Major Issues and Future Directions of Undergraduate Mathematics: Instructional Techniques in Freshman and Sophomore Math Courses

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Introduction

Major issues and future directions of instructional techniques in freshman and sophomore mathematics courses are briefly discussed. The following eight topics are considered:

Topics:

1. What are the strengths/drawbacks of various instructional methods?
2. How to choose and integrate various instructional methods?
3. Which methods best increase success of underrepresented groups?
4. How do the learning media affect reading, writing, and problem solving?
5. How to build theoretical understanding?
6. How to align the "achieved curriculum" and the "intended curriculum"?
7. What guiding principles arise from educational research?
8. Should calculus be a laboratory (discovery) course?

1. Strengths/drawbacks of various instructional methods

The Preface to *Calculus – Single and Multivariable*, Second Edition [1] recommends several general features that instructional methods should have:

- Focus on a small number of key concepts ? emphasize depth rather than breadth of understanding
- Encourage active learning
- Employ multiple representations ? geometric, numerical, analytical, and verbal;
  For example, represent functions graphically (picture), numerically (table of values), and algebraically (formula)

Instructional methods for freshman and sophomore mathematics courses include the following [2]:

1. Traditional lectures
2. Use of computer algebra systems (CAS) and electronic notebooks in place of a traditional textbook
3. Pedagogical approaches based on a constructivist theoretical perspective of how mathematics is learned (described by a shift from a teacher-centered classroom, where lecture predominates, to a student-centered laboratory, where students make and test hypotheses and discover mathematical truths for themselves)
4. Reform calculus, etc., emphasizing real-world problems, hands-on activities, and discovery-based learning (involving conceptual problems as well as computation)
5. In-class group work activities, projects, and student oral presentations

The tradeoffs between these methods are explicitly given by the descriptions of these methods.

Using case studies can be an effective means to motivate engineering and physics students to appreciate the role of mathematics in engineering and physics disciplines [3], [4], [5]. Once students see the relevance of mathematics to their disciplines, there is interest and incentive to pursue mathematics.

2. Choosing and integrating various instructional methods

The term "calculus reform" has various interpretations. However, there seems to be a consensus that "calculus reform" includes active student learning in the classroom ? students actively participate in their classroom activities, instead of assuming the "traditional" role as passive note-takers. Material is presented from multiple perspectives in order to encourage understanding of concepts. Technology is used to alleviate tedious computations, to enable students to attack more "real world" problems and "explore" for themselves, and to aid visualization. Math "diaries", group projects, and writing projects may also be components of "calculus reform" [6]. Issues involve what degree of each innovative aspect of "calculus reform" is to be utilized in a specific course and by a specific instructor.

CAS are now used with most freshman and sophomore math courses. Current issues center on the degree of use of CAS. At one extreme, some courses are taught with computers in the classroom [7]. On the other hand, CAS may be used only as a supplementary tool to perform numerical computations and obtain graphics. In any event, the use of CAS opens a whole new range of pedagogical questions. What is the role of such technology? How are course objectives best met with CAS? What criteria are to be used in assessment of student performance? And there are many other related questions.

One of the most touted benefits of CAS is the graphics capability, useful for visualization. This can be especially helpful in multivariable calculus [8].

There are also various degrees of "web-based courses" in which an assortment of course material is made available to students on the Internet [9], [10]. This can range from instructor comments to help clarify and emphasize (or de-emphasize) specific material in the course text, to large-scale use of the Internet to deliver homework problems, assignments, and projects. Students may submit their work electronically. Distance learning is another example of an instructional technique that can involve the Internet
(see Section 4).

Various instructional methods that "work" in large lectures, small classes, computer labs, small groups, or traditional classrooms, and with various technologies (from graphing calculators to complete CAS) should be chosen so as to fit the specific scenario.

Whatever decisions are made with respect to a specific course, the uses of and tradeoffs between various instructional techniques must be re-evaluated frequently. There are many forces for change [11]. These forces necessitate such evaluations in order to provide students with educational experiences that will serve them well in the rapidly changing scientific and engineering workplace.

