CORE MATHEMATICS AT USMA
AY 01-02

CONTENTS:

INTRODUCTION 3

SUPPORT OF USMA ACADEMIC GOAL 4

EDUCATIONAL PHILOSOPHY 8

STUDENT GROWTH GOALS 9

MATHEMATICAL THREAD OBJECTIVES 10

PRINCIPLES AND EXPECTATIONS FOR LEARNING: 12

PRECEPTS FOR INSTRUCTORS 13

FACULTY GROWTH 14

COURSE OBJECTIVES:
MA103 15
MA104 16
MA205 17
MA206 18
MA364 19

MATHEMATICS PORTFOLIOS 20

THE MATH CLINIC 21

THE MATH LIBRARY. 21

INTERDISCIPLINARY LIVELY APPLICATIONS PROJECTS (ILAP's) 22

LIAISON PROFESSORS 24

MATHEMATICAL RECALL KNOWLEDGE 25

REQUIRED SKILLS IN SCIENTIFIC COMPUTING 29

REQUIRED MATHEMATICAL SKILLS FOR ENTERING CADETS 33

HISTORY OF THE DEPT OF MATHEMATICAL SCIENCES AT USMA 35

THIS PAGE INTENTIONALLY LEFT BLANK

CORE MATHEMATICS AT USMA
AY 01-02

INTRODUCTION

This document is designed to inform USMA faculty in the Math/Science/Engineering (MSE) Departments (and interested others) about the core mathematics program. Inside you will learn what mathematical skills and concepts you can expect from your cadets as they progress from admission to the end of the core math sequence. You will see some of the philosophy and growth goals that we have adopted in order to develop a diverse group of high school graduates into college juniors who are prepared to succeed in an engineering stem or in higher-level disciplinary study. You will read about our Interdisciplinary Lively Applications Projects (ILAP) and Liaison Professor programs. These programs aim to achieve a more integrated MSE experience for cadets by promoting coordination and collaboration between the Department of Mathematical Sciences and other academic departments. You will also learn the details of our program for identifying and reinforcing required mathematical skills for entering cadets.

We have included a detailed summary of course objectives for each core math course for this academic year. As part of our educational philosophy, we recognize that progression of mastering the conceptual knowledge is an inefficient process requiring periodic review, practice, and consolidation. Therefore, we recommend that courses that rely heavily on portions of this conceptual material identify those portions to the cadet at the beginning of the course, and then reinforce student understanding as appropriate.

Also of special interest is the list of Mathematical Recall Knowledge. This is a modest list of basic facts that the MSE Committee has decided should be memorized by each cadet. Within the core math program, cadets encounter tests of current and accumulated recall knowledge. For MSE courses that rely heavily on some subset of these recall skills, we recommend that you identify them to your cadets at the beginning of the course, and then reinforce (and test) them as appropriate.

Mathematics is the process of transforming one form into another form in order to reveal additional insight. This can be as simple as transforming an array of numbers into a sum; a function into its derivative; or a computer network into a bipartite graph. A historian who can transform the data of secondary sources into information that reveals the interactions of an earlier society is a mathematician in disguise! It is essential that we provide our cadets an experience where they can appreciate and apply the power of transformations to solve real problems.

Technologies are providing new transformations and have made other transformations obsolete. Hence, mathematics is changing! While new technologies allow us do innovative things today, experience strongly suggests that much of it will be only partly relevant, sometimes misleading, and occasionally wrong. We know that the fundamentals change more slowly. Our role is to continually find the appropriate balance between technology and the fundamentals as the mathematics evolves.

We hope you find the information in this booklet useful in understanding your cadets. We welcome your feedback on how we can better coordinate our programs, and on what information we can include here in order to help you succeed as a teacher who uses the tools from core mathematics. The Department of Mathematical Sciences updates this document annually in coordination with the MSE Committee. Please direct any comments to the former for inclusion in the next edition.

GARY W. KRAHN
COL, Professor USMA
Head, Department of Mathematical Sciences

DISTRIBUTION:

3 Dean of the Academic Board
5 Dept of Behavioral Sciences & Leadership
35 Dept of Chemistry
55 Dept of Civil and Mechanical Engineering
40 Dept of Electrical Engineering & Computer Science
10 Dept of Geography and Environmental Engineering
100 Dept of Mathematical Sciences
35 Dept of Physics
10 Dept of Social Sciences
40 Dept of Systems Engineering
30 Faculty in Advanced Civil Schooling
30 Interview Candidates @ MAA/AMS Joint Meetings

SUPPORT OF USMA ACADEMIC GOAL

The Role of Core Mathematics in the USMA Education

The core math program supports several of the Academy academic program goals. Its primary contribution, however, is in support of the Math, Science, and Technology Academic Program Goal:

Graduates understand and apply the mathematical, physical, and computer sciences to reason scientifically, solve quantitative problems, and use technology.

The Learning Model

Structure of cadet experience (domain of cadet learning): Core mathematics and science education at USMA include both the acquisition of a body of knowledge and the development of thought processes judged essential to a cadet's understanding of the fundamental ideas and principles in mathematics, science, technology, and engineering. This foundation affords opportunities for graduates to progress in their development as life-long learners who are able to formulate intelligent questions, research answers, reach logical conclusions, make informed decisions, and study other mathematical and scientific-based disciplines, such as engineering and economics. This foundation is critical in the development of future leaders for the highly technical Army envisioned in Joint Vision 2020 and for our nation's technological development.

Process of cadet experiences (planned activities): The emphases of the learning model for this goal are at the conceptual and fundamental skill levels, where the main thrusts are for cadets to internalize the unifying framework of mathematical and scientific concepts and to learn the basic skills to apply these concepts to problem solving. Cadets learn to integrate theory and practice, moving from an initial emphasis on theory to a greater emphasis on practical application. To enhance conceptual understanding, cadets study major scientific principles and concepts from several perspectives and at different levels. Cadets learn these principles and concepts through various means such as reading, discussion, laboratory discovery, and computer manipulation. This conceptual understanding, along with knowledge of the essential information of the scientific disciplines (terminology, facts, notation, procedures, etc.) and the development of critical thinking, facilitates the presentation of higher-level concepts in later courses. Skill development and retention requires practice and reinforcement at all stages of student growth. Cadets constantly interconnect and apply concepts to representative problems from mathematics; computing; physical, social, behavioral, earth, and life sciences; and engineering. Both disciplinary and interdisciplinary application problems develop cadet experience in the use of technology, problem solving, mathematical modeling, scientific reasoning, and technical communications (written and oral). This growth of cadets' learning from basic knowledge, through concepts, thinking, and problem solving, enables the learning model and its program design to be progressive and hierarchical. Therefore, the learning processes and connections of the courses and their goals and objectives are critical for cadets to accomplish this academic goal.

Content of cadet experiences (substance of material): Cadets who successfully complete the core mathematics and science program understand the fundamental principles and thought processes underlying: discrete and continuous, linear and nonlinear, and deterministic and stochastic mathematics; the theory and practice of computing; the concepts of earth sciences including terrain and atmospheric analysis; physical, organic, inorganic, and analytical chemistry; classical mechanics; electromagnetism;

optics; and economics. Cadets also develop a curious and experimental disposition; possess the ability to formulate intelligent questions; learn how to find and use appropriate information, references, and technology to research answers and make decisions; perform critical and creative thinking; and effectively communicate their ideas and results. Cadets perform these activities in disciplinary, interdisciplinary, and multidisciplinary settings. Achievement of this academic goal explicitly prepares cadets in many underlying areas of engineering education and therefore, supports the goal of learning the engineering thought process. In particular, cadets experiences in the core mathematics program contribute to the following areas of engineering education as outlined in the ABET Engineering Criteria 2000: apply knowledge of mathematics, science and engineering; design and conduct experiments, analyze and interpret data; identify, formulate and solve problems; understand professional and ethical responsibilities; effectively communicate; recognize the need for and ability to engage in life-long learning; and a knowledge of contemporary issues.

Learning Model Summary

The learning to achieve this goal is progressive and this progression can be divided into levels of understanding that create a hierarchical structure. In summation, the hierarchical learning model to support this goal requires cadets to:

- learn the essential information and fundamental skills associated with the core undergraduate subjects studied in mathematics, computer science, chemistry, physics, economics, and earth science through drill, practice and reinforcement.

- develop logical and critical thinking skills in scientific reasoning and mathematical modeling through creative work in solving complex problems.

- build a strong foundation in mathematical and scientific concepts, processes and structure through experimentation, discovery, discussion and reinforcement and appreciate the value of building on this foundation as life-long learners.

- learn to formulate and solve applied problems and connect principles and concepts from different scientific disciplines, including moral-ethical concerns, to obtain an interdisciplinary approach to problem solving.

The two major underlying threads that support and connect the learning model require cadets to:

- communicate effectively their ideas and results that involve mathematics, science, and technology.

- use technology (computers, calculators, and laboratory instruments) to analyze, experiment, discover, and resolve scientific and quantitative principles.

Major Goals of the Core Mathematics Program

To support this effort, the major goals of the core math program are as follows:

MA103 -- Discrete Dynamical Systems and an Introduction to Calculus:

?? Use skills to manipulate and simplify functions, expressions, and equations.
?? Model changing behavior with discrete dynamical systems.
?? Use the processes of iteration, equilibrium, stability, and behavior analysis to understand and solve problems.
?? Use technology to discover, experiment, and solve applied problems.
?? Explain the concepts of linearity, function, and limit.
?? Appreciate the relevance of discrete mathematics in problem solving by investigating interdisciplinary application problems and presenting results.

MA104 -- Calculus I

?? Use the concepts of continuous change and accumulation to solve applied problems.
?? Model changing behavior with differential equations.
?? Use and relate the symbolic, graphic, numeric, and word representations of behaviors, functions, and models.
?? Use technology appropriately to gain insight into a solution of a problem.
?? Appreciate the relevance of continuous mathematics in problem solving by investigating interdisciplinary application problems and presenting results.

MA205 -- Calculus II

?? Use visualization of vectors, curves, and surfaces to help investigate problems.
?? Use coordinate representations, parameterization, and vector algebra to model problems in $?^3$
?? Extend the ideas of change, optimization, and accumulation to $?^3$.
?? Survey the mathematics needed uniquely in each cadet's Major/FOS (for MSE majors/FOSs) or Engineering Stem (for others).
?? Use appropriate analytic, technological, and software tools to solve interdisciplinary applied problems and present the results to others.

MA206 -- Probability and Statistics

?? Explain the basic concepts of probability, statistics, and random variables.
?? Use the fundamental probability distributions to model applied problems.
?? Question results critically, make informed decisions, and present technical arguments.
?? Appreciate the relevance of stochastic mathematics in problem solving by investigating interdisciplinary case studies and presenting results.

