Problem 8: Double Counting

USMA D/Math Problem of the Week

Submission Deadline: November 8, 2007 at 1600

Circle one:	cadet	faculty	non-usma student	non-usma faculty	other

Problem Statement:

The number of ways to arrange n sets of parentheses so that they are well-matched is the same as the number of ways to subdivide a regular polygon with $n+2$ sides into triangles. As an example, when $n = 3$, there are five parenthetical matchings:

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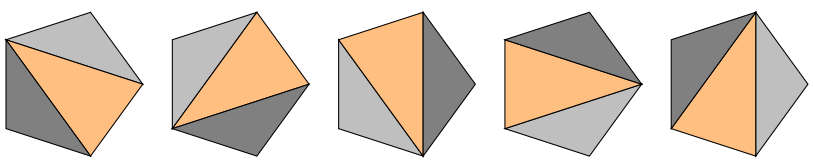
The coloring is shown for emphasis. A non-example is ( )) (( ). There are also five subdivisions of a polygon into triangles:



(a) Show this is true by finding a *one-to-one* correspondence between the set of matchings of n sets of parentheses and the set of triangulations of an $(n+2)$-sided polygon. (b) How many ways are there to subdivide a polygon with 15 sides into triangles?

Submit your answer to Dr. Elisha Peterson at ae3263@usma.edu as an attachment to your email, with the subject line **WP POTW**. Or drop your solution off in my mailbox or on my desk (with date and time please!)