3. Methods to increase success of underrepresented groups

An article, "Research finds advantages in classes of 13 to 17 pupils", appeared in the April 30, 1999 edition of the **New York Times**. The research discussed in this article involved students in

grades K-12, but it could be argued that the results of the study would also apply to college students. Some observations were that students in these smaller classes had higher grades, better graduation rates, and were more likely to attend college than those from larger classes. Quoting from the article, "The small-class study also found that minority and poor students were helped even more by small classes in some areas than other groups…"

Role models, diversity in the administration, faculty, and student body, and an appropriate balance of competition versus cooperation are just a few of the important factors to consider with respect to encouraging underrepresented groups in math, engineering, and physics.

4. The effect of learning media on reading, writing, and problem solving

There are tradeoffs between various learning media. For example, Greg Reynolds of New Mexico State University has had good results teaching via a computer projection system in the classroom and the Internet [12]. Students are given hard copies of the class notes, which they can annotate as these notes are presented using a computer projection system in the classroom. There are links from the notes, which are available on the Internet, to homework problems and solution modules. Computer simulations are included in the class presentations. The claim is that this more repetitive and visual approach promotes more discussion and interactions between students, resulting in improved interest and learning of the material.

Phil Smith, also of New Mexico State University, has presented courses using television and the Internet [13]. At this stage it is unclear what the pedagogical tradeoffs of such a "distance" learning mode are.

Some studies have shown a difference in abilities of students who have taken "traditional" calculus courses versus students who have taken "reform" calculus courses.
One such study [14] showed a distinct difference in approaches to solving engineering mechanics problems that involve calculus. "Calculus and *Mathematica*® students, who learned calculus with a conceptual emphasis, were found to be more likely to solve problems from a conceptual viewpoint than were the traditional students, who were more likely to focus on procedures." For several other studies, see [6, pp. 5-9].

5. Building theoretical understanding

There are many factors involved in promoting a deep understanding of material covered in freshman and sophomore mathematics courses, in particular an understanding of the underlying theory. Some of these factors are that [15]

- "the students have the prerequisite skills and knowledge,
- the instructor provides clarity of goals and standards,
- there is perceived relevance of this theoretical course material,
- there is sufficient student practice and instructor feedback,
- testing includes these theoretical topics,
- the instructor has knowledge, preparation, enthusiasm, and empathy,
- there is a reasonable student workload, and
- there exists a match between an instructor's teaching style(s) and a student's learning style(s)."

A student should feel some degree of "ownership" in the course. That is, a student should be able to feel that she or he developed some ideas or carried out some solution on her or his own. This

instills confidence. Another advantage is that is that an individual remembers ideas better when he or she has discovered them on his or her own
[16, Introduction].

6. Aligning the "achieved curriculum" and the "intended curriculum"

Is there a core curriculum? The following is a quote from the spring 1999 Newsletter of the SEFI (European Society for Engineering Education) Mathematics Working Group [17]. "In the 1990s it has come to be recognized that specialisations of engineering have continued to diversify, possibly to such an extent that a totally common core of mathematics at the degree level is becoming increasingly difficult to define, though there remain large interlocking clusters of common substance."

Traditionally, freshman and sophomore mathematics courses have a diverse student clientele: mathematics majors, engineers, physics students, etc. Some students will require a more theoretical approach emphasizing logical structure and symbolic work. Others will require a course that is perhaps more modeling-oriented with emphasis on analyzing the real world and the use of technology. Each course must be made useful to this diverse selection of students. The course must be flexible enough to accommodate diverse student needs. Theory, practical understanding, skill building, and applications all must be considered.

7. Guiding principles arising from educational research

Regardless of what guiding principles arise from educational research, since many of the researchers who study engineering (math, physics) student learning are not themselves engineers (mathematicians, physicists), the research is usually published in specialized journals that engineering (math, physics) educators do not read. However, since much of this work is of direct interest to engineering (math, physics) educators, there is a need to make this work more accessible to these engineering educators who can apply it [17]. (Also see Section 1, Item 3.)

