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INTRODUCTION

This document is designed to inform cadets about the elective mathematics program. Inside you will learn about the opportunities available within the department of Mathematical Sciences. You will see some of the philosophy and growth goals that we have adopted in order to develop cadets in competent, confident problem solvers. We have included a detailed summary of course descriptions for each math elective course, the structure of the mathematics and OR programs, academic opportunities, and resources for this academic year.

Mathematics is the process of transforming one form into another form in order to reveal additional insight. This can be as simple as transforming an array of numbers into a sum; a function into its derivative; or a computer network into a bipartite graph. A historian who can transform the data of secondary sources into information that reveals the interactions of an earlier society is a mathematician in disguise! It is essential that we provide our cadets an experience where they can appreciate and apply the power of transformations to solve real problems.

Technologies are providing new transformations and have made other transformations obsolete. Hence, mathematics is changing! While new technologies allow us do innovative things today, experience strongly suggests that much of it will be only partly relevant, sometimes misleading, and occasionally wrong. We know that the fundamentals change more slowly. Our role is to continually find the appropriate balance between technology and the fundamentals as the mathematics evolves.

We hope you find the information in this booklet useful in understanding our program. This document is updated annually by the Department of Mathematical Sciences. Please direct any comments to the former for inclusion in the next edition.

GARY W. KRAHN
COL, Professor USMA
Head, Department of Mathematical Sciences

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Electives in Mathematics Program at West Point

EDUCATIONAL PHILOSOPHY: The Role of Elective Mathematics Education at USMA

Inherently, education is an inefficient process and time must be provided for experimentation, discovery, and reflection so that each student may achieve progressive intellectual development. Viewed from this perspective, the elective mathematics experience at USMA is not a terminal process wherein a requisite subset of mathematics knowledge is mastered. Rather, it is a vital step in an educational process that enables the student to acquire more sophisticated knowledge more independently. The review, practice, reinforcement, and consolidation of mathematical skills and concepts are necessary and appropriate, both within the elective math program and in other science and engineering courses.

STUDENT GROWTH GOALS: Transforming College Juniors into 2LT Problem Solvers

When students enter elective mathematics, they have just completed (or are about to complete) the core math program. They possess mixed insights about the utility of mathematics, are unable to read the details of a math book for meaning; are used to relying on the instructor to tell them everything they need to know; have little idea what a rigorous mathematical argument is; and represent a variety of backgrounds, levels of preparation, and attitudes toward problem-solving. At the end of their elective math program, these same students will enter an Army that expects them to be able to apply mathematical concepts and tools at the journeyman level in order to analyze quantitative scenarios of interest arising from management and engineering, to work in multidisciplinary teams and settings to accomplish specified missions, and to communicate technical results and implications to non-technical audiences. For students from other academic majors, the service elective courses must take these core math graduates and develop in them the qualities expected by their discipline: competence in the mathematical concepts and tools used by that discipline, the ability to translate back and forth between the quantifiable scenarios common in the discipline and mathematical representations, and the confidence to independently seek and use appropriate resources to analyze mathematical problems at hand. We feel that success in the MSE program at the Academy requires that, over a period of time, each entering student must mature in attitude toward the nature of problem-solving and mathematics, that each must develop and grow confidence in the processes that allow them to successfully learn and use mathematics, and that each must develop facility in the skills and arts that allow them to apply mathematics and to collaborate with others. Our vehicle for doing this is the set of student growth goals in the following outline.

Attitudes

Mathematics is a Language
- Possesses numerical, graphical, and symbolic aspects
- Adds structure to ideas
- Facilitates synthesis

Mathematics is Thinking
- Is logical
- Requires only a few principles to be internalized

Learning Mathematics is an individual responsibility
- Requires effort
- Requires time
- Requires interaction with others
- Requires proficiency with a variety of resources

Learning Mathematics requires a curious mind
- Requires the motivation to understand and learn

Learning Mathematics requires an experimental disposition
- Requires the ability to recognize patterns
- Requires the ability to conjecture
- Requires the ability to reason by analogy

Processes

Learn to be confident and aggressive problem-solvers
Learn to think mathematically (discrete, continuous, deterministic, stochastic)
Learn good scholarly habits toward progressive student independence
Learn to apply math to the real world

Skills/Arts

Communicate mathematics orally and in writing
Learn modeling as the art of describing the real world with mathematical concepts
Use the computer and calculator as tools for learning and problem-solving

The service elective courses, as well as the math and operations research major/FOS programs are carefully coordinated so that the student can grow in each of these areas. The result is a student prepared to perform the synthesis and meet the open-ended challenges required by their discipline and by the Army. It is the responsibility of the elective math program to move this diverse group of core graduates from the former state to the latter.