EDUCATIONAL PHILOSOPHY

The Role of Core Mathematics Education at USMA

The mind is not a vessel to be filled but a flame to be kindled. -- Plutarch

Core mathematics education at USMA includes both acquiring a body of knowledge and developing thought processes judged fundamental to a cadet's understanding of basic ideas in mathematics, science, and engineering. Equally important, this educational process in mathematics affords opportunities for cadets to progress in their development as life-long learners who are able to formulate intelligent questions and research answers independently and interactively.

At the mechanical level, the core math program seeks to minimize the memorization of a disjoint set of facts. The emphasis of the program is at the conceptual level, where the goal is for cadets to internalize the unifying framework of mathematical concepts. To enhance understanding of course objectives, major concepts are interpreted both numerically and graphically to reinforce the symbolic presentations. This visceral understanding facilitates the use of these concepts in downstream science and engineering courses.

Concepts are applied to representative problems from science, engineering, and the social sciences. These applications develop cadet experience in modeling, and provide immediate motivation for developing a sound mathematical foundation for future studies. Additionally, these applications demonstrate future opportunities for the cadets and aid them in developing their interests.

Inherently, progressive cadet development dictates that education is an inefficient process and we must provide the cadet time for experimentation, discovery, and reflection. Viewed from this perspective, the core mathematics experience at USMA is not a terminal process wherein a requisite subset of mathematics knowledge is mastered. Rather, it is a vital step in an educational process that enables the cadet to acquire more sophisticated knowledge more independently. Within this setting, review, practice, reinforcement, and consolidation of mathematical skills and concepts are both necessary and appropriate, both within the core math program and in later science and engineering courses.

The cadet who successfully completes the USMA core mathematics program should have a firm grasp of the fundamental thought processes underlying both discrete & continuous mathematics, both linear & nonlinear mathematics, and both deterministic & stochastic mathematics. The cadet should also possess a curious and experimental disposition, as well as the scholarship to formulate intelligent questions, to seek appropriate references, and to independently and interactively research answers. Most importantly, cadets should understand the role of mathematics – the power of transformations.

STUDENT GROWTH GOALS

Transforming High School Graduates into College Juniors

The greatest good you can do for [students] is not just to share your riches but to reveal to [them their] own. -- Benjamin Disraeli

When cadets enter the core math sequence, they are only a few months past their high school graduation. They represent a variety of backgrounds, levels of preparation, and attitudes toward problem solving. At the end of the core math sequence, these same cadets will have chosen an upper-division engineering sequence and a major (or field of study), will be halfway through the requirements for a BS degree, and will be aggressively and successfully tackling the more synthetic and open-ended challenges of their engineering science and engineering design courses. One of the responsibilities of the core math program is to move this diverse group of entering cadets from the former state to the latter. Our vehicle for doing this is the set of student growth goals outlined below. We feel that success in the MSE program at the Academy requires that, over a period of time, each entering cadet must mature in attitude toward the nature of problem-solving and mathematics, that each must develop and grow confidence in the processes that allow them to successfully learn and use mathematics, and that each must develop facility in the skills and arts that allow them to apply mathematics and to collaborate with others. Careful coordination of the four core math courses allows a cadet to grow in each of the following areas. The result is a cadet who is prepared to perform the synthesis and meet the open-ended challenges required by their chosen engineering stem.

ATTITUDES:

Mathematics is a <u>language that</u>:
- possesses numerical, graphical, and symbolic aspects
- adds structure to ideas
- facilitates synthesis

Mathematics is <u>thinking that</u>:
- is logical
- requires only a few principles to be internalized

Learning Mathematics is an <u>individual</u> responsibility that:
- requires the motivation to learn
- requires effort
- requires time
- requires interaction with others

Learning Mathematics requires an <u>experimental disposition</u> that:
- requires a curious mind
- requires the ability to recognize patterns
- requires the ability to conjecture
- requires the ability to reason by analogy

PROCESSES:

In studying Mathematics cadets learn:
- to be confident and aggressive problem-solvers
- to think mathematically (discrete, continuous, deterministic, stochastic)
- good scholarly habits toward progressive cadet independence
- to apply math to the real world

SKILLS/ART:

In studying Mathematics cadets learn:
- to communicate mathematics orally and in writing
- modeling as the art of applying mathematics to the real world
- to use the computer and calculator as tools for learning and problem solving

MATHEMATICAL THREAD OBJECTIVES

A Framework for Student Growth

One of the ways in which we weave together the four core math courses to attain our student growth goals (previous page) is through the use of Mathematical Threads. These threads allow us to incorporate the student growth goals by translating them into specific, measurable course objectives. Thread objectives fall into two sets: learning threads and content threads.

LEARNING THREAD OBJECTIVES

We have established learning thread objectives in five areas: Mathematical Reasoning, Mathematical Modeling, Scientific Computing, Communicating Mathematics, and History of Mathematics. The goals for the core math program in each of these threads are listed below. Each core course builds upon these threads in a progressive and integrated fashion; thread objectives for each core course are summarized at the bottom of pages 14 through 18.

MATHEMATICAL MODELING:

- Recognize when a quantitative model may be useful
- Apply the modeling process (identify the problem, make assumptions, define variables and their relationships, solve the model, verify the model)
- Test the conclusions from a model for sensitivity
- Test a model for significance

MATHEMATICAL REASONING:

- Apply mathematical concepts
    - Recognize recurring ideas in different settings
    - Apply an appropriate model and solution process
    - Interrelate numerical, graphical, and symbolic representations
- Analyze results
    - Synthesize; Conjecture; Deduce
    - Infer in situations of uncertainty

SCIENTIFIC COMPUTING:

- Appreciate the role of machines as aids in computing and in learning (exploration and discovery)
- Manipulate and analyze data
- Interrelate symbolic, numerical, and graphical representations
- Recognize the capabilities and limitations of computational aids
- Perform simple programming

COMMUNICATING MATHEMATICS:

- Communicate mathematics effectively through reading, writing, and speaking
- Express ideas clearly and logically
- Model real world problems with mathematics; interpret mathematics as information about the real world phenomenon
- Use mathematical notation and terminology effectively and accurately
- Use electronic networks to find/transmit timely knowledge from/to others

HISTORY OF MATHEMATICS:

- Appreciate mathematics as an ongoing human endeavor
- Motivate the further study of mathematics
- Know key chronology and personalities involved
- Understand the service role of mathematics

CONTENT THREADS

In an effort to link student growth through the sequential development of common mathematical content, we have identified nine content threads as concepts that we strive to weave throughout the four-course core program. These content threads are as follows:

The concept of **function**, both in discrete and continuous mathematics, including single variable, multivariable, and vector-valued applications, is the fundamental basis of exploring relations among objects. A function is a relation between two sets that associates a unique element of the second set with each element of the first set.

Limits should take a sequential approach and build on the ideas of a stable equilibrium value or a long-term value. A limit must be viewed as the value that is approached increasingly close by a function or a sequence. Also, the content needs to support single variable and multivariate limits, and limits of accumulation.

Rate of change is a major topic throughout MA103, MA104 and MA205. Discrete Dynamical Systems facilitates the development of the derivative from a discrete approach. In calculus, cadets model continuous change using derivatives and differential equations. The core content also addresses a multivariate approach to include the gradient and directional derivative.

Accumulation should cover both discrete and continuous perspectives, whether in single variable or multivariable settings. Required topics are Riemann integrals, numerical integration, the Fundamental Theorem of Calculus (relating integrals and derivatives of continuous functions) and iterated integrals.

Vectors, in both two- and multi-dimensions, should be explored throughout the four-course program. Taking a multivariate approach from the beginning facilitates adept applications of vectors in future courses

Approximation should be supported by graphical, numerical and technological approaches to estimation, and to solutions of problems without closed-form solutions.

Visualization should be motivated with exemplary graphics in the text, application of technology, and problems of an exploratory nature.

Solution representations should include graphic, numeric, analytic, and descriptive (word) forms throughout.

Models are abstractions of processes or phenomena. The mathematical content threads form the basis of the mathematical concepts applied in modeling. The mathematical learning threads are the processes used in analyzing, solving and communicating the solution of the model. Cadets in core mathematics will experience models categorized as:

- discrete and continuous (difference equations and ordinary differential equations)
- deterministic and stochastic (probability and statistics)
- linear and nonlinear (analytic, graphical, and numerical representations)
- single variable and multivariate (linear systems and multivariate equations).

PRINCIPLES AND EXPECTATIONS FOR LEARNING

Cadets & Instructors

Teaching Principles: The following principles guide the entire core math program:

Small section sizes (15 to 18 cadets)
Interactive teaching
Active classroom participation
Daily preparation by each cadet
Frequent and timely feedback to each cadet
Availability of additional instruction
High standards of performance required of each cadet

Instructor Expectations of Cadets: As cadets progress through the core mathematics program, they grow both mathematically and academically. To help the student grow in this manner, we hold the following expectations of each cadet:

Cadet Responsibility for Learning:

Possess a foundation in the basics
Come prepared to class
Put forth a scholarly effort
Work to potential
Participate in the instruction and discussion
Seek assistance when needed
Learn one thing about mathematics every day

Student Growth and Development:

Manage one's time effectively
Organize work - to include course portfolios
Meet suspenses of turn-in requirements
Submit scholastic work of quality - to include spell-checking
Document properly
Reason and think through problems
Communicate - to include checking e-mail
Use technology effectively
Display appropriate standard of conduct (military courtesy, appearance, etc.)
Connect math to everyday problem-solving and to other coursework

Cadet Expectations of Instructors: The instructor certainly has a role in student growth. Here is what each cadet can expect from his or her core math instructor:

Facilitation of discussion and learning
Challenging problems
Applications that demonstrate the relevance of the cadet's mathematical education
Activities that allow the cadet to participate in learning
Timely feedback
Fairness in grading
Assistance/help
Enforcement of standards

PRECEPTS FOR INSTRUCTORS

The following precepts typify the ideas that each mathematics instructor is asked to reflect on as he or she sets the tone in the classroom and leads the cadets through the our math program.

Support the missions of USMA and the Department of Mathematical Sciences. This is accomplished by understanding and working toward the goals of USMA and the Department.

Exemplify the highest standards of your profession. Instructors are the professional, intellectual, and ethical role models that we expect our cadets to want to emulate.

Develop these same high standards in the cadets that you teach, coach, or mentor. Our responsibilities go beyond the teaching of mathematics to support the total development of our cadets as future military officers.

