The Machine is the Textbook

Charles M. Patton
MathTech Services
blister0@nic.cerfnet.com

Textbook publishers are just now discovering that the decade-long slide in textbook sales is not just due to the high prices attendant on high quality production (their first theory), nor to lackluster, hippie professors ignoring the textbooks (their second theory), nor to cut-throat competition from unscrupulous used book sellers (their third theory), nor to "the Internet" (their latest theory). While all these factors may contribute, the underlying cause is that students, the publisher's ultimate customers, are placing a lower and lower value on textbooks. Be it cultural evolution, or cultural degeneration, students are looking more and more to "the machine" (calculators, computers, the Internet, video, gameboys, etc.) for guidance, insight, and entertainment.

Virtually all the math software available today is built on an architectural plan laid down in the "textbook era". It is not at all obvious that any of it can meet the challenges, take advantage of the opportunities, or live up to the responsibilities which obtain as we leave the textbook behind.

The responsibilities and opportunities fall into roughly three areas: discovery, communication, and assessment.

Discovery

> The intellectual scaffolding necessary to allow students to explore, make connections, gain experience.

Communication

> The infrastructure necessary to allow for this most human of activities, to facilitate collaboration among students - when appropriate, and allow students to present their work naturally and efficiently.

Assessment

> The system necessary to enable a student's **act** to become an **artifact**: to be observed, recorded, reflected on, and ultimately judged.

Discovery - Insight, Not Answers

The promise of student discovery has been a driving force for the adoption of technology in the classroom at all levels, especially in the Calculus. Graphs, tables, formulae, numeric experiments, integration, differentiation - all provide for an environment in which the student can experience a range of Calculus concepts for themselves, and in their own way.

Yet you may have noticed that the advance of discovery tools for Calculus, which was quite rapid in the late '80s and early '90s, has slowed down considerably in recent years. In part, this is because the tools available to advance the state of the art were designed first and foremost to get answers. Up to a point, you can use an answer-oriented tool to help with insight by "throttling down" the power of the answer engine, making it go more slowly - perhaps even step by step, but adding more and more sophisticated answer-finding capability is unlikely to add more insight power at all.

PUFM

Roger Howe [HOWE99] in his review of *Knowing And Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States* [MA99] highlights Ma's notion of a "Profound Understanding of Fundamental Mathematics" and while Ma's book targets primary and secondary school mathematics, Howe clearly believes that PUFM is just as relevant in college teaching.

One aspect of PUFM is the knowledge of the connections among and analogies between mathematical, and physical, concepts. So, for example, a teacher would be able to relate the notion of derivative with approximate local linearity, which builds on both the model of linear functions and interpolation, which in turn builds on the notions of functions, composition, and proportionality, which builds on addition, subtraction, multiplication, and division. In turn, higher dimensional (even infinite dimensional) versions of the derivative are built on this same notion.

The only way technology could help students discover these roots and draw mathematical nourishment from them would be to have these connections present and manifest at the deep levels of the software itself.
Although no software system in use meets this requirement today, several technologies, long in the works, were ushered in during 1998 and 1999 and hold great promise as a starting point for implementing a system with this kind of depth.

One of these is the MathML specification [ION99] which, among other things, provides an unambiguous, machine-readable grammar for describing the intended meaning of a mathematical expression, including its context, annotations, and links to other conceptual structures.

At the same time, work is progressing on laying out conceptual structures which could capture some of the richness of mathematical connections. In particular, the Math MetaData project [ROBSON99] aims to create an extensible, inter-linked classification system that both incorporates the AMS Subject Classification, and extends into the undergraduate and K-12 learning environment, while providing for appropriate concept, topic, subject, and class interconnections.

The dream is that as these two efforts progress, it will be possible for a student looking at the derivative of a composition of functions to ask of the system "is there anything like this I might have seen before?" and being shown the slope of the composition of two linear functions. Repeating with the query with the new expression, they might be shown that if A is proportional to B and B is proportional to C, then A is proportional to C.

Multiple, Multiple Representations

A recurring theme arising both in the NCTM Standards efforts, and in the Calculus Reform efforts is that of linked multiple representations of mathematical concepts - often graphical, numeric, and symbolic. While these are all important and well represented in current technology, they don't go far enough. For example, a significant impediment to integrating mathematics and engineering instruction is simply the difference in representation of the underlying mathematics. Something as simple as Newton vs. Leibniz notation can derail joint understanding. So can ***i*** vs ***j***.