MATHEMATICAL THREAD OBJECTIVES: Framework for Student Growth

One of the ways in which we weave together the courses of the elective math program to attain our student growth goals is through the use of Mathematical Threads. These threads allow us to operationalize and achieve the student growth goals by translating them into specific, measurable course objectives. Thread objectives fall into two sets: learning threads and content threads.

Learning Thread Objectives

We have established learning thread objectives in five areas: Mathematical Reasoning, Mathematical Modeling, Scientific Computing, Communicating Mathematics, and

History of Mathematics. Listed below are the goals for the elective math program in each of these threads. Each elective course builds upon these threads in a progressive and integrated fashion.

Mathematical Reasoning

Interrelate numerical, graphical, and symbolic representations
Conjecture
Recognize valid and invalid logical arguments
Make rigorous arguments using deduction, induction, contradiction, and counterexample
Infer in situations of uncertainty
Grow at all intellectual levels:
- retain proficiency in fundamental skills
- practice problem solving skills
- grow more sophisticated in modeling skills
- recognize the recurring ideas of mathematics as they occur in different fields and settings
- synthesize the major themes and concepts of mathematics

Mathematical Modeling

Recognize when a quantitative model may be useful
Apply the modeling process
Identify assumptions in a model
Test the conclusions from a model for sensitivity
Test a model for significance
Interpret and articulate the implications of a model solution

Scientific Computing

Appreciate the role of machines as aids in learning (exploration and discovery) and in doing (tedious calculations, more sophisticated problems, future uses of technology)
Manipulate and analyze data
Recognize the capabilities and limitations of computational aids
Perform simple programming

Communicating Mathematics

Communicate mathematics effectively through reading, writing, and speaking
Express ideas clearly and logically
Model English with mathematics; interpret mathematics into English
Use mathematical notation and terminology effectively and accurately
Use electronic networks to find/transmit timely knowledge from/to others

The next area is viewed more as a target of opportunity than an objective area. Each course employs math history at appropriate points as a tool to motivate students by uncovering the elements it reveals of human discovery, meaning, and significance.

History of Mathematics

Appreciate mathematics as an ongoing human endeavor
Motivate the further study of mathematics
Know key chronology and personalities involved
Understand the service role of mathematics

Content Thread Objectives

In an effort to link student growth through the sequential development of common mathematical content, we have identified nine content threads as concepts that we strive to weave throughout the elective program. These content threads are as follows:

Functions

The modeling course (MA391) picks up with both discrete and continuous functions, and both deterministic and stochastic functions. Linear Algebra (MA371) continues with including multivariable, and vector-valued functions. Both tracks (Applied Mathematics and Statistics/Optimization) extend these themes with discrete and continuous stochastic functions in the Stats/Optimization track and univariate and multivariate functions in the Applied track.

Limits

Limits are presented through a sequential approach in Analysis (MA387), and ideas of behavior at singularities and at improper limits, limiting orbits, etc., are extended in the Applied track. The idea of a stable equilibrium value occurs in both tracks.

Rates of change

Rate of change is a major topic in modeling (MA391) and analysis (MA387). MA387 develops the derivative from a discrete approach, as does numerical analysis.

Accumulation

Accumulation covers both discrete and continuous perspectives, in both single variable or multivariable settings. Topics treated are Riemann integrals, numerical integration, and the Fundamental Theorem of Calculus (all in the Applied track).

Models

Models are used to introduce new majors to the various areas of mathematics in MA391. This modeling emphasis is prominent through ODE's & vector calculus (MA363), linear algebra (MA371), and numerical analysis (MA386). The learning thread common between these different courses is the process of coming up with the model, solving the model with the tools of the course, interpreting the result, and using feedback to modify the model. The models themselves are the experiences the student retains for use in later problem solving opportunities. These models can be categorized as the following:

- discrete vs. continuous (difference equations, linear algebra, and discrete RV's vs. ODE's and continuous RV's)
- deterministic vs. stochastic (applied analysis vs. probability and statistics)
- linear vs. nonlinear (linear algebra and ODE's vs. numerical analysis)
- single variable vs. multivariate (ODE's and numerical analysis vs. linear algebra).

The program seeks opportunities to use, compare, and contrast all of these models.

Vectors

Vectors are a thread that picks up in the vector calculus portion of MA363 and continues in linear algebra.

Approximation

Approximation is supported by numerical and technological approaches to estimation and solutions to problems without closed-form solutions in ODE's and numerical analysis.

Visualization

Visualization is motivated throughout the program with exemplary graphics and problems of an exploratory nature.

Solution

Solution representations include graphic, numeric, analytic, and descriptive (word) forms throughout.

