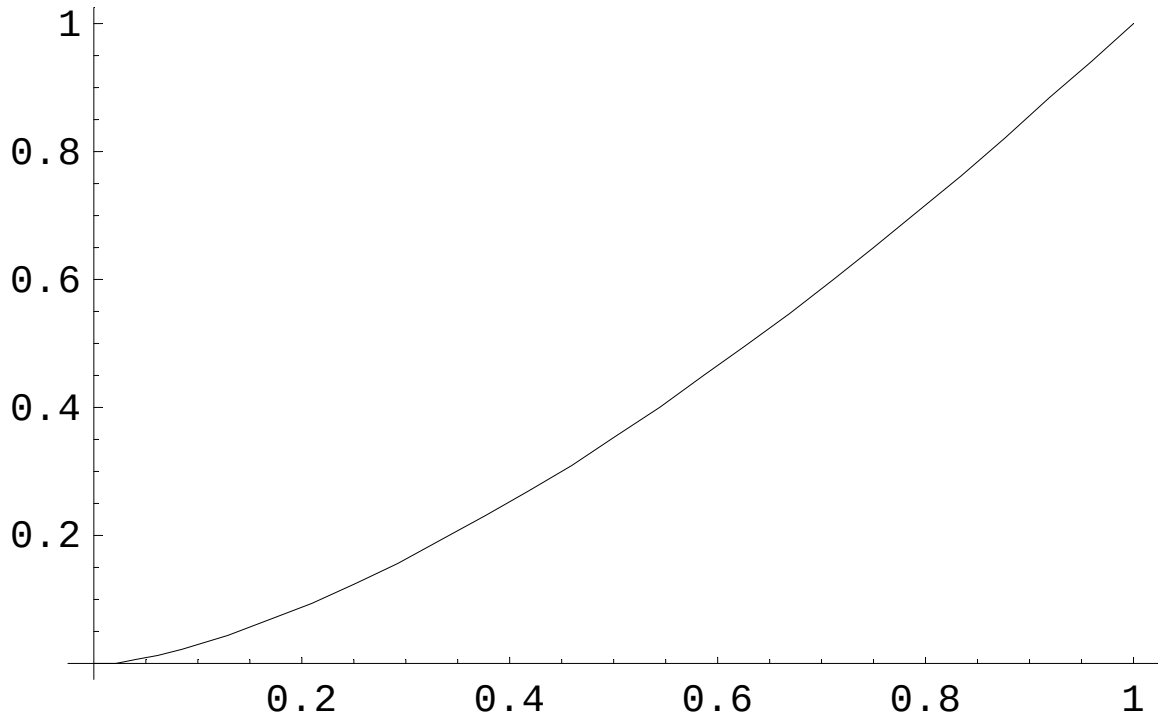
CDT Scott Baker
Co. H4, 2009

The given curve is plotted below using Mathematica:

```
f[x_] = √x^3;
 Plot[{f[x]},{x,0,1}]
```



For the line tangent to the curve to be 45˚, it must have a slope of 1, meaning the standard equation for a line, y=mx+b, is simply y=x+b. The only remaining question is where the line tangent to f(x), at point (x,y), crosses the y axis (the value for b).

Since the slope of the line tangent to the curve is equal to the derivative for any value in the domain, 1 must be both the slope of the tangent line and the value of the derivative for some value of *x*. To find this value, I set the derivative equal to 1, and solved for *x* using mathematica:

$$f'[x] = \frac{3x^2}{2\sqrt{x^3}}$$

```
Solve[f'[x]==1]
```

$$\left\{\left\{x \to \frac{4}{9}\right\}\right\}$$

The last step is to determine the length of the curve from *x*=0 to x=4/9. These values will become the limits of the integral in the Arc Length formula:

CDT Scott Baker
Co. H4, 2009

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

With the given and calculated values substituted, the answer becomes:

$$L = \int_0^{\frac{4}{9}} \sqrt{1 + \left(\frac{3x^2}{2\sqrt{x^3}}\right)^2}\, dx = \frac{8}{27}(-1 * 2\sqrt{2}) \approx 0.541756$$

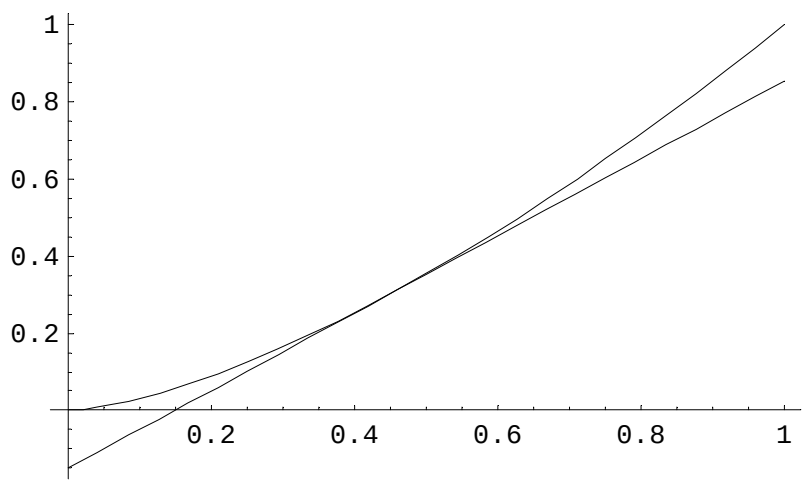
So, the final answer is about 0.541756.

<u>Extra Note</u>:

Knowing that the *x* value of the tangent point is 4/9, we now have half of the point (x,y) (see above). Substituting this value into the original function, we get $f(\frac{4}{9}) = \sqrt{(\frac{4}{9})^3}$. Using this point, $\left(\frac{4}{9}, \sqrt{(\frac{4}{9})^3}\right)$ with a slope of 1, the point-slope formula is used to determine the equation of a line tangent to a curve:

$y - \frac{4}{9} = 1 * (x - \sqrt{(\frac{4}{9})^3})$ which can also be written as $y = x - \frac{4}{9} + \sqrt{(\frac{4}{9})^3}$.

Graphically, this curve and the original curve can be seen below:

```
In[17]:= tangent[x_] = x - 4/9 + Sqrt[(4/9)^3]
```

```
In[27]:= Plot[{f[x], tangent[x]}, {x, 0, 1}]
```