Support and strengthen the Honor Code. The code provides the foundation upon which to develop each cadet.

Be an expert in the material that you teach. Thorough knowledge and preparation is paramount to being an effective instructor.

Use mathematics instruction to develop each cadet's reasoning power, independence of thought, and effectiveness of written and oral expression. These attributes are required for life-long learners.

Aid in student growth. Know where cadets are in terms of skills, attitudes, and scholarly habits, and help them progress in these areas.

Develop cadet interest in mathematics by expanding your own mathematical interest, sharing your mathematical work and experiences, and demonstrating the applications of mathematics.

Continuously evaluate each cadet and provide accurate, frequent feedback. This is the very purpose of having small classes and is that aspect of the "Thayer System" that has endured.

Grade constructively. Grading is an inherently subjective endeavor and should be used to recognize good work, as well as to identify poor work. A goal in grading is to provide constructive feedback. It is not to be used as a disciplinary measure.

Maintain a positive, interesting and professional atmosphere in the classroom. The Classroom environment molds and reflects cadet attitudes for learning. Make it a lively center for active student learning. Classroom time is precious – make it useful!

Help cadets recognize that mathematics can be useful and fun. The fun comes from hard work, enlightenment, and satisfaction of accomplishment.

Work to develop the academic potential of each cadet. These young men and women are here because of their potential. Inspire them to be successful and to achieve their goals.

FACULTY GROWTH

The Department of Mathematical Sciences' mission is two-fold: to provide cadets with a broad mathematical education and to provide for the professional growth and development of its faculty. A majority of our instructors are new MS or Ph.D. degree-holders that are only with us about three years. Our mission is to provide them the opportunity to grow as teachers and researchers. We have five faculty development workshops (FDW) that support faculty teaching the four core courses and those designated as course directors. These workshops vary in length from a six week first summer FDW to a two-afternoon course director/course leader FDW. The course FDWs include policy discussions, strategic overviews, technology familiarization, and explanations of "how" that course fits into the student growth model, as the instructor prepares to teach that course. The department provides Instructors with research opportunities in applied mathematics or operations research with the Army or other government agency. Typically, instructors can also dedicate one of their summers solely to research activities.

Our teaching faculty consists of 67 professors, associate professors, assistant professors, and instructors. The majority of our faculty members are rotating military instructors. These individuals usually have a newly acquired master's level degree in applied mathematics or operations research. Their instructor load is three sections of a 4.5 credit hour course, or four sections of a 3.0 credit hour course. The section sizes in our core courses are small; about 16-18 cadets.

We currently have seven rotating civilian positions. These positions consist of both Title 10 and Davies Fellows. Most of these individuals have a recently earned doctorate level degree. These faculty members are hired on three year, non-renewable contracts and serve as assistant professors. The fellowships are administered in conjunction with the National Research Council (NRC) and the Army Research Laboratories (ARL). These assistant professors are involved in our Center for Faculty Development in Mathematics as they establish themselves in the mathematical community in terms of research and teaching. The Title 10 and Davies Fellows teach primarily in our core courses.

Our associate and full professors are our core course leaders and mathematics elective leaders. Each of the core courses is under the supervision of one of our Senior Faculty members (assistant, associate, or full professor). In addition to supervising the daily operation of the core course, they provide a vision for the course; identifying how it should change over time and how it fits in with other programs in the department and at the academy. They are also responsible for faculty development. This is an especially great challenge given the diverse backgrounds and diverse needs of the department's faculty. Additional responsibilities include maintaining fairness and objectivity of their course, preparing projects and testing instruments, and implementing and executing end-of-course surveys. Senior professors direct the Center for Faculty Development.

Each year the department selects rotating military faculty members (current or former) for sponsorship in civil schooling in pursuit of a Ph.D. Upon completion of their doctorate, these individuals return to the Department for a second three-year tour. These officers generally already hold a technical functional area specialty (such as Operations Research or Research & Development Management) within the Army. Their doctoral program is designed to support both a second tour in the Department and more specialized service to the Army in subsequent Army tours. The department often asks second tour military PhD faculty to execute some of the department's senior faculty duties, as well as assist in teaching in both the core and the electives programs.

Our senior faculty usually teaches the math electives. The class size is usually smaller than that of the core -- from 2 -16 cadets per section. All of our electives are 3.0 credit hours. The typical load is three or four sections. Electives are taught to approximately 300 cadets per year comprised of our mathematics majors (about 15-25 per year), our operations research majors (about 20-30 per year), and cadets from our serviced departments.

MA103 - DISCRETE DYNAMICAL SYSTEMS
AND INTRODUCTION TO CALCULUS
AY 01-02 - Class of 2005
COURSE OBJECTIVES

1. Introduce the cadets to mathematical modeling and reasoning, and lay the foundation for calculus and differential equations. This first course in the core mathematics sequence uses discrete structures to introduce the idea of a limit and to introduce the derivative as an instantaneous rate of change. The course is broken down into three primary blocks of instruction. The block objectives are as follows.

A. Discrete Dynamical Systems (DDS).
(1) Model problems involving change at discrete intervals (populations, decay, interest rates).
(2) Solve DDS numerically (through iteration), graphically, and analytically.
(3) Analyze the long-term behavior of a system.

B. Systems of DDSs with Matrix Algebra.
(1) Model problems involving related or competing structures (voting trends, market share, predator-prey).
(2) Familiarize cadets with the linear algebra skills required to solve these problems (matrix multiplication, inverses, determinants, eigenvalues and eigenvectors, and elementary row operations).
(3) Solve systems of DDSs numerically through iteration, graphically, and analytically.
(4) Analyze the stability of a system of DDS's and interpret its long-term behavior.

C. Introduction to Differential Calculus.
(1) Model continuous problems involving growth and motion.
(2) Understand the derivative as a limit, as an instantaneous rate of change, and as the slope of the tangent line to a curve at a point.
(3) Use basic rules of differentiation, to include those involving powers, constant multiples, sums, and exponentials.
(4) Solve first order initial value problems as an introduction to differential equations.

2. Develop cadets' modeling abilities through classroom activities and through group projects, the latter requiring from 48 hours of work by each group member. Each project requires cadets to analyze a system, make assumptions, model the system, solve the model, and then verify and analyze the results.

3. Develop cadets' abilities to employ calculators and computers as analytical aids.
A. Calculator (TI-89): Graph functions; solve equations; find determinants, eigenvalues and eigenvectors of matrices; solve linear systems of equations.
B. Computer Algebra System (Mathcad): Iterate solutions of DDSs; create tables of function values; graph functions; solve systems of linear equations; find determinants, eigenvalues and eigenvectors; compute limits; symbolically evaluate derivatives.

4. Practice technical expression through student writing. Articulate and interpret technical results on projects in formatted technical reports. Complete writing requirements throughout the semester, to include essays on written examinations.

MA104 - CALCULUS I
AY 01-02 - Class of 2005
COURSE OBJECTIVES

1. Expose cadets to problems in the continuous domain involving a single independent variable, and introduce them to methods for analyzing and solving these types of problems.

A. Differential Calculus
(1) Model problems involving motion, related rates, and optimization.
(2) Analyze properties of functions using the 1st & 2nd derivatives.
(3) Refresh understanding of limits, continuity, and derivatives.

B. Integral Calculus
(1) Model accumulation problems involving motion and area.
(2) Recognize the definite integral as a limit, and approximate via numerical integration.
(3) Understand the Fundamental Theorem of Calculus.
(4) Solve integration problems using the technique of substitution.

C. Differential Equations (DEs)
(1) Model problems involving growth/decay, motion, spring-mass systems, heating/cooling, and mixing.
(2) Be acquainted with graphical (slope and direction fields) and numerical (Euler's method) techniques for analyzing DE's.
(3) Analytically solve 1st order DE's via separation of variables.
(4) Use the method of eigenvalues and eigenvectors to analytically solve systems of linear, constant coefficient DE's.
(5) Analytically solve linear, constant coefficient, homogeneous DE's via the characteristic equation.
(6) Analytically solve linear, constant coefficient, nonhomogeneous DE's via the method of undetermined coefficients.

2. Develop cadets' modeling abilities through classroom applications, homework, and mini-projects, as well as through one or two major projects each requiring 6-8 hours of work. Each project requires understanding a realistic situation, making assumptions, using the computer, and writing a technical report. The technical report includes a summary of results, as well as appendices of the process of problem solving using analytical, graphical, numerical and verbal support.

3. Develop cadets' abilities to employ calculators and computers as analytical aids.
A. Calculator (TI-89): graph algebraic and elementary transcendental functions, evaluate definite integrals, symbolically differentiate and integrate, find roots, plot slope fields of DE's.
B. Computer Algebra System (Mathcad): tabulate data for approximating solutions to definite integrals and differential equations, find limits, graph algebraic and elementary transcendental functions, evaluate definite integrals, symbolically differentiate and integrate, find roots, plot slope fields of DE's, solve systems of equations - complete two graded homework mini-projects using Mathcad.

4. Develop cadets' written and verbal skills. Analyze, interpret, and articulate results of course projects (#2 above) in written technical reports. Present analysis and results of problems and projects verbally in class. Complete essays during the semester and one course-end essay.

MA205 - CALCULUS II
AY 01-02 - Class of 2004
COURSE OBJECTIVES

1. Analyze problems in the continuous domain involving one or more independent variables.
   A. 3D Space Geometry (aka Vector Algebra)
      (1) Recognize and be able to sketch conic sections.
      (2) Parameterize curves and analyze parameterized functions.
      (3) Understand vectors and be proficient in their use.
      (4) Understand and be able to apply the concepts of dot product and cross product.
      (5) Visualize and be able to find the equations of lines and planes in $?^3$.
      (6) Be able to find the distance between points, lines, and planes in $?^3$.
      (7) Gain skill, experience and understanding in visualizing different mathematical representations.

   B. Motion in 3D Space (aka Vector Calculus)
      (1) Become proficient in vector parametrization.
      (2) Understand, compare, and contrast the notions of path intersection and object collision.
      (3) Apply single variable calculus to vector functions.
      (4) Use vector functions to model and analyze motion problems, to include distance traveled.

   C. Change in More than One Variable (aka Multivariate Calculus - Differentiation)
      (1) Extend the concepts of calculus to multivariable functions.
      (2) Understand and visualize surfaces, gradients, and directional derivatives.
      (3) Find and classify extrema.
      (4) Optimize multivariable functions subject to equality constraints.