PRINCIPLES AND EXPECTATIONS FOR LEARNING: Students and Instructors

Teaching Principles

The following principles guide how the department views student and faculty participation in the entire elective math program:

Faculty have a high degree of subject matter expertise in the class material
Increased assumption of academic and personal maturity on the part of students
Daily preparation for class by each student
Small section sizes for personal attention by the faculty
Interactive teaching; active classroom participation by students
Frequent formal and informal feedback to each student
Availability of additional instruction
An expectation of high performance and desire to learn on the part of students

Instructor Expectations of Students

As students progress through the mathematics electives program, they grow both mathematically and academically. The department expects the following from its students:

Student Responsibility for Learning
- Foundation in the basics -- capable of self-remediation
- Prepared for class; a professional, scholarly effort
- Work to potential, not accepting mediocre performance
- Participate in the instruction and discussion
- Take the initiative; seek assistance if needed

Student Growth and Development (linked to officer skills)
- Improvement in time management skills; meet all suspenses
- Better organization of techniques
- Improvement in reasoning/thought process
- Proper documentation of all academic work
- Improvement in communication skills, both oral and written
- High standards of conduct (military courtesy, lectures, etc.)
- Increased ability to recognize applications of mathematics in everyday events

Student Expectations of Faculty

Students may expect their faculty to take an active role in facilitating student growth:

Problems that require full understanding of material
Ability to facilitate discussion and student learning
Ability to establish relevance for the material being discussed
Activities that allow the student to participate in learning
Timely feedback on personal progress
Fairness in grading; availability for assistance/help
Establishing high standards for student performance

Why Elect More Mathematics?

The Information Age is tremendously empowering to those who are quantitatively literate. "Quantitatively literate people are capable of manipulating aspects of mathematical knowledge to understand, predict, and control situations important to their lives. Such people have the ability to reason in numerical, data, spatial, and chance settings; to integrate and apply mathematical concepts and procedural skills; and to develop and interpret models related to the problems they encounter."[1]

What Will I Be Doing in the Army With a Math Degree?

Depending upon the branch of commissioning that you select prior to graduation, you will initially be "doing" mathematics either very little or not at all. However, you will be "using" mathematical thinking right from the start in the way that you organize and understand tactical operations, personnel management, schedule training, maintenance planning and operations, and budget management, just to name a few. In its most basic state, mathematics conditions the way that you view the world around you. Perhaps more importantly, the Army's downsizing effort over the past several years has increased the amount of outsourcing it does to civilian corporations for technical advice, analysis, and assistance. While this may be expedient in the short run, it increases the need for the Army to have technically savvy "green suiters" who can act as quality control agents in the interest of the government. Math and Operations Research majors are extremely valuable to the Army. These are technically competent officers who understand the links among mathematics, engineering, operations, and complex systems.

What Advanced Education Is Available for a Math Major?

With a BS degree in mathematics, you have not yet committed to specialization, a fact that later works in your favor because mathematics is often the preferred undergraduate major for disciplines such as those listed below:

Field	**Army Career Field**
Operations Research	Operations Research/Systems Analyst (FA49)
Modeling and Simulation	Modeling and Simulation Officer (FA57)
Science	Army Scientist Program, Nuclear Research and Operations Officer (FA52)
Acquisition Corps	Research and Development Officer (FA51)
Space	Army Astronaut Program, Space Operations Officer (FA40)
Management	Contracting and Industrial Management (FA97), Human Resource Management (FA43), Comptroller (FA45),
Computers	Systems Automation Officer (FA53)
Intelligence	Information Operations (FA30), Strategic Intelligence (FA34)
Cryptology	Information Operations (FA30), Strategic Intelligence (FA34)

Graduate level education opportunities are more easily accessible for math majors than for any other major. A degree in mathematics communicates to graduate school entrance

[1] John Dossey, Distinguished University Professor of Mathematics, Illinois State University, Visiting Professor USMA: 1993-94.

committees that here is a person who has talent in analytical and logical thinking, problem solving, and who has knowledge of the language used to formulate and solve problems in management, science, and engineering. For this reason, math majors are highly competitive and successful in graduate school in such diverse areas as:

Applied Mathematics	Statistics
Economics	Business Administration
Operations Research	Industrial Engineering
Computer Science	Physics
Most Engineering disciplines	Life Sciences
Law School	

Even for those majoring in other areas, more advanced mathematical work is a clear sign of the graduate who has sought a tough, rigorous, and complete education; this is often the deciding factor for graduate schools choosing from among applicants whose records otherwise look much the same.

