

Problem 7: Aquaville

USMA D/Math Problem of the Week

Submission Deadline: November 1, 2007 at 1600

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Problem Statement:

A biological agent has been released in a country with four major cities: Aquaville, Gotham, Wanington, and Biltmore. Authorities are trying to figure out where and when the agent was released. The citizens of the country follow very precise travel patterns. They only travel on Sundays, and the percentage who travel between various cities is exactly the same every week, as shown in the following diagram:

Currently, authorities have found 15917 people in Aquaville, 16686 in Gotham, 12754 in Wanington, and 4643 in Biltmore who were exposed to the agent. Determine where and when the agent was released.

Answer: Five weeks ago, the agent was released in both Aquaville and Wanington where there were 25,000 people each.

Solution: The matrix representing people's movement is:

$$\mathbf{M} = \begin{bmatrix} 0.7 & 0.1 & 0.2 & 0.0 \\ 0.2 & 0.7 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.6 & 0.4 \\ 0.0 & 0.1 & 0.1 & 0.4 \end{bmatrix}$$

\mathbf{M} represents where people are after 1 week. \mathbf{M}^2 is, then, where people are after 2 weeks. \mathbf{M}^{-1} (\mathbf{M} inverse) represents on iteration backward through time. Starting with the final population vector (of those exposed), we can work backward until a "starting point" is found. The final population vector is:

$$\mathbf{P} = \begin{bmatrix} 15917 \\ 16686 \\ 12754 \\ 4643 \end{bmatrix}$$

Iterating backward, we find $(\mathbf{M}^{-1})^5 \mathbf{P} = \begin{bmatrix} 25000 \\ 0 \\ 25000 \\ 0 \end{bmatrix}$ which indicates that Aquaville and

Wanington each had 25,000 exposed, and Gotham and Biltmore each had 0 exposed 5 weeks ago.