8. Should calculus be a laboratory (discovery) course?

If calculus is taught as a "laboratory course", emphasis can be placed on open-ended problems for which there is more than one solution approach and more than one correct solution. Common sense ideas can be brought into the picture. Working in small groups can be helpful. With computer technology, related problems could be solved, each time varying a specific parameter and observing the solution dependence upon that parameter. This use of technology can enhance basic understanding.

Summary

The issues are many, and they are complex. There are many schools of thought on the topics that have been raised [18]. The biggest difficulty that many instructors have in trying to implement any of these approaches is finding the time required. This workshop will help us to address this and related topics.

Bibliography

[1] D. Hughes-Hallett, A. Gleason, W. McCallum, et al., *Calculus – Single and Multivariable*, Second Edition, Wiley, New York, (1998) iii-xi.

[2] http://forum.swarthmore.edu/mathed/calculus.reform.html, (Aug. 5, 1999).

[3] L.R. Mustoe and A.C. Croft, Motivating engineering students by using modern case studies, *International Journal of Engineering Education (IJEE)*, to appear in *Vol. 15*, No. 6, (2000).

[4] R. Marchand and T.J. McDevitt, Learning differential equations by exploring earthquake induced structural vibrations: a case study, *IJEE*, to appear in *Vol. 15*, No. 6, (2000).

[5] D.B. Meade and A.A. Struthers, Differential equations in the new millennium: the parachute problem, *IJEE*, to appear in *Vol. 15*, No. 6, (2000).

[6] http://www.mste.uiuc.edu/murphy/papers/CalcReformPaper.html, (Aug. 5, 1999).

[7] G.D. Allen, J. Herod, M. Holmes, V. Ervin, R.J. Lopez, J. Marlin, D.B. Meade, and D. Sanchez, Strategies and guidelines for using a computer algebra system in the classroom, *IJEE*, to appear in *Vol. 15*, No. 6, (2000).

[8] T.J. Murphy, R.E. Goodman, and J.J. White, Using the web in multivariable calculus to enhance visualization, *IJEE*, to appear in *Vol. 15*, No. 6, (2000).

[9] J. Rehig, Developing web-based courses using an online development guide and templates, *ASEE Computers in Education Journal*, *Vol. IX*, No. 2, (April-June 1999) 51-55.

[10] S. Goldberg, Bridging the gap between instructor and textbook, *ASEE Computers in Education Journal*, *Vol. IX*, No. 2, (April-June 1999) 45-50.

[11] D. Small, Core mathematics for engineers, mathematicians, and scientists, *IJEE*, to appear in *Vol. 15*, No. 6, (2000)..

[12] G. Reynolds, and A.T. Hanson, A computer-based approach for delivery of equation-rich material, Presentation at the 1999 ASEE Conference and Exposition, Charlotte, N.C., (June 23, 1999).

[13] P. Smith, Teaching a graduate engineering analysis course by combining TV and the internet, Presentation at the 1999 ASEE Conference and Exposition, Charlotte, N.C., (June 23, 1999).

[14] C. Roddick, How students use their knowledge of calculus in an engineering mechanics course, Presentation at the Seventeenth Annual Meeting for the Psychology of Mathematics Education, (October 21-24, 1995).

[15] R.M. Felder and R. Brent, *Effective Teaching: A Workshop*, Thirty-first Midwest Section ASEE Conference, Tulsa, OK, (April 10-11, 1996) F5.

[16] W. Roberts, Editor, *Calculus: the Dynamics of Change*, Mathematical Association of America, CRAFTY subcommittee, MAA Notes No. 39, (1996).

[17] N. Steel, Editor, *Newsletter of SEFI (European Society for Engineering Education) Working Group on Mathematics in Engineering Education*, (Spring 1999) 10-11; 15-16.

[18] L.R. Mustoe and S. Hibberd, *Mathematical Education of Engineers*, Clarendon Press, Oxford, (1995).