U.S. News and World Report made the following comment in its Nov 25, 1985 article *Advice that Can Help You Succeed On Campus*:

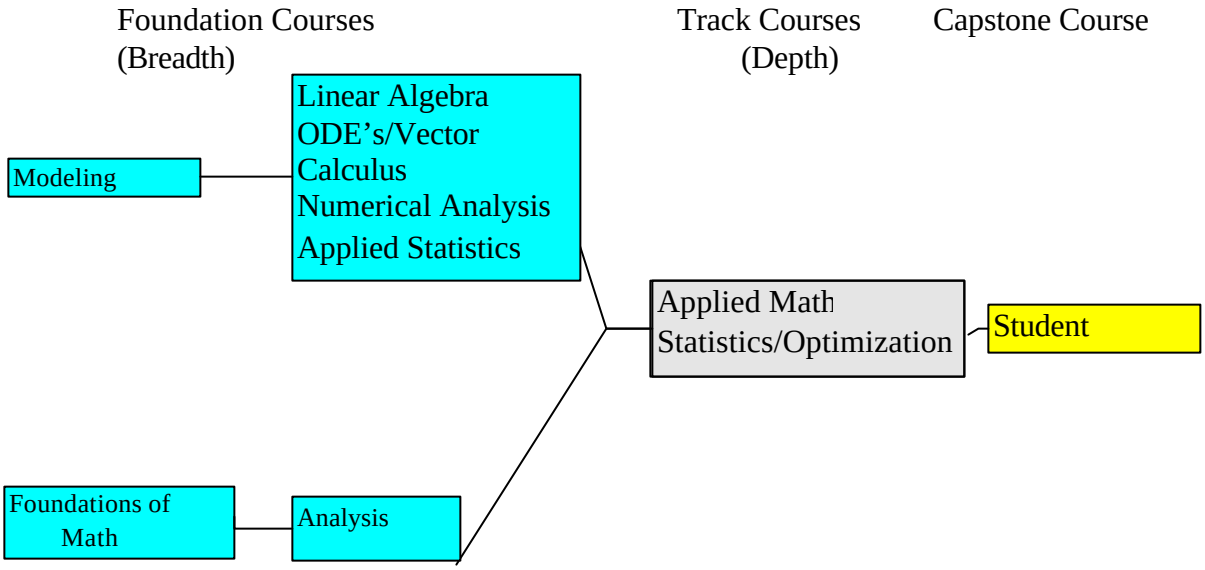
"It is quite appropriate to view college as a broadening experience, a preparation for life. Indeed, many college students do not select their ultimate career path until after they graduate. So take occupational courses if you like, but don't feel compelled to mold your major to the market."

A math major keeps you at the intersection of all the other quantitative disciplines. That means: it keeps your options open. Your choice of major is a wonderful opportunity to create your own destiny. So pick something you like and are good at!

Mathematics Major/Field of Study Structure

Cadets interested in studying mathematics and its applications may pursue either a field of study or a major in Mathematical Sciences. Either track allows the student to choose any available core engineering sequence. The Mathematical Sciences major consists of 8 specified courses and 3 courses selected by the student (8+3); the Math Sciences field of study consists of 6 specified foundation courses and 3 courses selected by the student (6+3). Cadets who have started a major may revert to the field of study by dropping two courses. The foundation courses provide breadth over the areas covered by practically every math major program across the country. The 3 courses selected by the student are normally aligned to achieve depth in either Applied Mathematics or Statistics/Optimization.

The Math Major at USMA is designed to follow the Program B recommendations of the Mathematical Association of America's Committee on Undergraduate Programs in Mathematics. This means that our program has very much of an applied orientation, but is axiomatic enough to provide appropriate preparation for future studies.



The Modeling course (MA391) and the Foundations of Math course (MA383) serve to introduce the student to the field of mathematics, and are normally the first courses taken in the program. The Modeling course introduces students to basic techniques in modeling from a variety of scenarios (deterministic and stochastic, discrete and continuous, linear and nonlinear) that are intentionally chosen to expose the students to most of the stem areas of mathematics. The Foundations of Math course introduces students to the axiomatic method, rigorous thinking, and to the various methods of proof that are used in mathematics. Because of the early placement of these two courses in the

students' programs and because of the survey component present in each, these courses give students the experience upon which to select follow-on courses in their programs.

Linear Algebra (MA371), Ordinary Differential Equations & Vector Calculus (MA363), Applied Statistics (MA376), and Numerical Analysis (MA386) are the other foundation courses, which include a significant modeling component, in addition to their significant conceptual component. These courses cover the basic theory and concepts of the material, but often use applications and ILAP's to help demonstrate the meaning and significance of the ideas involved. Mathematical Analysis (MA387) is the next axiomatic course after Foundations of Math. It begins with the basic topology of the real line and rigorously constructs from it the calculus. These foundation courses provide breadth over the same areas covered by practically every math major program across the country.

After the foundation courses, the student selects three courses of interest, usually concentrated in either Applied Mathematics or Statistics/Optimization. The courses comprising each of these two tracks are described below. Concentrating elective choices within one of these tracks allows the student to achieve depth in a chosen area of mathematics. Faculty advisors can assist with these types of choices.

The capstone course of the program is a Student Research course (MA491), normally taken in the student's last semester. Faculty propose a list of both applied and pure topics; students usually select a topic of interest from this list, but occasionally propose and have approved a topic of their own. Student topics are usually related to their track choice, and the students' faculty counselors often serve as research advisors as well. This research experience is an opportunity for students to synthesize previous material in tackling a question of their choice, and provides experience in presenting technical results both orally and in a technical report.