   D. Accumulation in More than One Variable (aka Multivariate Calculus – Integration)
      (1) Use multiple integrals to model and analyze simple physical problems
      (2) Evaluate double integrals.
      (3) Be able to convert points and equations between Cartesian and polar forms (e.g., to evaluate double integrals)

   E. Mathematics Specific to Cadets' Majors and Engineering Stems - Cadets are sectioned by Major/FOS (MSE) or Engineering Stem (HPA), and taught from one of six syllabi developed in conjunction with partner disciplines.

2. Develop cadets' modeling abilities through classroom activities and two group projects each requiring 6 to 8 hours of work per cadet. Each project poses open-ended questions, requiring cadets to use mathematical modeling techniques and technology to solve real-world problems.

3. Develop cadets' abilities to employ calculators and computers as analytical aids.
   A. Calculator (TI-89): Evaluate derivatives, definite integrals; plot multivariable functions; calculate dot and cross products, solve nonlinear systems of equations.
   B. Computer Algebra System (Mathcad): Plot surfaces, and space curves; calculate partial derivatives (1st, 2d, and mixed partials); solve systems of linear and nonlinear algebraic equations; evaluate definite multiple integrals; and calculate dot and cross products and vector magnitudes.

4. Practice technical expression through student writing and oral reports. Articulate and interpret technical results from projects (#2 above) in formatted technical reports and technical oral reports, and complete essay assignments and a reflective summary during the semester.

MA206 - PROBABILITY & STATISTICS
AY 01-02 - Class of 2004
COURSE OBJECTIVES

1. Expose cadets to uncertainty and its management from a mathematical perspective.

   A. Descriptive Statistics
      (1) Recognize and interpret of different types of data (categorical or numerical)
      (2) Use visual techniques for representing data (histogram, dotplot, stem-and-leaf, boxplot).
      (3) Understand the relationships between a population and a sample.
      (4) Describe a data set (mean, mode, median, variance, standard deviation).

   B. Classical Probability
      (1) Understand sample spaces and events.
      (2) Understand the laws and axioms of probability (to include representations via calculus).
      (3) Understand mutually exclusive, collectively exhaustive, and independent events.
      (4) Understand and apply Venn diagrams and Tree Diagrams.
      (5) Understand and apply basic counting techniques.
      (6) Understand Bayes' Theorem and conditional probability.

   C. Random Variables
      (1) Define and classify random variables (discrete, continuous, and joint).
      (2) Understand and use PDFs and CDFs (uniform, binomial, Poisson, exponential, normal, *t*, $?^2$).
      (3) Use PDFs, CDFs, calculator, and computer software to find probabilities.
      (4) Find expected values and variances of random variables.
      (5) Apply knowledge of binomial, Poisson, and exponential random variables to reliability modeling.
      (6) Understand the fundamental concepts of multivariate probability: covariance, correlation, and independence.
      (7) Understand and apply the Central Limit Theorem.

   D. Point Estimation, Interval Estimation, and Hypothesis Testing
      (1) Understand and apply the basic concepts of point estimation.
      (2) Under appropriate conditions, calculate and interpret confidence intervals for the mean and/or variance.
      (3) Set up and perform hypothesis tests; understand how to calculate and interpret level of significance, *P*-values, and power.

2. Develop cadets' modeling abilities through classroom applications (#1 above) and through group and /or individual projects requiring up to 6 hours of work. Each project typically requires student interpretation of a realistic situation, requires the cadet to identify any simplifying assumptions, asks open-ended questions, requires model refinement, and requires a written technical report.

3. Develop cadets' abilities to employ technology in the form of calculators and computers as analytical aids in solving problems. Emphasis is placed on determining the appropriate calculation to perform, understanding why the calculation is performed, and interpreting the results of the calculation.

4. Improve cadets' abilities to transform raw data into information for decision makers. Further develop their skills at explaining technical material, both verbally and in writing.

MA364 - ENGINEERING MATHEMATICS
AY 00-01 - Classes of 2002 and 2003
COURSE OBJECTIVES

1. Equip cadets with some of the basic mathematical tools to be used in their engineering studies and develop their expertise in the applied mathematics process.

A. Vector Calculus
(1) Understand the utility and applications of vector fields and of vector differential and integral operations.
(2) Visualize vector fields and operations and describe them in terms of a physical example.
(3) Describe and interpret the vector differential operators and the vector integral theorems.

B. Laplace Transforms
(1) Model simple mechanical and electrical systems with differential equations.
(2) Understand the utility of the Laplace transform.
(3) Solve initial value problems with Laplace transforms.

C. Systems of Ordinary Differential Equations
(1) Understand the origins of systems of ODE's.
(2) Solve systems of ODE's by Laplace transforms and by eigenvalues.
(3) Use the solutions obtained from systems of ODE's to predict system behavior.

D. Introduction to Partial Differential Equations
(1) Understand the derivation of the heat equation.
(2) Understand the utility of Fourier series representations.
(3) Solve the diffusion equation using separation of variables.
(4) Use numerical methods to approximate the solution of the heat equation.

E. Linear Algebra
(1) Solve systems of equations.
(2) Find eigenvalues and eigenvectors.
(3) Understand conditions for diagonalizability of an n x n matrix; find the diagonalizing matrix.

2. Develop cadets' modeling abilities through course projects. These projects are in applied settings and are coordinated with the Departments of Civil and Mechanical Engineering and Electrical Engineering and Computer Science to ensure that the setting is realistic and complementary to design efforts the cadets will encounter later. Each project is undertaken in 2-3 person groups and requires that a written report be submitted including adequate supporting work (calculations, derivations, tables, and graphs) as necessary.

3. Develop cadets' abilities to use their computers as tools in the problem solving process to obtain and verify solutions, as well as to present understandable results. In the fall term, Mathcad is used for algebraic manipulation, evaluating integrals, determining eigenvalues and eigenvectors, operating on vectors, and graphing results. Matlab is introduced to Mechanical and Electrical Engineering majors in the spring term to complement the mathematics, as well as to facilitate future work in their majors.

4. Improve cadets' ability to effectively communicate the results of their work. The reports submitted for each project are required to be legible, readable, and well organized. Summary comments and supporting work are expected to be arranged to present clearly to the reader the purpose of the project, procedure used, results obtained, conclusions drawn, and recommendations made.

5. Typically, the fall semester addresses the mathematical needs of cadets majoring in Civil Engineering while the spring term addresses the needs of Mechanical and Electrical Engineering majors.

MATHEMATICS PORTFOLIOS

The Department of Mathematical Sciences requires all core course cadets to produce a portfolio of their course work from that semester. The purpose of these mathematics portfolios is two-fold:

- ?? Each portfolio is the cadet's "one-source" study guide for the term end exam for that semester.
- ?? The cadet's accumulated set of portfolios is a mathematical reference that he or she will build upon throughout the core math program, in future MSE courses, and in the core engineering stem.

In order to help achieve these objectives, portfolios typically contain at least the following:

- ?? Portfolio Title Page.
- ?? Table of Contents.
- ?? Course syllabus.
- ?? Gateway Exams/Fundamental Skills Exams.
- ?? Projects/Problem Sets.
- ?? Written Partial Reviews.
- ?? Other items (reflective summary, suggested problems, quizzes, writing assignments, etc.), as determined by the Program Director.
- ?? Other items, clearly marked and neatly organized, that the cadet would like to maintain.

These portfolios may be collected and evaluated by the instructor as a small part of the cadet's course grade.

Cadets are required to maintain all of their math portfolios throughout their academic careers at USMA. Therefore, downstream MSE Departments are encouraged to make reference to and build on the material contained in cadets' math portfolios. If cadets develop mathematical amnesia, instructors may require them to bring in their appropriate math portfolios and brief you on how they used the idea at issue in core mathematics. Instructors may also require them to rework and resubmit problems from their math portfolios that are similar to the idea at issue in your class.

Cadets find the collected set of math portfolios to be a unique documentary of their intellectual growth, as well as a valuable summary of their mathematical repertoire. In addition, we believe that downstream MSE instructors can use these math portfolios to help improve student retention and performance of math skills in your program.

THE MATH CLINIC

The Math Clinic, located in TH226, is a significant resource for cadets. It has been designed to provide a place where cadets can join other cadets to study mathematics. For many cadets, group study is a valuable form of collaborative learning.

The Math Clinic has been resourced with equipment and materials to assist cadets in their study. Some of these resources include:

- Carrels for individual study.
- Tables and blackboards for group study.
- Computers with all course software and TI-89 calculator graph links.
- Course and instructor solution books (for courses/instructors that maintain such a book).
- Reference books for core courses.
- WPR solutions (posted for some finite period of time after the WPR). The Math Clinic is not intended to replace the usual AI between cadets and their instructors. Rather, as part of the student growth model, we expect cadets to take increasingly more responsibility for their own learning, and we provide the Math Clinic as a daily study resource to facilitate this growth.

The Math Clinic is also available to cadets with mathematical difficulties in other Departments. If you, as an instructor in another MSE Department, have a cadet with a mathematical problem (either remedial or advanced) that you think can be best addressed by a math instructor, please refer the cadet to the Math Clinic for assistance. This referral process often works best if you close the loop by contacting your departmental Liaison Professor (p.23) to discuss the nature of the need and its resolution.

THE MATH LIBRARY

The Department of Mathematical Sciences library is located in Thayer Hall, Room 256. The library holds over 5500 volumes and numerous periodicals. There is also a reserved book section in the departmental library holdings. Cadets may use the library at any time during the duty day. Cadets may work at the individual study carrels or at large tables. Computers in the library have all course software and TI-89 calculator graph links.

INTERDISCIPLINARY LIVELY APPLICATIONS PROJECTS (ILAP's)
AY 01-02

The Department of Mathematical Sciences believes that cadets learn mathematics by doing mathematics. Traditionally, this has meant daily work that enables cadets to practice and explain their problem-solving skills, which enables instructors to gauge the cadet's understanding. While this practice continues, the advent of calculators, computers, and associated software now provides opportunities for cadets and teachers to create and explore more sophisticated problems. In developing these "more sophisticated problems" mathematics instructors here, as elsewhere, have naturally turned to mathematical applications in nature, science, engineering, economics, as well as to other fields including physical education and psychology. At the same time, teachers in these disciplines who were obtaining these new tools were concerned about cadet proficiency with these new tools. Thus, mathematics faculty are often engaged in conversations on how to best prepare cadets to wield these new weapons of discovery and learning. We formalized this process in our Interdisciplinary Lively Applications Project (ILAP) program.

ILAPs have been a part of the core mathematics program since AY 92-93. The goals of the ILAP program are as follows:

- Excite cadets with the power of mathematics as a tool in describing, investigating, and solving applied problems.
- Excite cadets about the fields that become accessible to them as they master more mathematics.
- Acquaint downstream departments with their future customers, allow these departments to demonstrate the utility of mathematics within their discipline, and involve these departments in exciting these cadets at an early stage of their development.
- Enhance inter-departmental cooperation and make the four-year cadet experience more cohesive.

