

Problem of the Week, Fall 2006 12 October 2006

Circle One: Cadet Faculty  Student (non USMA)  Faculty (non USMA)  Other

Last Name (Please Print): Faber, Case Western Reserve University

Problem 6

Let  $a$  be the first number you obtain in two consecutive fair die throws and  $b$  the second. What is the probability that the graph of the function  $y = (x - a)(x - b) + 1$  does not intersect the x-axis?

Solution:

First, expand:

$$y = x^2 - ax - bx + ab + 1 = x^2 - (a + b)x + (ab + 1)$$

This function will not intersect the x-axis when the discriminant is negative.

$$[-(a + b)]^2 - 4(ab + 1) < 0$$

$$a^2 + 2ab + b^2 - 4ab - 4 < 0$$

$$a^2 - 2ab + b^2 - 4 < 0$$

$$(a - b)^2 < 4$$

$$|a - b| < 2$$

This occurs for pairs:

$$(a, b) =$$

$$(1, 1), (2, 1), (1, 2), (2, 2)$$

$$(3, 2), (2, 3), (3, 3), (4, 3)$$

$$(3, 4), (4, 4), (5, 4), (4, 5)$$

$$(5, 5), (5, 6), (6, 5), (6, 6)$$

16 pairs total out of 36 possible a,b pairs; a probability of 4/9.