More information about the major and field of study programs can be found in the online Redbook at http://www-internal.dean.usma.edu/dean/docs/redbook.htm .

MATHEMATICS MAJOR SCHEDULING TEMPLATE: Applied Mathematics

Terms

	4	5	6	7	8
1	PH202	ENG SEQ	ENG SEQ	ENG SEQ	ENG SEQ
2	MA206	ENG SEQ	MA363	MA386	MA485*
3	LX2XX	MA376	MA371	MA484*	MA491
4	SS20X	MA383	MA387	MA487*	MA492C*
5	EV203	SS307	EN302	HI301	HI302
6	MA 391	PL300		LW403	

* Recommended Elective (only three electives required)

MATHEMATICS MAJOR SCHEDULING TEMPLATE: Stats/Optimization

Terms

	4	5	6	7	8
1	PH202	ENG SEQ	ENG SEQ	ENG SEQ	ENG SEQ
2	MA206	ENG SEQ	MA363	MA386	MA476*
3	LX2XX	MA376	MA371	MA381*	MA481*
4	SS20X	MA383	MA387	MA492D*	MA491
5	EV203	SS307	EN302	HI301	HI302
6	MA 391	PL300		LW403	

* Recommended Elective (only three electives required)

FOUNDATION COURSES FOR MATH MAJOR/ FOS

MA 363 VECTOR CALCULUS AND ORDINARY DIFFERENTIAL EQUATIONS

SCOPE: This course continues the study of vector calculus from MA205 through the remainder of the vector differential operations, line and surface integrals, and the vector integral theorems of Green, Gauss, and Stokes. The focus then turns to series solutions of ordinary differential equations and solving systems of ordinary differential equations. Emphasis is placed upon analyzing a variety of practical applications that give rise to ordinary differential equations. Numerical methods of solution are also studied.

MA 371 LINEAR ALGEBRA

SCOPE: This course emphasizes both the computational and theoretical aspects of linear algebra one encounters in many subjects ranging from economics to engineering. The course covers solutions of linear systems of equations, the algebra of matrices, and the utility of determinants. The foundational aspects of vector spaces and linear transformations to include linear dependence and independence, subspaces, bases and dimension, inner products, and orthonormalization are developed. This is rounded out with a detailed investigation of eigenvalues and eigenvectors as they relate to diagonalization, quadratic equations, and systems of difference equations. A software package is used to compute solutions to problems. Applications of the course material are included in the form of special problems to illustrate its wide scope.

MA 376 APPLIED STATISTICS

SCOPE: This course builds on the foundations presented in the core probability and statistics course to provide a broad introduction to some of the most common models and techniques in applied statistics. The mathematical basis for each of the models and techniques is presented with particular emphasis on the development of the required test statistics and their distributions. The texts are the same as for the core course. Topics covered include hypothesis testing, analysis of variance, categorical data analysis, regression analysis, and nonparametric methods. For the special problem, students can choose an application in their own field.

MA 383 FOUNDATIONS OF MATHEMATICS

SCOPE: This course introduces the student to the methods and language of upper division mathematics. It presents formal set theory, and introduces the student to the methods of formulating and writing mathematical proofs. Finally, it provides the student a rigorous introduction to the theory of relations, functions, and infinite sets.

MA 386 INTRODUCTION TO NUMERICAL ANALYSIS

SCOPE: This course develops an understanding of the methods for solving mathematical problems using a digital computer. Algorithms leading to solution of mathematical problems will be examined for consistency, stability, and convergence. After a brief review of calculus theory, a study of error analysis and computer arithmetic

will provide the framework for the study of the following topics: solutions of equations of one variable, solutions of linear and nonlinear systems of equations, the use of polynomials to approximate discrete data, curve fitting, numerical integration and differentiation, and the approximation of continuous functions. Special problems will incorporate computer graphics and the use of mathematical software libraries to produce numerical solutions of applied problems.

MA 387 MATHEMATICAL ANALYSIS I

SCOPE: A one-semester course providing a rigorous introduction to the calculus of a single variable. The course is designed to introduce the student to the foundations of the calculus necessary for advanced undergraduate and graduate studies in applied mathematics and engineering. Course coverage includes a treatment of the structure of the real number system, sequences, continuous functions, and differentiation.

MA 391 MATHEMATICAL MODELING

SCOPE: This course is designed to give the student the opportunity to develop skills in model construction and model analysis while addressing interesting scenarios with practical applications from a wide variety of fields. The course is divided into nine major blocks of instruction. In the first two blocks, the student is introduced to modeling using graphical methods and the modeling process. During the next block of lessons, the student learns the techniques of modeling using proportionality and constructs models which will be tested and fitted in the next block of lessons, model fitting. Next, the student studies several scenarios, which lead to models requiring optimal solutions. The course then addresses empirical model construction. In this block, real data will be used as the basis to construct a model. Next, the student uses dimensional analysis to assist in model building. A short block of lessons on simulation follows. The course concludes with the modeling of dynamic scenarios. The student learns how to use the derivative to model and analyze time dependent situations such as competitive hunter scenarios.