The ILAP program operates under the supervision of the Academy's Math, Science, and Engineering (MSE) Committee as part of a National Science Foundation project involving a consortium of 10 schools. Each year, the MSE Committee designates downstream ("application") Departments and particular mathematics core courses to coordinate project development. Faculty members from the two departments formulate a suitable project in an applied area of interest in which a particular mathematical concept being studied in the core course is used.

Every core math course has one to three projects during the course of the semester that demonstrate realistic applications of the mathematics being studied. In some cases, when the project is assigned to the class, the application department presents an introductory lecture (or video) explaining the project. When the project has been completed, the application department returns to present a concluding lecture (or video) reviewing the formulation and problem solving process, highlighting the important concepts being used, and illuminating extensions of the problem in their discipline. ILAPs that have been used in the core mathematics courses include the following:

MA103:

1-D Heat Transfer in a Bar – D/CME (Thermo) – 931
Pollution Levels in Lake Shasta – D/GEnE – 931
Tank Battle Direct Fire Simulation – D/SE – 931
Pollution Levels in the Great Lakes – D/GEnE – 941/942
Smog in the LA Basin – D/Chem – 941
Car Financing – D/SocSci (Econ) – 942
Pollution Levels in the Great Lakes – D/GEnE – 951
Car Financing – D/SocSci (Econ) – 951
Making Water in Space – D/Chem – 971
Brusselator – D/Chem – 981
Bioaccumulation of methylmercury – D/Chem – 001

MA104:

Aircraft Ranges under Various Flight Strategies – D/CME (Aero) – 932
From Bungee Cords to the Trebuchet – D/CME (Vibes) – 932
Oxygen Consumption and Lactic Acid Production – DPE – 942
From Wing Resonance to Basketball Rims – D/CME (Vibes) – 942
Patient Scheduling & Profit Management – D/SocSci (Econ) – 952
Car Suspension System – D/C&ME (Vibes) – 952
Fighting Forest Fires – D/GEnE – 951/952
Flow Rates of A Reservoir Over a Dam – D/CME – 951/952
Parachute Jumping – D/Physics – 951/952
Modeling Engineer Operations in a Nation Building Scenario – D/CME – 971/972
Cut/Fill and Bridge Abutment/Span Computation – D/C&ME – 971/972
Railway Headwall Design – D/CME – 971/972
Water Contamination and Treatment – D/GEnE – 981/982
Forest Fire / Water Tank Properties – D/CME – 991/992
Earthquake Analysis of Water Tank – D/CME – 991/992
Dissolved Oxygen – D/GEnE – 011/012

MA205:

Vehicular Collisions – D/Physics – 951
Aircraft Utilization – D/SE – 951
Airborne Parachutists – D/Physics – 952
Patient Scheduling & Profit Management – D/SocSci (Econ) – 952
Tank Production – D/SocSci (Econ) – 961
Parachute Panic – D/Physics – 961
Math, the National Pastime? (Baseball) – D/Physics – 971
Cobb–Douglas Problem – D/SocSci (Econ) – 971
LAPES (Drop Zone) Problem – D/Physics – 971
Shuttle Problem – D/Physics – 981
Satellite Problem – D/CME – 981
Vector analysis of truss members – D/CME – 991
Modeling Bangalore Production (DDS /SV &MV Calculus) – D/SocSci (Econ) – 991
MTA Metrorail Connections – D/Soc Sci (Econ) – 992
Storage Tank Optimization – D/GEnE – 992
Flight of a TOW Missile – D/Physics – 001
Least Squares Regression of Data – D/Physics – 001
Counterbattery Target Acquisition – D/Physics – 011
Modernizing Camera Product Lines – D/SocSci (Econ) – 011
Incoming Interception in BMD – D/Physics – 021
Treating Motor Pool Runoff – D/GEnE -- 021

MA206:

Pollution Levels in the Great Lakes – D/GEnE – 951/952
Statistical Analysis of Hudson River Pollution Data – D/GEnE – 981
Markov Chain Model of Dow Jones Industrial Average – D/SocSci– 982
Statistical Analysis of Elementary Circuits – D/Physics – 982*
Call for Artillery – D/EECS - 981
The Dam Problem – D/CME – 992*

*MST ILAP: a traditional ILAP that is also a capstone project helping the Academy assess student growth near the end of the core Math, Science, and Technology program.

The ILAP program embodies the belief that early mastery of mathematics skills produces in cadets the realization that they can indeed formulate and analyze interesting problems arising in engineering, science, business, and many other fields. Our experience is that this realization increases cadet motivation. We and the application departments have seen benefits of this increased motivation as cadets move from core mathematics into their disciplines.

Our ILAP material is public domain – any reader interested in these materials can find many of the ILAPs at the Department of Mathematical Science's web page (www.dean.usma.edu/math/), or can contact the ILAP coordinator, LTC Michael Jaye, for more information.

LIAISON PROFESSORS

"Servicing What We Sell"

Most departments at USMA rely on at least some portion of the core mathematics program to prepare cadets for studies in their Department. Additionally, the Department of Mathematical Sciences has found that the study of mathematical principles and methods is almost always made easier by considering problems in applied settings with realistic scenarios. In order to facilitate success in both of these areas, the Department of Mathematical Sciences has instituted a liaison program in which a tenured faculty member is designated as the Liaison Professor primarily responsible for coordination with a particular client Department. The designations for this academic year are as follows:

DEPARTMENT	LIAISON PROFESSOR	PHONE	OFFICE
BS&L (Human Factors)	LTC Steve Horton	x5610	TH237
Chemistry	MAJ Alex Heidenberg	x5608	TH225
C&ME	LTC Mike Jaye	x5218	TH250
EE&CS	LTC Mike Phillips	x5870	TH238A
GEnE	LTC Mike Jaye	x5218	TH250
Physics	COL Joe Myers	x5611	TH233
Soc Sci (Econ)	LTC Steve Horton	x5610	TH237
Systems Eng	LTC Andy Glenn	x5988	TH251

The electives program director, LTC Michael Jaye has primary responsibility for oversight of the Liaison Professor program.

The major focus of this program is to achieve a more integrated student MSE experience by promoting coordination and collaboration between the Department of Mathematical Sciences and the other MSE Departments. The role of the Liaison Professor is to serve as the principal point of contact for members of his client department. The Liaison Professor fields questions, accepts suggestions, and works issues from the client department with respect to course material, procedures, timing, and any other matters of mutual interest. Additionally, the Liaison Professor is a first point of contact for course directors in the Department of Mathematical Sciences who are looking for examples and applications appropriate to their course and need referrals.

This program is not intended to preclude closer coordination (for instance, at the course director or instructor level), but rather is intended to provide a continuing source of information and input at the senior faculty level who can ensure that inter-departmental cooperation is accomplished across several courses in a consistent fashion, as required.

UNITED STATES MILITARY ACADEMY
MATHEMATICAL RECALL KNOWLEDGE

The following constitutes a basic mathematical vocabulary that will be built upon during each cadet's four-semester core mathematics experience and in his or her future math/science/engineering courses. Once each of these basic ideas has been covered in class, each cadet can be required to reproduce, upon demand in any future lesson of any math/science/engineering course, that idea exactly as shown here. Annotated beside each heading or item are the course number and lesson number that denotes the exact point in time when the cadet is responsible for each item (e.g.: 103/1 means that each cadet is responsible for knowing that item the first lesson of MA103). These items are recall knowledge - cadets are also required to be proficient in the more conceptual, less-verbatim ideas and skills reflected in each core math course's Course Objectives section of this document.

ALGEBRA (103/1)

1. $ax^2 ? bx ? c = 0 \Rightarrow x = \dfrac{?b ? \sqrt{b^2 ? 4ac}}{2a}$
2. $a^b \cdot a^c = a^{b+c}$
3. $(a^b)^c = a^{bc}$
4. $\dfrac{a^b}{a^c} = a^{b?c}$
5. $y = \log_b x \;\; => \;\; x = b^y$
6. $\log_b b^x = b^{\log_b x} = x$
7. $\log_b x^a = a \log_b x$
8. $\log_b ac = \log_b a ? \log_b c$
9. $\log_b \dfrac{a}{c} = \log_b a - \log_b c$
10. $\log_b a = \dfrac{\log_c a}{\log_c b}$

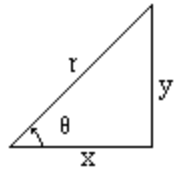
ANALYTIC GEOMETRY (103/1)

Rectangle:	Area = lw	Perimeter = $2l + 2w$
Circle:	Area = $?r^2$	Circumference = $2?r$
Rectangular Solid:	Volume = lwh	Surface Area = $2lw + 2lh + 2hw$
Cylinder:	Volume = $?r^2\, l$	Surface Area = $2?r^2 + 2?rl$
Sphere:	Volume = $\frac{4}{3} ? r^3$	Surface Area = $4 ? r^2$

Distance between (x_1, y_1) and (x_2, y_2) = $\sqrt{(x_1 ? x_2)^2 ? (y_1 ? y_2)^2}$

TRIGONOMETRY (103/1)

With reference to the right triangle:



2? radians = 360 degrees

$\sin ? = \dfrac{y}{r}$	$\cos ? = \dfrac{x}{r}$	$\tan ? = \dfrac{y}{x}$
$\tan ? = \dfrac{\sin ?}{\cos ?}$	$x^2 + y^2 = r^2$	$\sin^2 ? + \cos^2 ? = 1$
$\cot ? = \dfrac{1}{\tan ?}$	$\sec ? = \dfrac{1}{\cos ?}$	$\csc ? = \dfrac{1}{\sin ?}$

RELATIONSHIPS (MA103/1)

Corresponding sides of similar triangles are proportional

Distance = average rate ? time

DIFFERENTIATION (MA 103-104)

1. $\frac{d}{dx}(a) = 0$ (103/47)

2. $\frac{d}{dx}(x) = 1$ (103/47)

3. $\frac{d}{dx}(au) = a\frac{du}{dx}$ (103/47)

4 $\frac{d}{dx}(u ? v) = \frac{du}{dx} + \frac{dv}{dx}$ (103/47)

5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ Product Rule (103/50)

6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} ? u\frac{dv}{dx}}{v^2}$ Quotient Rule (103/50)

7. $\frac{d}{dx}(u^n) = nu^{n?1}\frac{du}{dx}$ Power Rule (103/49)

8. $\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$ Chain Rule (103/54)

9. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$ (103/51)

