum X_i$ are **approximately** normally distributed. (206/26)