Problem 17: Prediction Markets

USMA D/Math Problem of the Week - AY2008

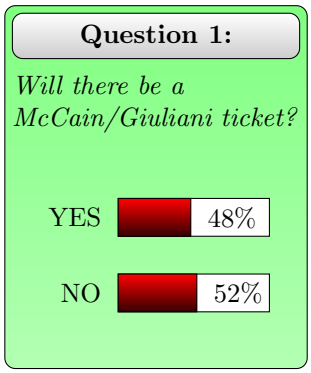
Submission Deadline: April 3, 2008 at 1600

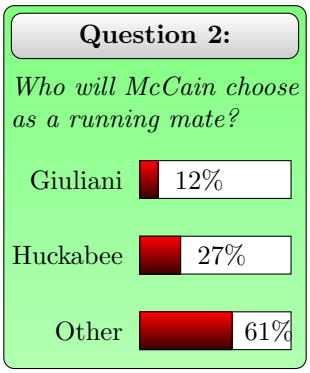
Solution by: MAJ Carlos K. Fernandez - [faculty]

Problem Statement:

In the online prediction market www.hubdub.com, you can place bets on particular event outcomes to earn "Hubdub dollars". If you place a bet of B on an outcome with current probability p, you earn $B\frac{1}{p}$ if the outcome actually occurs, and 0 otherwise. For example, if the current probability of an event is 30% and you place 30 H$ on the "YES" option, you receive 100 H$ if the event occurs, for a net gain of 70 H$. The probabilities fluctuate as users place bets on particular outcomes.

In one such market, the probability that there will be a McCain/Giuliani ticket stands at 48 percent. In another, the probability that out of several choices Giuliani will be the Republican Vice Presidential candidate stands at 12 percent.





What is the minimum amount you must bet in order to be *guaranteed* a net gain of 100 Hubdub dollars, *regardless of whether Giuliani is selected or not*?

Solution:

We begin by defining our variables.

$$
\begin{aligned}
N_1 &\rightarrow \text{Net profit from bet one.}\\
N_2 &\rightarrow \text{Net profit from bet two.}\\
B_1 &\rightarrow \text{Dollar amount of bet one.}\\
B_2 &\rightarrow \text{Dollar amount of bet two.}\\
P_1 &\rightarrow \text{Probability that bet one wins.}\\
P_2 &\rightarrow \text{Probability that bet two wins.}
\end{aligned}
$$

Now we define the equations for our net profit taking into account that we must recoup our costs for bets one and two.

$$N_1 = B_1\left(\frac{1}{P_1}\right) - B_1 - B_2 \tag{1}$$

$$N_2 = B_2\left(\frac{1}{P_2}\right) - B_1 - B_2 \tag{2}$$

Setting equations 1 and 2 equal to each other and solving for B_2, we get

$$B_2 = B_1 \left(\frac{P_2}{P_1} \right). \tag{3}$$

Replacing B_2 in equation 1 and solving for B_1, we find

$$\begin{aligned} N_1 &= B_1 \left(\frac{1}{P_1} \right) - B_1 - B_1 \left(\frac{P_2}{P_1} \right) \\ &= B_1 \left(\frac{1}{P_1} - 1 - \frac{P_2}{P_1} \right) \\ &= B_1 \left(\frac{1 - (P_1 + P_2)}{P_1} \right) \\ B_1 &= N_1 \left(\frac{P_1}{1 - (P_1 + P_2)} \right). \end{aligned} \tag{4}$$

Note that this strategy only has solutions if $0 < P_1 + P_2 < 1$. Replacing N_1, P_1, and P_2 with \$100, 52%, and 12% respectively, we find that

$$\begin{aligned} B_1 &= 100 \left(\frac{0.52}{1 - (0.52 + 0.12)} \right) \\ &= \$33.34, \end{aligned}$$

and

$$\begin{aligned} B_2 &= 33.34 \left(\frac{0.12}{0.52} \right) \\ &= \$144.45. \end{aligned}$$

We check our solution by substituting these values back into equations 1 and 2, and our net profit is in fact \$100.00.