Major Issues and Future Directions of Undergraduate Mathematics: Instructional Techniques in Freshman and Sophomore Mathematics Courses

Elizabeth J. Teles
Program Director for Mathematics
Division of Undergraduate Education
National Science Foundation
Arlington, Virginia 22230
Eteles@nsf.gov

Recently program directors at the National Science Foundation were asked to identify the most significant achievements in science, engineering, mathematics, and technology (SMET) research and education in the 20th century and the rich areas anticipated for development in the 21st century. The top three educational achievements recognized by the Division of Undergraduate Education (DUE) have vital implications for instructional strategies that are employed in SMET education.

The first achievement is a paradigm shift from "teacher-centered" to "learner-centered" for undergraduate education. The difficult task of moving higher education to focus on what students actually learn fosters examination and re-formulation of the content and processes of undergraduate SMET education. The result has been a greater recognition by institutions of higher education that:

- instruction is enhanced by a research base;
- learning occurs in a large variety of curricular and extracurricular settings;
- with sufficient motivation, introduction, and time, all students can learn complex concepts;
- information can be effectively presented in an integrated context which is interdisciplinary and tied to actual students' interests and real-world applications;
- authentic assessment is a critical component of instruction; and
- faculty must be well-prepared for their roles as teachers.

The second achievement is the recognition that balance must be achieved between the learning of "facts" and the learning of "processes." This is the true value of the integration of research and education. Such learning must incorporate personal experience if it is to be effective. Students must have opportunities to see, hear, do, and teach.

The third achievement is the exploitation of various technologies (smart laboratory instruments, computers, calculators, modeling and visualization tools, the Internet) to allow students to:

- explore theories and concepts without getting bogged down in tedious calculations or manipulations; and/or
- learn outside the confines of a particular time and classroom or laboratory setting. This relates both to learning "anytime, anywhere" and to engaging students in the observation or simulation of processes normally too large, too small, too fast, too slow, or too dangerous for direct interaction.

How is the mathematics community involved in these activities and instructional strategies? How do these achievements intersect issues facing mathematics - particularly the way mathematics interacts with other disciplines? How will educational changes related to these achievements be supported by faculty, administrators, departments, institutions, and funding agencies?

The three achievements identified for undergraduate SMET education closely parallel the recommendations of *Crossroads in Mathematics* (the AMATYC Standards) and the goals and objectives of calculus and other mathematics "reform" texts. Proposals to DUE programs increasingly discuss not only content of the materials to be developed but also the instructional strategies to be used. Recently funded proposals include instructional strategies such as: (a) the incorporation of exercises that involve critical thinking and concept development, (b) cooperative activities, (c) the use of technology as a means of student construction and manipulation of concepts, (d) the use of reading and writing in addition to traditional mathematics skills, and (e) multiple representations of concepts.

The guidelines for pedagogy described in *Crossroads in Mathematics* [4] include active involvement of students, technology to aid in concept development, problem solving and multistep problems, mathematical reasoning, conceptual understanding, use of realistic problems, integrated curriculum developed in context, multiple approaches to problem solving, diverse and frequent assessment, open-ended problems, oral and written communications, and use of a variety of teaching strategies.

The article "Visions of Calculus" in *Calculus: The Dynamics of Change* [1] mentions several common instructional strategies in the reform calculus movement. These encompass multiple representations of concepts, the use of technology, student projects, writing about mathematics, and concept development. In *Assessing Calculus Reform Efforts* [2], Tucker and Leitzel note similar strategies and their impact on the teaching of calculus. The Statistical Abstract of *Undergraduate Programs in the Mathematical Sciences in the United States: Fall 1995 CBMS Survey* reports that almost 30% of calculus was being taught using "reform" texts; however, over 40% of were using computer assignments and other "reform" instructional strategies. [3]

The article "Linking Teaching with Learning" in *Science Teaching Reconsidered* [5] makes a case for scientific research as a model for learning and teaching – specifically for *active* learning and *active* teaching as opposed to methods that college teachers have traditionally used, such a lectures, assigned readings, problem sets, and closely supervised laboratory work. The authors recommend methods such as engaging students in communities of learning, establishing a context for exploration, proposing explanations, and reading and writing for understanding. They give numerous research references that indicate that traditional methods are less effective than once thought in developing understanding.