10. $\frac{d}{dx}(\cos u) = ?\sin u\frac{du}{dx}$ (103/51)

11. $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$ (103/49)

12. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ (104/4)

INTEGRATION (MA 104)

13. $\int a\,dx = ax + C$ (104/24)

14. $\int (u + v)\,dx = \int u\,dx + \int v\,dx$ (104/22)

15. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n ? ? 1)$ (104/24)

16. $\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$ (104/32)

17. Understand and be able to apply the Substitution Rule (104/32)

18. $\int \frac{du}{u} = \ln|u| + C$ (104/32)

19. $\int \sin(ax)\,dx = ?\frac{1}{a}\cos(ax) + C$ (104/30)

20. $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$ (104/30)

21. If f is integrable on $[a, b]$, then $\int_a^b f(x)dx ? F(b) ? F(a)$ where $\frac{dF}{dx} ? f(x)$

(Fundamental Theorem of Calculus)

VECTOR CALCULUS (MA205)

22. $|\vec{A}| = \sqrt{a_i^2 + a_j^2 + a_k^2}$ (205/6)

23. $\vec{A} ? \vec{B} ? a_i b_i ? a_j b_j ? a_k b_k ? |\vec{A}||\vec{B}|\cos ?$ (205/7)

24. $|\vec{A} ? \vec{B}| ? |\vec{A}||\vec{B}||\sin ??$ (205/8)

26. $\vec{A} ? \vec{B} ? \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix}$ (205 / 8)

25. $? f = \frac{?f}{?x}\hat{i} + \frac{?f}{?y}\hat{j} + \frac{?f}{?z}\hat{k}$ (205/33)

PROBABILITY & STATISTICS (MA 206)

27. If an event A is one of n possible outcomes among the total number N of all possible outcomes (all being mutually exclusive and equally likely), then the probability of event A is defined to be n/N and the range of $P(A)$ is [0,1], (i.e., $0 \le P(A) \le 1$). (206/3)

28. PDFs and CDFs: (206/16)

Discrete: PDF: $P(X = x) = p(x)$.

CDF: $P(X \le x) = \sum_{y \le x} p(y)$.

Continuous: PDF: $f(x)$ is used to find probabilities - $P(a \le X \le b) = \int_a^b f(x)dx$.

CDF: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy$.

29. The total accumulation of a probability distribution function is 1. (206/15)

Discrete: $\sum_{\forall x} p(x) = 1$ Continuous: $\int_{-\infty}^{\infty} f(x)dx = 1$

30. Inclusion-Exclusion (simple form): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (206/3)

31. Intersection & Conditional Probability:

$$P(A \cap B) = \begin{cases} P(A|B)\cdot P(B) & \text{(In General)} \\ P(A)\cdot P(B) & (A \text{ and } B \text{ Independent}) \\ 0 & (A \text{ and } B \text{ Mutually Exclusive}) \end{cases}$$

???????

32. Calculate and interpret the expected value (mean) of a random variable. (206/16)

Discrete: $E(X) = \sum_{\forall x} x \cdot p(x)$ Continuous: $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$

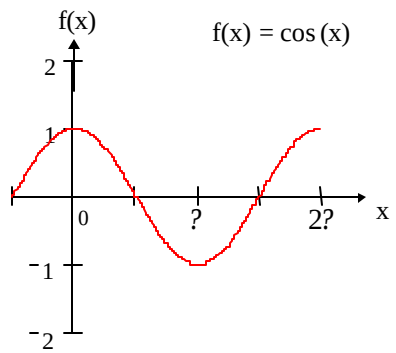
33. Calculate and interpret the variance of a random variable. (206/16)

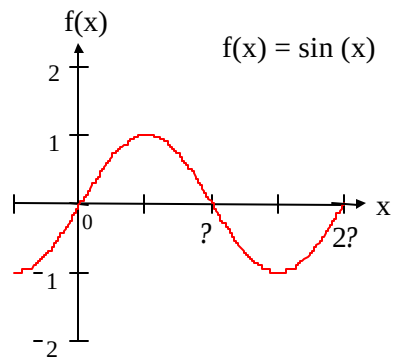
The variance is an expected value: $V(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$.

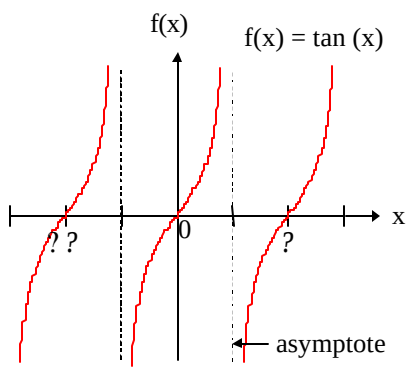
34. Percentiles of random variables including the median (50th percentile). (206/17)

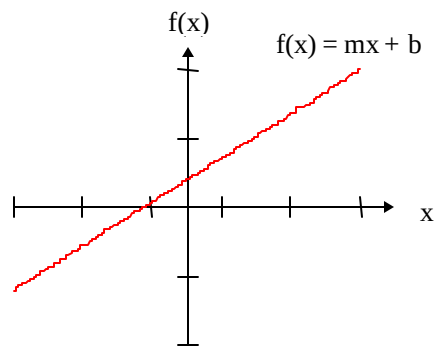
35. Central Limit Theorem: Let $X_1, X_2, \ldots, X_n$ be a random sample from *any* population with a finite mean and variance. Then, if n is sufficiently large, $\overline{X} = \sum X_i / n$ and $T_0 = \sum X_i$ are **approximately** normally distributed. (206/26)

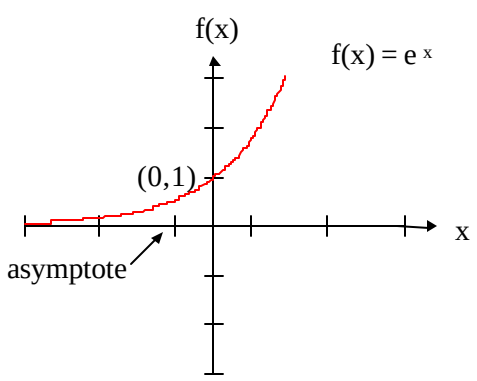
PROPERTIES OF FUNCTIONS (103/1)

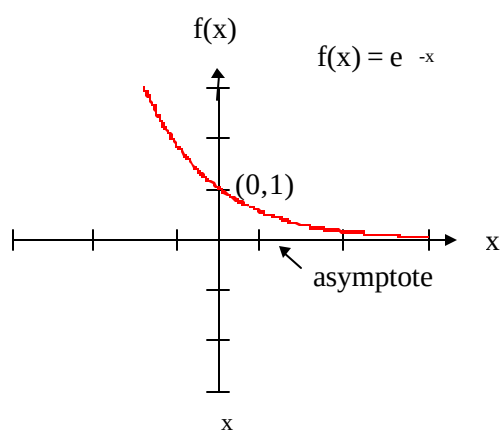


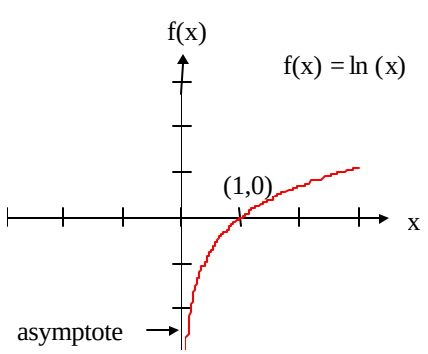












UNITED STATES MILITARY ACADEMY
REQUIRED SKILLS IN SCIENTIFIC COMPUTING

CALCULATOR SKILLS (TI - 89 / or equivalent) - MATHCAD SKILLS – MS EXCEL SKILLS

The following list constitutes the basic, required scientific computing skills that you will learn and use during your four-semester core mathematics experience and in your future math/science/engineering courses. Once each of the listed topics is covered in class, you must be able to execute this skill upon demand using both (unless otherwise specified) the calculator and Mathcad in any future lesson or course. In MA206, you must be able to execute the skills designated in Microsoft Excel. Reference books, for the respective computing system, are the chief source of information for learning these skills. Annotated beside each item are the course and lesson numbers, which denotes the exact point in time you are responsible for that particular item (e.g. 103/1 indicates responsibility for that skill following the first lesson of MA103).

1. Program a first or second order discrete dynamical system (DDS) and evaluate analytically, numerically and graphically:

E.g. Given the DDS $a(n+1) = 3\,a(n) + n^2$ (103/11)

a. Find a(100).
b. Display a table of "n" and "a(n)" values.
c. Graph the time-series plot of a(n).

2. Perform basic matrix and vector operations

a. Given 2 matrices, compute (if defined) their sum and product. (103/26)
b. Find the transpose a given matrix. (103/27)
c. Find the inverse of a given square matrix (if it exists). (103/28)
d. Find the magnitude of a given vector. (205/8)
e. Given 2 vectors, compute their dot product and cross product. (205/8)
f. Given 3 vectors, compute their triple scalar product. (205/8)

3. Perform matrix operations on linear systems of equations in two and three variables:

E.g. Given the linear system of equations,

$$\begin{bmatrix} a(n-1) \\ b(n-1) \\ c(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 2 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} a(n) \\ b(n) \\ c(n) \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

a. Find the determinant of the coefficient matrix. (103/26)
b. Find the inverse of the coefficient matrix. (103/28)
c. Find the eigenvalues and eigenvectors of the coefficient matrix. (103/32)

E.g. Given the linear system of equations,

$$\begin{bmatrix} 2 & 5 & 6 \\ 7 & 8 & 4 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$$

a. Solve the system of equations. (103/25)
b. Perform row reduction operations on the augmented matrix. (103/25)
c. Find the determinant of the coefficient matrix. (103/26)

4. Find the roots of a polynomial or transcendental function (both real and complex):

E.g. Find the roots of the functions (103/36)

a. $r^3 - 7r^2 - 14r - 8 = 0$

b. $e^x - 2 = 0$

5. Graph algebraic and elementary transcendental functions:

E.g. Graph the particular solution (103/11)

$a(k) = 2.3 \cdot 4^k - 5k^2 - 2, 0 \le k \le 25$.

E.g. Graph the following functions individually and together on the same axes. (103/8)

$y_1 = 2x^2 + 3x - 24\, e^{2x}$

$y_2 = \sin(3x) + e^x - 2\ln(x)$

6. Graph in 3-Dimensions using Mathcad (surfaces and space curves; level curves):

E.g. Graph the space curve (205/14)

$\vec{r}(t) = t^2\,\vec{i} + 2t\,\vec{j} + \sin(t)\,\vec{k}, t = 0, \ldots$

E.g. Graph the surface $z = \dfrac{x^2}{4} - \dfrac{y^2}{16}$. (205/26)

E.g. Graph the level curves for $x^2 + y^2 = k$ 205/26)

7. Find limits: (103/46)

E.g. Evaluate $\lim\limits_{x \to 5} \dfrac{x - 5}{x^2 - 25}$.

