

Problem 9: Numerical Solutions

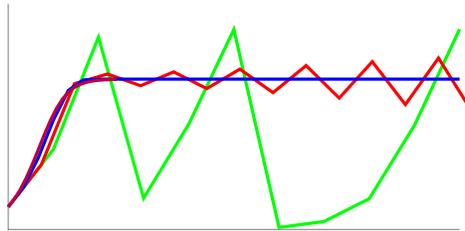
USMA D/Math Problem of the Week

Submission Deadline: November 15, 2007 at 1600

Circle one:	cadet	faculty	non-usma student	non-usma faculty	other
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Problem Statement:

The simplest approximation to a difference equation $\frac{dy}{dt} = f(y)$ is the recursion equation $y_{n+1} = y_n + f(y_n)\Delta t$ for some Δt . The solutions to this difference equation may exhibit drastically different behavior for various Δt , as shown below for $f(y) = \frac{y(E-y)}{c}$.



Consider $\frac{dy}{dt} = \frac{y(E-y)}{c}$ with corresponding approximation

$$y_{n+1} = y_n + \frac{y_n(E - y_n)}{c} \Delta t.$$

- (i) If $0 < y_0 < E$, for which choices of Δt does y_n *never* cross the line $y = E$?
 - (ii) If $0 < y_0 < E$, for which Δt is y_{n+1} *always closer* to E than y_n ?
- (For partial credit, answer the question for $E = 20$ and $c = 1$.)

Submit your answer to Dr. Elisha Peterson at ae3263@usma.edu as an attachment to your email, with the subject line **WP POTW**. Or drop your solution off in my mailbox or on my desk (with date and time please!)