Beyond that, there are many interesting and useful representations which, for historical reasons, are not in widespread use (my favorite example is the nomograph representation [NOMO]). Because they weren't in widespread use, they weren't integrated into base software, and with the old-style software architectures they continued to be widely unavailable.

But today, everyone is used to the notion that it is the *viewer* who decides which representation they would like to see - which paragraph and heading style they'd like, which browser plug-in to associate with a particular kind of data, and so on. This constitutes a quiet revolution, and has fostered the birth of efforts like the Educational Object Economy [EOE]. Combining the strengths of markup language and style sheet technologies with a widespread, unambiguous means of describing mathematical content will allow the proliferation of new ways to view old constructs.

Communication

Besides just replacing the textbook, technology provides opportunities to address issues that textbooks could not address effectively. One of those issues is communication, especially communication in the classroom, especially structured conversations for learning.

Structured Conversations

If you look around your classroom you will note that much of what goes on there is a structured conversation. It is structured both in form - who converses with whom and under what conditions, and in content. In the pure lecture-style class, any technology beyond what is already widespread is probably not very necessary. However, as we move from "Sage on the Stage" to "Guide on the Side", from individual to collaborative efforts, from passive listening to active response, the logistical and administrative costs of the structured conversation rise significantly, and so does the opportunity for technology to make a difference.

Supposing that the technology could provide for conversation structured in form (who converses with whom under what conditions) and in content ("live" mathematical objects, dialog and commentary), what could be done with such a system?

Emergent Understanding from Emergent Behavior

There are many situations where a deeper insight about a mathematical process might be obtained by having each student focus on an individual local problem, while observing how their local solution contributes to a more complicated global solution [WILENSKY99]. A simple example is a group effort approximate solution to a differential equation using Euler's method. In this structured conversation, each student is assigned a step in the Euler process and need to choose, on the basis

of the results posted by the owner of the previous step, which of the family of parallel line segments matches up to continue the solution. . The first segment, of course, has no previous step to match with, providing the opportunity for an initial value discussion. The students can see the entire solution arising piece by piece from their individual choices, and can see the effects of a change of initial value rippling through the solution, forcing them to make a new (but related!) choice.

A related, very interesting example which I have used with 4th and 5th graders (but without benefit of the kind of technology envisioned here) is the problem of how the eye perceives shape from a shaded surface [PATTON99].

Plugging Into Rich Environments

The Internet (and the Web, in particular), like the real world, has a wonderful cornucopia of rich, immersive, experiences. But, like the real world, many of those experiences are too rich, too open-ended, with too many pitfalls and dead ends for everyday classroom use.

Imagine that to learn about a battle, you could actually participate in it, or to learn about a city you could actually go there, or to learn about jet propulsion you could fly a jet. These, and more, are becoming possibilities over the Internet. But, does it make sense to learn about every city by going there, every battle by participating?

Some tour groups go to a cathedral for the overall, immersive, experience. Other groups go to focus solely on the art of the stonework. Others go to pray. How very useful it would be to be able to actively guide an audience through a web site, filtering out the distractions, providing a focus and commentary, providing an experience shaped to the needs of the audience. This same concern applies to other data sources, including census statistics, financial and stock market data, data from simulations and web-published papers, etc.

While there are a number of experimental systems which attempt this kind of tight integration and "re-purposing" of external content [CRIT], it is currently neither as successful, nor as easy as it will need to be in the future especially to, for example, pull engineering content into a math classroom, or vice versa.

Assessment

Of all the areas in which technology - both communication technology and computation technology could have a significant impact, assessment, both formative (feedback and guidance) and summative (examinations, inventories, and rankings) holds the greatest promise and is currently the least well developed.

Scarcity of Time - Do the math

An instructor typically has about 6 or 7 minutes per week per student which can be allocated to assessment, feedback, and grading. It's no wonder that multiple choice is so popular, and so little homework and other out-of-class work is assigned. While some might argue that multiple choice

questions, carefully constructed and used appropriately, can be adequate to the task, no one would argue that they are ideal from a learning perspective. They are, in the current context, a simple necessity.

At the same time that you hear complaints of the form "they reach for the calculator to add two and seven", it would seem to be a simple matter of programming to build into the calculator the capability to recognize the situation and react with "That's easy. You try it." More generally, technology could be configured to recognize, at least in a rudimentary way, a variety of "learning moments" and react appropriately, effectively extending an instructor's reach, both in space and in time.