E.g. Evaluate $\lim\limits_{x \to \infty} \dfrac{2\sqrt{x} - x^{-1}}{3x - 7}$.

8. Evaluate derivatives:

E.g. Given the function $y = \dfrac{1}{x} + \sin(2x^3 - x)$,

a. Find the derivative. (103/53)

b. Plot the derivative. (103/53)

E.g. Given the function $f(x, y) = \dfrac{1}{y} + \sin(2x^3 - y)$, find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$. (205/27)

E.g. Given the function $f(x, y) = x^2 y - xy^2$,

a. Find the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial^2 f}{\partial x \partial y}, \dfrac{\partial^2 f}{\partial x^2}$. (205/27)

b. Find the gradient of $f(x,y)$ at the point (2,3). (205/33)

c. Find the directional derivative of $f(x,y)$ at the point (2,3) in the direction of $2\vec{i} - 4\vec{j}$. (205/33)

9. Evaluate definite and indefinite integrals:

E.g. Evaluate $\int_0^{?} (x^2 ? \cos(2x))\,dx$. (104/20)

E.g. Evaluate $\int (x\ln(x) ? e^x)\,dx$ (104/21)

E.g. Evaluate $\int_0^3 \int_0^2 (x^2 y)\,dy\,dx$ (205/45)

E.g. Evaluate $\int_0^1 \int_0^{y^2} (3y^3 e^{xy})\,dx\,dy$. (205/47)

E.g. Evaluate $\int_0^3 \sin\left(\frac{(2n ? 1)(x)}{6}\right) \sin\left(\frac{(2m ? 1)(x)}{6}\right) dx$, (364/33)

for $m = n$ and $m ? n$.

10. Differential Equations (DE):

E.g. Given the DE $\frac{dy}{dx} ? \cos(?x) ? e^{?x} ? 2$,

a. Plot the direction (slope) field of the DE. (104/35)
b. Graphically approximate a solution curve to the DE for $y(0) = 2$. (104/35)
c. Find a numerical approximation for $y(3)$ given $y(0) = 2$. (104/37)
d. Verify a given solution to the DE (104/31)

11. Probability and Statistics:

a. Fit a regression line to given data. (205/38)
b. Compute the sample and population mean, median, variance, and standard deviation: (206/3)

E.g. Given the following data: 3.0 2.5 3.7 2.7 3.3, compute the mean, median, variance, and standard deviation.

c. Find probabilities and percentiles for binomial, Poisson, uniform, exponential, normal, t, and $?^2$ distributions. (206/10-19, 28, 30).

E.g. Given that X follows a binomial distribution with parameters $n = 14$ and $p = .72$, find $P(X ? 11)$.

E.g. Find the 85th percentile of a normal distribution with parameters $? = 9$ and $?^2 = 25$.

E.g. Find $P(?^2 ? 15.2)$ for the case where $? = 11$ degrees of freedom.

d. Evaluate probabilities and expectations for univariate and multivariate probability distribution functions. (206/21)

E.g. Given the joint PDF

$$f(x, y) = \begin{cases} \frac{6}{5}(x ? y^2) & 0 ? x ? 1,\ 0 ? y ? 1 \\ 0 & \text{otherwise} \end{cases}$$

FIND: $P\left(0 ? x ? \frac{1}{4},\ 0 ? y ? \frac{1}{4}\right) ? \int_0^{1/4} \int_0^{1/4} f(x, y)\,dy\,dx ? \int_0^{1/4} \int_0^{1/4} \frac{6}{5}(x ? y^2)\,dy\,dx ? 0.0109$

e. Generate confidence interval for the population mean, proportion, and variance. (206/27-33).

f. Test Hypothesis on the population mean. (206/28-33).

12. Solve nonlinear systems of equations:
    a. Solving for relative extrema (205/37)
    b. Constrained optimization (Lagrange) (205/40)

13. Convert complex numbers between polar and rectangular forms: (364/12)

E.g. Express $7 ? 3i$ in polar form.

E.g. Express $6e^{i\frac{?}{3}}$ in rectangular form.

14. Compute the divergence, and curl: (364/35)

E.g. Calculate the divergence and curl of $\vec{F} ? 2xy\vec{i} ? e^{y}\vec{j} ? 2z\vec{k}$

15. Create a formal document in electronic form that includes textual, graphical, numerical, and analytical modeling and analysis. (103/11)

Microsoft Excel

1. Analyze data with descriptive statistics and descriptive plots. (206/01)

2. Simulate data (generate pseudo-random data) (206/09)

3. Compute probabilities using Excel functions or macros (206/10, 11, 16-18)

4. Perform and interpret tests for normality (206/20)

5. Construct and interpret confidence intervals (206/27-30)

6. Perform and interpret hypothesis tests (206/32-34)

REQUIRED MATHEMATICAL SKILLS FOR ENTERING CADETS

Cadets enter the United States Military Academy (USMA) with different backgrounds in mathematics. To successfully begin work in your math/science/engineering (MSE) courses at USMA, you must arrive with the knowledge of certain mathematical skills and concepts. The MSE Committee identified the items below as critical fundamental skills and concepts that all entering cadets must possess. Early in Cadet Basic Training, the Department of Mathematical Sciences will administer a "Gateway" exam to assess your fundamental skills. Therefore, we encourage you to evaluate your mathematics skills and work at remediation of any deficiencies prior to arriving at USMA.

The Department of Mathematical Sciences administers two additional "Gateway" exams during the first semester in MA103 (Discrete Dynamical Systems and Introduction to Calculus). Self-paced texts and software files offer cadets opportunities for self-remediation. Failure to pass at least one "Gateway" exam at mastery level (? 80%) may result in placement in MA100 (Precalculus) during your second semester instead of continuing into MA104 (Calculus and Differential Equations). Cadets enrolled in MA100 who do not demonstrate proficiency in the fundamental mathematics skills by the end of their first year may be separated from USMA.

Almost any high school algebra book is a suitable reference for these fundamental skills. You can obtain a DOS-based disk tutorial (3.5" format) by writing to "Department of Mathematical Sciences, USMA, West Point, NY 10996," or by calling (845) 938-4603/5673.

Note: All calculations must be done <u>without</u> the use of technology (i.e., calculator). Some examples of skills are provided in parentheses.

1. Algebra and Real Numbers.

a. Use symbols and operators to represent ideas and objects and the relationships existing between them.

b. Understand the relationship between measures of the physical world. (Velocity, distance and time: On a 40-mile car trip to Middletown, NY, you drive the first twenty miles at 40 mph and the last twenty miles at 60 mph. What is your average speed during the trip?)

c. Know and apply the following algebraic properties of the real number system: identity, associative, commutative, inverse, and distributive.

d. Express numbers using scientific notation. (Express 0.004312 in scientific notation.)

2. Radicals and Exponents.

a. Convert between radical and rational exponent form. (Transform $\frac{1}{\sqrt{x ? 2}}$ to the rational exponent form $(x ? 2)^{?\frac{1}{2}}$.)

b. Manipulate algebraic expressions that contain integer and rational exponents. (Simplify $4^{?\frac{3}{2}} ?27^{?\frac{2}{3}}$.)

3. Algebraic Expressions.

a. Add, subtract, multiply, and divide algebraic expressions. (Find the remainder when $x^3 - 7x^2 + 9x$ is divided by $x - 2$.)

b. Simplify algebraic expressions. (Expand and simplify $(x\text{-}3)(x\text{-}2)(x\text{-}1)$.)

4. Factoring / Prime Numbers

a. Write a number as the product of factors. (Write 42 as the product of prime factors.)

b. Solve for the roots of a polynomial by factoring. (Find the roots of $x^2 - 5x + 6 = 0$.)

5. Linear Equations, Inequalities and Absolute Values.

a. Solve 2 simultaneous linear equations by graphing and by substitution. (Use a graph to estimate the point of intersection of the lines $2x + 3y = 7$ and $-x + y = 4$. Verify your result using back substitution.)

b. Solve linear equations and inequalities [graphically and algebraically]. (Solve $5(3\text{-}x) > 2(3\text{-}2x)$ for x.)

c. Solve linear equations and inequalities with absolute values. (Solve $|x - 4|$? 3 for x.)

6. Polynomials and Rational Inequalities.

a. Solve simple polynomial inequalities. (Solve $x^2 + 3x + 6 > x - 4$ for x.)

b. Solve simple rational inequalities. (Solve $\frac{x \ ? \ 3}{x \ ? \ 1}$? 2 for x.)

7. Straight Lines.

a. Determine the equation of a line. (Find the equation of a straight line passing through the points (2, 1) and (5, 4).)

b. Determine the equation of a line that is parallel or perpendicular to a given line. (Find the equation of a line parallel to the line $2y - 3x = 7$ and passing through the point (1, 2).)

8. Functions.

a. Identify the independent and dependent variables of a function.

b. Determine the domain and range of a real valued function. (Find the domain and range of the real valued function $g(x) = \frac{1}{x^2 \ ? \ 2}$.)

c. Evaluate a function at a point. (Given $f(x) = 3x^2 - 2x + 4$, find $f(2a)$.)

d. Evaluate composite functions. (Given $h(r) = 3r^2$ and $g(s) = 2s$, find $h(a+2) - g(2a)$.)

9. Quadratic Equations and Systems

a. Solve for real and complex roots using the quadratic formula. (Find the roots of $3x^2 + 2x = -1$.)

b. Solve a system of quadratic equations in 2 variables by substitution. (Solve the system $y = 3-x^2$ and $y = 4 + 2x^2 - 2x$.)

10. Trigonometric Functions.

a. Define each of the 6 trigonometric functions (sin(?), cos(?), tan(?), cot(?), sec(?), csc(?)) in terms of the sides of a right triangle. (cos(?) = x/r where x is the adjacent side and r is the hypotenuse.)

b. Define each of the 6 trigonometric functions in terms of sin(?) and cos(?). (tan(?) = $\frac{\sin \ ?}{\cos \ ?}$.)

c. Know the domain and ranges for the sine, cosine, and tangent functions.

d. Translate angle measures between degrees and radians. (Write 120 degrees as a radian measure.)

e. Memorize and use the 30/60/90 and 45/45/90 degree reference triangles.

f. Know and apply the trigonometric identity $\sin^2(?) + \cos^2(?)=1$. (Simplify the expression $2\cos^2(?) + \sin^2(?) - 1$.)

