Problem of the Week, Fall 2006 10 November 2006

Circle One: **Cadet** Faculty Student (non USMA) Faculty (non USMA) Other

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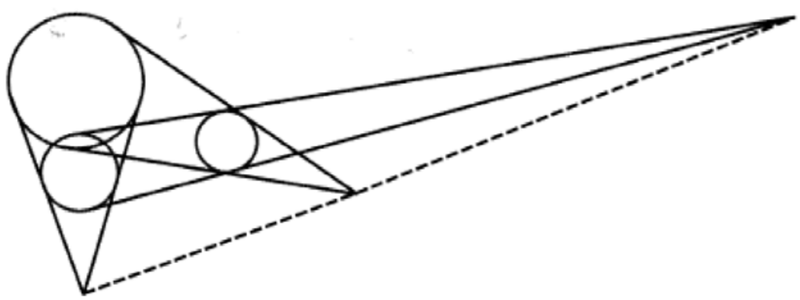
Problem 10

The "focus" of two circles of different size is the intersection of two lines, each of which is tangent to both circles, but does not pass between them. Thus three circles of different radii (but none contained completely in another) determine three foci. Prove that the three foci lie on a line.



This theorem is called the Monge and d'Alembert Three circles Theorm 1 and can be explained using Desargues' Theorem. Let the centers of the circles be names A,B,C, forming a triangle. Let 1, 2, 3, be the names of the intersections of the common tangents to Triangle ABC. Inside triangle 123 the lines A1,B1,C1 serve as bisectors of angles 1, 2, 3, respectively. Therefore, the three lines intersect at the center of triangle 1,2,3. This shows that the two triangles ABC and 123 are perspective form a point. By Desrgues' Theorem, they are also perspective from a line.

Works Cited

1. J. McCleary, An Application of Desargues' Theorem, *Mathematics Magazine*, Vol. 55, No. 4 (Spet., 1982), 233-235.