Drill and Kill, or Extra Credit

Writing good problems is difficult. Writing enough variants of good problems to provide a solid practice environment for a diverse population of learners is even more difficult. Allocating enough time to correct those problems and offer feedback is impossible. Technology of any sort will not make writing good problems any easier. But if those good problems were designed as templates, technology could be used to produce unlimited numbers of variants of those problems on demand, as well as close the feedback loop on the learner's interaction with those problems.

In Appendix A. are three screen shots of auto-generated versions of an Everyday Math® Study Link [ELC97]. The versions differ in the "snakiness" of Snakey Lane. An important feature of the range of generated versions is that in some versions, Snakey Lane is nearly a straight line and hence, as hypotenuse, shorter than the alternate route while in other cases, Snakey Lane lives up to its name and is, in fact, longer than the alternative route.

In this web-browser delivered example, the main image is generated from a PostScript® file with curve coefficients generated randomly (but with bounded range) each time the page is requested and inserted into the document before it is passed to an image rasterizer (Ghostscript here) to produce an image viewable on a web browser.

The "correct" answers are generated from the same coefficients (exercise: compute the arc length in pixels of a curve segment generated by a ***curveto*** command as a function of its input parameters). When the completed page is submitted, the answers entered are compared with the "correct" answers, with allowances made for the approximation process.

What Kind of Math Engine Does It Take?

At this point there is, unfortunately, no math engine well suited to all the requirements necessary to allow this kind of automated practice and assessment widespread throughout education. Some of these requirements include:

Can be operated as a server in a request-response manner, handling multiple requests simultaneously.

Stateless, so that every request and response can be viewed as independent of every other.

Flexible, extensible syntax which on every request can be tailored to the subject, level, and approach.

Extensible domain-specific mathematical semantics which can reflect the semantics of the subject, level, and approach.

Flexible, extensible result formatting which covers the wide variety found in current documents.

Can recognize functional or semantic equivalences (or "near equivalences") in a variety of domains

Can generate random elements from implicitly characterized sets, e.g. 'a random pair of two-digit integers whose product is less than 2000'

Indefinite precision numeric evaluation of elementary functions.

The usual repertoire of symbolic computations.

References and Notes

[HOWE99] R.Howe, Knowing and Teaching Elementary Mathematics (book review), *Notices of the AMS* **46** (1999), 881-887

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[ROBSON99] R.Robson, A Mathematician's Guide to IMS Metatdata,Draft, http://robby.orst.edu/papers/guide_v2

[NOMO] A nomograph-style rendering of x->1/x:

[EOE] http://www.eoe.org/

[WILENSKY99] Wilensky and Stroup, *Learning through Participatory Simulations: Network-based Design for Systems Learning in Classrooms* Proceedings of CSCL '99, Computer-Supported Collaborative Learning, Stanford University

[PATTON99] Patton, *Shared Dataspace: A Frontier Scout's Report*, Presented at the USMA, November 1999

[CRIT] Ka-Ping Yee, *CritSuite, Critical Discussion Tools for the Web* Forsight Institute, http://crit.org/http://crit.org/index.html

[ELC97] Max Bell, et al, *Everyday Mathematics, Fifth Grade* Everyday Learning Corporation, Chicago 1997

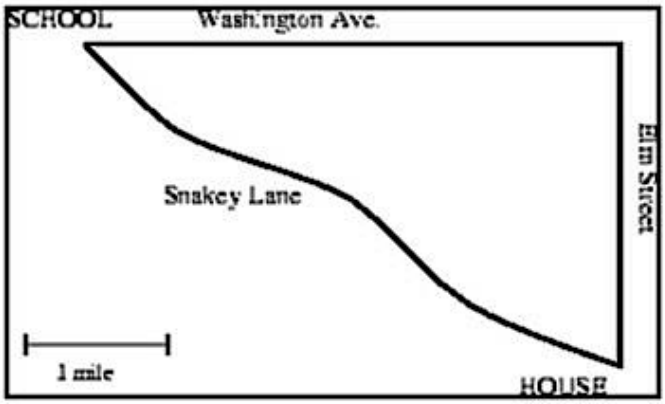
Appendix A. Screen Shots of Auto-generated Study Link

Name: Web_Meister **Date:** Mon Sep 20 09:42:53 PDT 1999

Study Link 25

There are two ways to get from Josephina's house to school. She can either take Elm Street to Washington Avenue, or she can take Snakey Lane.