MA 491 RESEARCH SEMINAR IN APPLIED MATH PROJECTS

SCOPE: The student integrates the mathematical concepts and techniques learned in previous courses with the principles developed in the whole USMA Curriculum to solve a current problem of interest to the Academy or other agencies in the Department of the Army. Students may select problems from a list of suitable projects provided by the Department of Mathematics. Students will be assigned a faculty advisor who has an interest and background in the problem. Students may work individually or in small teams, depending on the nature of the research. Regular workshop sessions will be held and presentations periodically made by the investigators. Course will culminate with a student presentation and a paper prepared to send to the "using" agency.

APPLIED MATHEMATICS TRACK COURSES FOR MATH MAJOR/FOS

MA 372 INTRODUCTION TO DISCRETE MATHEMATICS

SCOPE: The purpose of this course is to introduce topics in Discrete Mathematics, providing a foundation for further study and application. The topics covered are useful to both the applied mathematician and the computer scientist. They include propositional logic, elements of set theory, combinatorics, relations, functions, partitions, methods of proof, induction and recursion, digraphs, trees, finite state machines, and algebraic systems. Specific applications to computer science are presented.

MA 385 CHAOS AND FRACTALS

SCOPE: This course introduces topics in fractal geometry and chaotic dynamical systems, providing a foundation for applications and further study. The topics from fractal geometry are useful to the military with applications to image analysis and data storage, as well as to applications of biologists, meteorologists, and engineers. The chaotic dynamical systems studied in the course are one-, two-, and three-dimensional, nonlinear, discrete and continuous dynamical systems. Other topics studied are the logistics equation, the Henon attractor, the Lorenz equations, bifurcation theory, Julia sets, and the Mandelbrot set. These topics have applications in all fields of science, and examples from social science that involve nonlinear dynamics are explored. The course integrates concepts introduced in the core mathematics courses.

MA 396 NUMERICAL METHODS FOR SOLUTION OF DIFFERENTIAL EQNS

SCOPE: The focus of this course is to find numerical solutions of differential equations that result when modeling physical phenomena. The numerical solution of both initial value problems and boundary-value problems that arise with ordinary differential equations are covered. Techniques for solving partial differential equations are introduced. The special problem incorporates mathematical software libraries in developing solutions to applied problems.

MA 484 PARTIAL DIFFERENTIAL EQUATIONS

SCOPE: The course is devoted to the solution of the classical partial differential equations of mathematical physics and most engineering fields. For example, these equations describe such diverse phenomena as the flow of heat in a metal plate, the gravitational field of the solar system, the vibration of a structural beam, and the energy levels of the hydrogen atom. The subject matter has application in many fields and should be of interest to mathematics, science, and engineering concentrators. Specific topics covered are the heat, wave, and potential equations, Fourier series, series solutions to ordinary differential equations, special functions, and boundary value problems.

MA 485 APPLIED COMPLEX ANALYSIS

SCOPE: This course presents a logical development of complex variable theory sufficient for the development and solution of a number of interesting and practical problems. Residue theory is developed and applied to problems in integration and in the solution of partial differential equations via transform techniques. Conformal mapping theory is used to solve partial differential equations for which the solution is a harmonic function satisfying prescribed boundary conditions. These classical Dirichlet-Neumann problems model phenomena arising in the study of electrostatic potential, equilibrium thermodynamics, incompressible fluids, elasticity, and other areas of continuum mechanics.

MA 487 MATHEMATICAL ANALYSIS II

SCOPE: Continuation of MA387. Course coverage includes Riemann integration, infinite series, sequences and series of functions, uniform convergence, and power series.

MA 492A APPLIED COMBINATORICS

SCOPE: This course introduces the basic techniques and modes of combinatorial problem solving important to the field of computer science and mathematical sciences such as operations research. Applications of combinatorics are also related to fields such as genetics, organic chemistry, electrical engineering and political science. Combinatorial enumeration and logical structure are stressed. Applications and examples provide the structure of progression through topics, which include counting methods, generating functions, recurrence relations, and enumeration techniques.

MA 492B APPLIED ALGEBRA

SCOPE: This course introduces the underlying algebra of computer science structures. Topics include sets, set functions, induction, partial orders, order-preserving maps, Boolean algebra, finite state machines, groups, and modular arithmetic.

