The Role of New Technology in Undergraduate Mathematics and Integration of This Technology in Physics and Engineering Mechanics

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Introduction

A current movement to change undergraduate mathematics education is called "Calculus Reform". The motivation for the movement can best be summarized by the following quote from a calculus text by Bradley and Smith(1):

"The major issue driving calculus reform is the poor performance of students trying to master the concepts of calculus. Much of this failure can be attributed to ways in which students study and learn mathematics in high school. The high school mathematics textbooks published over the last twenty years have increasingly focused on reducing mathematics to a series of small repeatable steps. Process is stressed over insight and understanding. In these same books, we often find problems matched to worked-out sample problems, which encourages students to memorize a series of problem-solving algorithms. Unfortunately, this reduces mathematics to taxonomy: students show up for calculus generally willing to work hard; for many, this translates into working hard at rote memorization! The task of getting students *to think conceptually* falls to the teachers of calculus……..."

I will base this position paper on the prospective of a customer of the Mathematics Department. Our Engineering students have had a full year of calculus before taking any mechanics course. These first two courses include:

Calculus I: Limits, continuous functions, derivatives and their applications. Integrals and the fundamental theorem of calculus.

Calculus II: Applications of the integral and methods of integration. Improper integrals. Polar coordinates and parametric curves. Sequences and series. Power series.

During their second year, they will take the third and fourth calculus courses concurrently with their mechanics courses.

Multivariable Calculus: Vectors in space. Functions of several variables and partial differentiation. Multiple integrals. Line and surface integrals. Green's and Stokes's theorems.

Differential Equations: Topics chosen from separable and exact equations, linear equations and variation of parameters, series solutions, higher order linear equations. Laplace transforms, systems of first order linear equations, nonlinear equations and stability, introduction to partial differential equations.

As teachers of the basic engineering mechanics courses, Statics, Dynamics, and Mechanics of Materials, we realized that all the statements in the preface of the calculus book are directly applicable to the basic engineering mechanics courses. Although the basic engineering mechanics courses are dependent upon the mathematical topics of vector algebra and calculus, linear algebra, multivariable calculus, and differential equations, the students seem to have no conceptual feeling for this material. Although they can quote that to find the local maximum or minimum of a function of a single variable, "take the derivative and set it equal to zero", they do

not seem to conceptually understand that the derivative is the slope of the curve. When asked whether a function or a differential equation is linear or nonlinear, they seem to be at a loss as to the meaning of linear although they could quickly give the equation for a straight line. The concepts of linear independent and ill-conditioned system of only two linear equations has no conceptual aspect, even when plotting the two equations and examining the intercept shows these concepts.

The basic mechanics courses are analysis courses and therefore are presented in much the same way as calculus. The students, therefore, can fall into the same mode of rote memorization and trying to fit solution to a formula or a sample problem. It is important to convey conceptual understanding of the physics of the problem. The ideas of Calculus Reform need to be applied to Mechanics Reform (2). A reviewer of a new mechanics text in Dynamics wrote:

"Why are people choosing to write so much on a subject that does not have the half-life of other engineering material? How many people are doing research in basic mechanics? What substantially new material or approach is ever adopted? All the books are variations on a theme. That is not to say that the delivery of the material cannot be improved. The computer opened up all sorts of possibilities."

Technology can affect what and how we teach both mathematics and the courses that depend upon the mathematical concepts. Graphic calculators can make an impact on calculus education as functions, for example, can be plotted and the students gain a better understanding of the concepts of the derivative and the integral. Many of these calculators can invert small matrices and solve systems of linear equations. Now commercial computational software, such as Mathcad, MATLAB, Maple and Mathematica, is available to the students that can change the way mathematics and engineering analysis are presented in the classroom. Many classrooms are being changed such that the computer monitor can be projected onto a large screen like an overhead projection. This allows a more conceptual approach to the presentation of the material and increases the types of problems that can be solved. These new computational software packages are more than fast graphical calculators as they also include symbolic solvers. As calculators replaced tables of trigonometric functions, the symbolic manipulators will replace tables of integrals.

The basic mechanics courses are heavily dependent upon vector algebra and vector calculus. Most students avoid the full use of vectors, particularly if the cross product is involved in the calculation. Computational software eliminates any difficulties in numerical vector calculation as well as making symbolic calculations feasible. Many of the vector equations that arise can be solved directly without resolving them into their scalar components.

In the introductory course in Dynamics, the resulting differential equations are usually nonlinear. In the past, the student was asked to solve the problem by a quasi-static method, that is, for the acceleration at the moment or position shown. The student has no idea of how the particle or rigid body moves and this approach gives creditability to the reviewer's comment on the short half-life of the subject. These nonlinear differential equations can now be solved numerically using an Euler method (tangent line) or higher order Runga-Kutta methods. The student gains an excellent conceptual view of the motion if the trajectory is plotted and is able to examine the effects of the change of the parameters of the equation.

In Statics, the student is asked to solve linear systems of equations for each equilibrium problem. Although this may be justified for a few problems, it is not the purpose of the course and all advanced courses would use linear algebra. The computational software packages can easily solve systems up to 10 by 10 and the emphasis can be placed upon modeling and the formulation of the equilibrium equations.

Currently at Michigan State University, computational software is not used in any of the calculus courses and there appears to be resistance among the mathematics faculty to use any software program or graphic calculators. The College of Engineering is under heavy pressure from the professional societies to train all students in the use of computational software. In March 1993, the American Society of Mechanical Engineers Council on Engineers, Board on Engineering Education and M.E. Department Heads (3) recommended that undergraduates be exposed to mathematics software early in the mechanical engineering program. Those who hire or work with recent college graduates should expect them to arrive with a working knowledge of computer mathematics. As a result, the College of Engineering is considering introducing these mathematical software packages during the Freshman year. We currently introduce this software in the mechanics courses at the cost of some other topics.

What are the difficulties in having parallel courses in mathematics, that is, one taught the traditional way and one using computational software? I think that it will result in a greater disconnect between mathematics and engineering which will be harmful to the students understanding of mathematics. One thing is clear and that is that computer mathematical techniques will be integrated into the engineering program. I would hope that analytical techniques and numerical techniques would be equally emphasized. The students are far more computer literate now and will naturally gravitate to any methods that reduce labor. In this manner, I feel they may not see the beauty of mathematics.

1. G. L. Bradley and K. J. Smith, *Single-Variable Calculus*, p. xiii, Preface, Prentice-Hall, Englewood Cliffs, New Jersey (1995)
2. R. W. Soutas-Little and D. J. Inman, *Calculus Reform to Mechanics Reform,* International Journal of Engineering Education, **13** (6), 1997, pp 442-447.
3. H. A. Pattee, Selecting computer mathematics, *Mechanical Engineering,* **117** (9) (1995) pp. 82-84.