Use the map and scale below to answer the questions.



1. Josephina started walking along Elm Street.

a. How far would Josephina walk before she turned onto Washington Avenue?

b. How far would she be from school when she turned?

2.

a. If Josephina wants to take the shortest route to school, which way should she go?

b. What is this distance?

3. Josephina's father jogged to the school along Snakey Lane and back along Washington Avenue and ELm Street to his house. About how far did he jog?

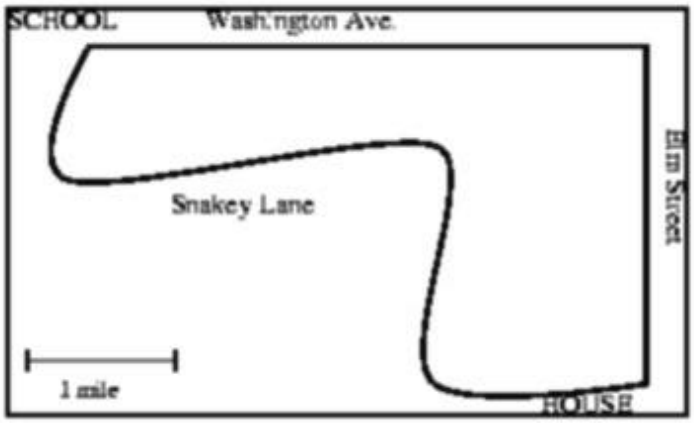
4. Josephina left school and walked along Snakey Lane. Her father left home at the same time and walked up Snakey Lane. Her father was walking twice as fast as Josephina. Click on the spot on road where they will meet.

Name: Web_Meister **Date:** Mon Sep 20 09:42:06 PDT 1999

Study Link 25

There are two ways to get from Josephina's house to school. She can either take Elm Street to Washington Avenue, or she can take Snakey Lane.

Use the map and scale below to answer the questions.



1. Josephina started walking along Elm Street.

a. How far would Josephina walk before she turned onto Washington Avenue?

b. How far would she be from school when she turned?

2.

a. If Josephina wants to take the shortest route to school, which way should she go?

b. What is this distance?

3. Josephina's father jogged to the school along Snakey Lane and back along Washington Avenue and ELm Street to his house. About how far did he jog?

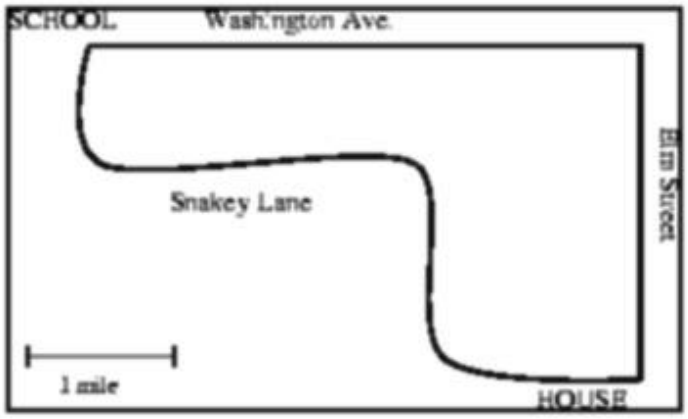
4. Josephina left school and walked along Snakey Lane. Her father left home at the same time and walked up Snakey Lane. Her father was walking twice as fast as Josephina. Click on the spot on road where they will meet.

Name: Web_Meister **Date:** Mon Sep 20 09:39:04 PDT 1999

Study Link 25

There are two ways to get from Josephina's house to school. She can either take Elm Street to Washington Avenue, or she can take Snakey Lane.

Use the map and scale below to answer the questions.



1. Josephina started walking along Elm Street.

a. How far would Josephina walk before she turned onto Washington Avenue?

b. How far would she be from school when she turned?

2.

a. If Josephina wants to take the shortest route to school, which way should she go?

b. What is this distance?

3. Josephina's father jogged to the school along Snakey Lane and back along Washington Avenue and ELm Street to his house. About how far did he jog?

4. Josephina left school and walked along Snakey Lane. Her father left house at the same time and walked up Snakey Lane. Her father was walking twice as fast as Josephina. Click on the spot on road where they will meet.