MA 492C ABSTRACT ALGEBRA

SCOPE: This is an introductory course in modern algebra for cadets who plan to do graduate work in mathematics or theoretical work in the physical sciences or engineering. The emphasis of the course is on group theory, considering such topics as cyclic and abelian groups, normal sub-groups and factor groups, series of groups, and solvable groups. Selected applications are interspersed with the material on group theory. The course concludes with an introduction to rings and fields. One special problem is provided to allow the student to do independent research in an area of the student's interest.

MA 493A OPERATIONAL CALCULUS AND TRANSFORMS

Former Title: Complex Analysis II

SCOPE: This course is the logical extension and synthesis of MA484 and MA485. It employs the integral calculus of complex functions and the theory of residues to investigate solutions to a number of partial differential equations arising from electrostatics, thermostatics, elasticity, gravitation, and other fields of continuum

mechanics. The Poisson-Integral Formula is applied to the solution of boundary-value problems. Fourier and Laplace transforms are studied in detail and are used to develop general techniques for the solution of many ordinary, partial, and integral equations that result from the above applications.

MA 493B REAL VARIABLE THEORY

SCOPE: Continuation of MA487. Topics include sequences and series of functions, equicontinuity power series, Fourier series, the exponential and logarithmic function, and the Gamma function. The last portion of the course will be devoted to individual research projects.

MA 493C TOPICS IN NUMERICAL ANALYSIS

SCOPE: A continuation of MA396. Topics include boundary-value problems for ordinary and/or partial differential equations.

MA 493D INTRODUCTION TO TOPOLOGY

SCOPE: The course begins with cardinality and the modern definition of a function. Then the basic properties of topological spaces--compactness, connectedness, and continuity--will be emphasized. Special attention will be given to metric topologies on Euclidean spaces. Complete metric spaces and function spaces will be introduced.

MA 493E TOPICS IN ANALYSIS

SCOPE: This course provides cadets the opportunity to pursue in detail subjects of special interest.

STATISTICS/OPTIMIZATION TRACK COURSES FOR MATH MAJOR/FOS

MA 381 NONLINEAR OPTIMIZATION

SCOPE: This course provides an undergraduate presentation of nonlinear topics in applied mathematics. These topics include a review of convex functions, minima and maxima of convex functions, Lagrange Multipliers, Kuhn-Tucker optimality conditions, search methods, and an introduction to the calculus of variations.

MA 476 MATHEMATICAL STATISTICS

SCOPE: This course builds on the foundation presented in the core probability and statistics course to provide a mathematical presentation of the important topics in mathematical statistics. The course begins with a review of probability concepts from the core course, adding additional topics such as transformations of random variables and moment generating functions. To provide the mathematical basis for much of statistical practice, certain limit theorems and sampling distributions are proven. The central themes of the course are decision theory, the theory of estimation, and the theory of hypothesis testing.

MA 481 MATHEMATICAL PROGRAMMING

SCOPE: This course emphasizes the applications of optimal solutions to linear algebraic systems using the simplex method of linear programming. This includes an in-depth development of the simplex method, an analysis of the dual problem, parametric programming, sensitivity analysis and the revised simplex procedure. Additional computational techniques that are applicable to specific mathematical models such as the transportation problem, assignment problem and network problems are also studied. Problems illustrating applications are emphasized throughout the course. Use of existing computer software to solve problems is also emphasized.

MA 489D ADVANCED INDIVIDUAL STUDY IN MATHEMATICS: DATA ANALYSIS

SCOPE: This is essentially a tutorial course or an individual project, offered only to a limited number of highly qualified cadets who have completed available mathematics elective courses and have expressed a wish to pursue advanced study in statistics-related area. The course work will be tailored to suit the individual needs.

MA 492D GRAPH THEORY AND NETWORKS

SCOPE: This course introduces the student to the techniques, algorithms, and structures used in graph theory and network flows in order to solve real world discrete optimization problems. Basic definitions relating to graphs and digraphs, together with a large number of examples and applications are provided. Cadets learn to implement new graph theory techniques in their area of study. Emphasis is on communication networks, flows, and scheduling.

GENERAL ELECTIVE COURSE FOR MATH MAJOR/FOS

MA 488 VISITING PROFESSOR'S COURSE

SCOPE: The Visiting Professor of Mathematics will conduct a course on a topic to be announced. (Offered every other year.)