Because of the widespread call for using alternative teaching strategies instead of depending almost totally on traditional lectures, there are - as there should be - frequent discussions of what students should know and be able to do. Most of the reports mentioned above recommend areas to de-emphasize and instructional strategies to use less frequently, as well as areas to emphasize and strategies to use. While there is some agreement that changes are needed, there is not at present universal agreement on what these should be. There has been some backlash against reform texts and methods of instruction. Movements such as "Back-to-Basics" even demand that high schools and colleges return to traditional methods of instruction. It is not uncommon to hear about "Math Wars" and lawsuits over the use of newer approaches.

What is needed to help college faculty and administrators decide what is best for students? One answer might be a high-level call within the mathematics community and related disciplines for the use of particular instructional strategies and knowledge. The Mathematical Association of America (MAA) has recently initiated a project entitled "Mathematics and Mathematical Sciences in 2010: What Should Graduates Know?" In this project, MAA hopes to develop in association

with other disciplines a consensus on many issues. Meetings such as this one at West Point and others planned for the coming year, which bring together mathematicians and those from other disciplines help, also develop consensus and plans. However, in these efforts, care should be taken that it is not just "preaching to the choir."

Another answer may be more research and evidence showing that students learn more or can perform better with the newer methods and materials. One well-known researcher at a recent meeting, however, made the observation that it is primarily in the United States where there is a constant demand for more research, more proof, and more accountability. In most other industrialized countries, involved constituencies such as academia, government, and labor sit at a common table and look toward the future, and decide what educational practices will best meet national goals. Perhaps this is due to a more centralized system of education in other countries. It may also be because many citizens of the United States seem to look more to tradition and espouse the philosophy, "If it ain't broke, don't fix it," when it comes to educational reform and ignore the evidence that is may indeed be broken.

A third answer may be increased funding by college administrators, state and local funding agencies, business and industry, and government agencies such as NSF. Departments need increased funds for technology, professional development for faculty, student assistants, laboratory space, and other related items. While just devoting resources to a projects will not assure success, the lack of resources can often doom it to failure.

A fourth potential answer is extensive support mechanisms and professional growth opportunities for faculty to assess and implement recently developed educational materials, emerging technologies, and teaching methods. [6] *Crossroads in Mathematics* [4] has a chapter on the implications of reform methods on faculty development, departmental considerations, advising and placement, laboratory and learning center facilities, technology, assessment of student outcomes, program evaluation, and articulation.

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Looking towards the future, the following two issues were identified by DUE as exciting and challenging prospects for undergraduate education for the 21st century.

First, the increased recognition of the value of a research-base to the understanding of learning will lead to answers to six key questions:

- What are the critical factors, optimal environments, necessary boundary conditions, and resultant indicators for effective instruction within the various SMET disciplines?
- What are the social, cultural, and institutional factors that affect participation in SMET fields by individuals and demographic groups; and how are they mitigated or optimized?
- What are the effects of teaching and learning technologies on instruction, student learning, and student critical thinking?
- What are the indicators of success and attainment in SMET education and how are these correlated with input and output measures?
- What new pedagogic theories and techniques might be effectively employed within SMET disciplines?
- How will educational research inform education for adult and life-long SMET learning?

Second, the focus on "educational complexity" must be extended from pre-college to undergraduate education. Our nation's increasingly complex social systems, the increasing

number of high school students who continue on to college, and the public desire to achieve accountability from a higher education system whose costs appear to spiral ever upward will result in the assignment of responsibilities that higher education has not previously had to assume. There will be greater demands for:

- the correlation of undergraduate education with employment outcomes, and
- social and support services as well as service-learning opportunities to provide greater attention to the affective component of learning, and coordination of these activities across academic departments, institutions, and levels (pre-college, two-year, baccalaureate, and graduate).

As we enter a new century, the time being dedicated to exploring where we have been and where we wish to go is a wise investment. The mathematics community should take the lead in articulating the major educational questions that need to be resolved and should do so in concert with other disciplines. Answers to these questions should be informed by past successes and a clear vision for the future.

References:

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