11. Logarithmic and Exponential Functions.

a. Know the relationship between logarithm and exponential functions [$y = \log_a x$, $a>0$, a? 1, is the inverse of the function $y = a^x$; $\log_a x=y$? $a^y=x$)] (Evaluate $\log_3 27$.)

b. Know the properties of the logarithmic and exponential functions and use them to simplify logarithmic expressions. (Express as a single logarithm: $.5 \log_{10} x - \log_{10} y$.)

c. Solve simple logarithmic and exponential equations. (Solve the equation $3^{x+4} = 4$. for x.)

12. Graphs and Graphing.

a. Graph equations and inequalities. (Sketch a graph of the function $f(x) = 3x^2 - 2x + 7$ for $1<x<5$.)

b. Properly label a graph (axes, intercepts, asymptotes, and roots).

c. Know the general characteristics and shapes of the graphs of polynomial, logarithm, exponential and trigonometric functions.

d. Transform the graph of a known function. (From the graph of $f(x)$, graph the function $g(x) = 2f(x) - 3$.)

13. Analytic Geometry.

a. Know and apply the distance formula between 2 points. (Find the distance between the two points A(1,2) and B(-5,-3).)

b. Know and apply the circumference and area formulas for circles, triangles, and rectangles. If you double the radius of a circle, what happens to its circumference?)

c. Know and apply the surface and volume formulas for cylinders, spheres and rectangular solids.

d. Know the relationship between similar triangles. (A rectangle with base x and height 5 is inscribed in an isosceles triangle with base 10 and height 20. Determine x.)

e. Know and apply the Pythagorean Theorem to simple geometric problems. (Given a rectangle that is 4 ft by 7 ft determine the length of the diagonal.)

HISTORY OF THE DEPT OF MATHEMATICAL SCIENCES AT USMA

A teacher affects eternity; he can never tell where his influence stops. -- Henry Adams

The Department of Mathematical Sciences, USMA, has had a rich history of contributing to the education of cadets as confident problem solvers and of developing its faculty as effective teachers, leaders, and researchers. The story of mathematical education at West Point is full of interest: faculty curriculum developments, teaching methods and tools, and technological equipment. Many of the Department's advances have been exported outside the Academy to be utilized by other civilian and military educational institutions.

EARLY BEGINNINGS: The actual teaching of mathematics at West Point dates from even before the Academy was established. In 1801, George Baron taught a few Cadets of Artillery and Engineers some of the fundamentals and applications of algebra. The Academy at West Point was instituted by act of Congress and signed into law by President Thomas Jefferson on 16 March 1802. The first acting Professors of Mathematics were Captains Jared Mansfield and William Barron. They taught the first few cadets topics in algebra, geometry, and surveying.

CONTRIBUTIONS TO THE NATION: Since the Academy was the first scientific and technical school in America, the early mathematics professors at USMA had the opportunity to make significant contributions not only to the Academy, but also to other American colleges. Perhaps the most prominent contributors were the early 19th century department heads Charles Davies and Albert E. Church. The work of these two professors had a significant impact on elementary schools, high schools, and colleges across the country. Davies became the Professor of Mathematics in 1823. He was one of the most prolific textbook authors of his day, writing over 30 books from elementary arithmetic to advanced college mathematics. His books were used in schools throughout the country from grade school to college. He had tremendous influence on the educational system of America throughout the 19th century. Albert Church succeeded Davies in 1837, and served as Department Head for the next 41 years. Another influential author, he published seven college mathematics textbooks.

PRODUCING LEADERS FOR THE NATION: Faculty from the Department have been notable military leaders for the country. Robert E. Lee was a standout cadet-instructor in the Department, Omar Bradley served as an Instructor for four years, Harris Jones and William Bessell were Deans of the Academic Board at USMA for a total of 15 years, and Department Heads Harris Jones, William Bessell, Charles Nicholas, John Dick, and Jack Pollin served impressively during two world wars.

The unique technical curriculum in place at the Academy during the middle of the 19th century produced many successful mathematicians and scientists for the country at large. West Point graduates Horace Webster, Edward Courtenay, Alexander Bache, James Clark, Francis Smith, Richard Smith, Henry Lockwood, Henry Eustis, Alexander Stewart, and William Peck filled positions as professors of mathematics or college presidents at other schools such as the U. S. Naval Academy, Geneva College, University of Virginia, University of Pennsylvania, University of Mississippi, Yale, Brown, Harvard, Columbia, Virginia Military Institute, Cooper Institute, and Brooklyn Polytechnic Institute. Two mathematics department heads became college presidents after leaving USMA; Alden Partridge founded and became the first president of Norwich University, and David Douglass served as president of Kenyon College in Ohio for four years. Jared Mansfield was appointed surveyor-general of the Northwest Territory, and Ferdinand Hassler became superintendent of the United States Coastal Survey. Capable individuals such as these exported the West Point model of undergraduate mathematics education throughout the nation.

While the faculty at USMA has been primarily military, the Department has benefited from civilian visiting professors since 1976. As part of the goal for civilianization of 25% of the faculty by 2002, begun in 1992, the Department established in 1994 a Center for Faculty Development in Mathematics. This Center establishes faculty development models and curricula and provides for the development of the "Davies Fellows", who serve as rotating civilian faculty members.

HISTORICAL AND LIBRARY HOLDINGS: Sylvanus Thayer's first task before assuming the Superintendancy in 1817 was to tour the technical institutions of Europe and assess what features USMA could use to advantage. One of Thayer's many accomplishments was to obtain numerous mathematics and science books from Europe. Thayer's book collection included many of the finest books available at that time. His books provided a solid foundation for the USMA library to build upon. Today, the West Point Library has one of the finest collections of pre-20th century mathematics books in the world. Also during the middle of the 19th century, the Academy instructors used elaborate physical models made by Theodore Oliver to explain the structures and concepts of geometry. This magnificent collection of string models is still in the Department today.

CURRICULAR DEVELOPMENT: After Thayer studied the military and educational systems of Europe, he reorganized the Academy according to the French system of the Ecole Polytechnic. The Department of Mathematics faculty (which included as Professor the distinguished scientist and surveyor Andrew Ellicott, and the famous French mathematician Claude Crozet whom Thayer recruited during his European trip to bring to USMA and America his expertise in Descriptive Geometry, advanced mathematics, and fortifications engineering) combined the French theoretical mathematics program with the practical methods of the English to establish a new model for America's program of undergraduate mathematics. This program of instruction in Mathematics grew over several decades and was emulated by many other schools in the country. The initial purpose of the Military Academy was to educate and train military engineers. Sylvanus Thayer, the "Father of the Military Academy" and Superintendent from 1817-1833, instituted a four-year curriculum with supporting pedagogy to fulfill this purpose. Thayer's curriculum was very heavy in mathematics; from Thayer's time to the late 1800's, cadets took the equivalent of 54 credit hours of mathematics courses. The topics covered in these courses were algebra, trigonometry, geometry, descriptive geometry (engineering drawing), analytic geometry, and calculus. Over the years, the entering cadets became better prepared and fewer of the elementary subjects were needed. During Davies' tenure (1823-37), calculus was introduced as a requirement for all cadets, and was used in the development of science and engineering courses. The time allotted for the mathematics curriculum decreased to 48 credit hours by 1940, and to 30 credit hours by 1950. During the 1940's, courses in probability & statistics and in differential equations were introduced into the core curriculum and a limited electives program was started for advanced students. In the 1960's, department head Charles Nicholas (previously one of the organizers of the Central Intelligence Agency) wrote a rigorous and comprehensive mathematics textbook (the "Green Death") that cadets used during their entire core mathematics program. With this text, he was able to adapt the mathematics program to keep up with the increasing demands of modern science and engineering. In the 1970's, Academy-wide curricular changes provided opportunities for cadets to major in mathematics.

During the 1980's, a mathematical sciences consulting element was established that allowed faculty members and cadets to support the research needs of the Army. This type of research activity continues today in the Army Research Laboratory (ARL) Mathematical Sciences Center of Excellence and in the Operations Research Center (ORCEN). In 1990 the Department introduced a new core mathematics curriculum that included a course in discrete dynamical systems, with embedded linear algebra. In that same year, the department changed its name to the Department of Mathematical Sciences to reflect broader interests in applied mathematics, operations research, and computation.

TECHNOLOGICAL DEVELOPMENT: USMA has a long history of technological innovation in the classroom. It was Crozet and other professors at USMA in the 1820's who were the first professors in the nation to use the blackboard as the primary tool of instruction. In 1944, the slide rule was issued to all cadets and was used in all plebe mathematics classes. During William Bessell's tenure (1947-1959), the mathematics classrooms in Thayer Hall (a converted riding stable) were modernized with overhead projectors and mechanical computers. Bessell was also instrumental in establishing a computer center at West Point. The hand held calculator was issued to all cadets beginning in 1975, and pre-configured computers were issued to all cadets beginning in 1986. In recent years, the department has established a UNIX workstation lab, an NSF-funded PC lab, and has run experimental sections with notebook computers and with multimedia.

NATIONAL COMPETITION: In the spring of 1933 West Point entered an interesting competition in mathematics. After USMA defeated Harvard at a football game the previous fall, a chance remark from President Lowell of Harvard to Superintendent Jones of USMA led the two schools to arrange a mathematics challenge match between the two schools with the two competing teams each composed of 12 second-year cadets. Army was the home team, so the Harvard competitors traveled by train to West Point. All the competitors took a test written by the president of the Mathematics Association of America. The **New York Times**, which had promoted the event with a series of articles in its sports section, reported the results. The West Point "mathletes" defeated Harvard in the competition that was the precursor to the national Putnam Competition. Since its inception in 1984, the Academy has entered two three-person teams in the International Mathematics Competition in Modeling. USMA won the top prize in 1988 and 1993.

During the first half of the 1990's, the Department of Mathematical Sciences at USMA became recognized as one of the more progressive mathematics programs in the country. The Department developed a strong "7 into 4" program that is exciting as well as innovative. Throughout the 1990's and into the 21st century, the Department of Mathematical Sciences has led the nation in curricular reform and in nurturing interdisciplinary cooperation and collaboration between and among partner disciplines. Project INTERMATH, a National Science Foundation grant to develop integrated curriculum and ILAPs, assists in supporting these efforts. The USMA mathematics program has had great influence on mathematics education in America throughout its history, and strives to continue contributing to the improvement of mathematical education in America.

For a more extensive exposition on the history of the Department, refer to
http://www-internal.dean.usma.edu/math/history/usma/index.htm