OPERATIONS RESEARCH MAJOR/ FIELD OF STUDY STRUCTURE

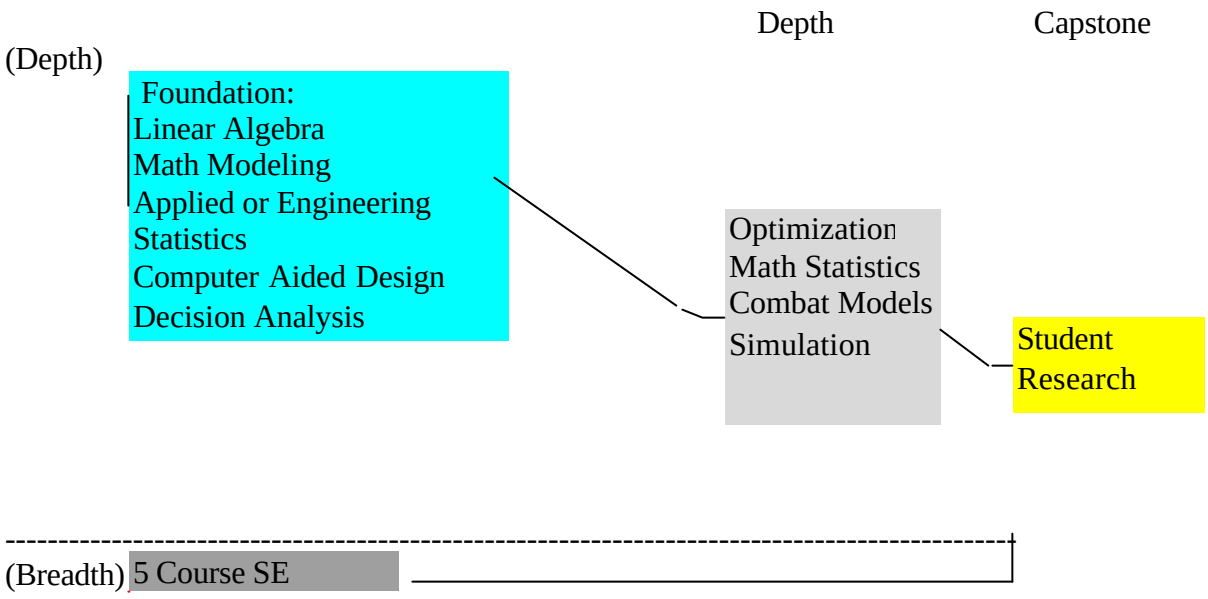
Cadets interested in studying operations research may pursue either a field of study or a major in Operations Research (OR). This program is an interdisciplinary program, jointly operated by the departments of Mathematical Sciences and Systems Engineering. The Systems Engineering five-course engineering sequence is mandatory for both the major and the field of study. In addition to the five course sequence, the OR major consists of 10 specified courses and 2 courses selected by the student (with a counselor) (10+2); the OR field of study consists of 9 elective courses (3 from a list of math electives, 3 from a list of system's electives, and 3 from a combined list with some additional economic and computer sciences courses). Cadets who have started a major may revert to a field of study by dropping 3 courses.

OR applies logic and quantitative methods to complex problems to give military commanders a sound basis for decision-making. Since WWII, OR has formally emerged as an applied science and engineering discipline, with broad application to many fields of study. The need for OR within the army is constantly growing as the demand for better utilization of limited resources continues. OR at USMA focuses on optimization methods, applications of probability and statistics, and modeling. OR prepares the cadets to meet the problem-solving challenges faced by company-grade officers in all branches of the army.

The 5 course sequence (Deterministic Models, Probabilistic Models, Engineering economics, Introduction to Systems Design, and Systems Design 1) serve to provide a broad generic background in OR and systems techniques as applied to real world systems.

The math electives -- math modeling (MA 391), Linear Algebra (MA 371), Nonlinear Optimization (MA 381), Mathematical Statistics (MA 476), and Math Programming (MA481) -- provide the foundation for understanding the principles of quantitative reasoning.

The Modeling course (MA391) serves to introduce the student to the field of mathematical modeling, and is normally the first course taken in the program. The Modeling course introduces students to basic techniques in modeling from a variety of scenarios (deterministic and stochastic, discrete and continuous, linear and nonlinear) that are intentionally chosen to expose the students to most of the stem areas of mathematics. Two of the 5 systems engineering core courses are modeling related courses: Deterministic and Probabilistic models. These two courses introduce students to the applied modeling process. Because of the early placement of these courses in the students' programs and because of the survey component present in each, these courses give students a little more experience in applying the modeling techniques.



Linear Algebra (MA371), Nonlinear Optimization (MA381), Math Programming (MA 481), and Applied Statistics (MA 376) are the other foundation courses that include a significant modeling component in addition to their significant conceptual component. These courses cover the basic theory and concepts of the material, but often use applications and ILAP's to help demonstrate the meaning and significance of the ideas involved. Math Statistics (MA476) is a rigorous and axiomatic course; it begins with the basic concepts from core probability and rigorously constructs from it applied theoretical concepts.

Coupled with these foundations courses, the student takes 4 systems courses: Computer Aided Systems Engineering (SE 370), Decision Analysis (SE 385), Systems Simulation (SE 481), and Combat Modeling (SE 485). These courses comprise a strong OR foundation.

The capstone course of the program is a Student Research course (MA491 or SE 491), normally taken in the student's last semester. Faculty propose a list of both applied and pure topics; students usually select a topic of interest from this list, but occasionally propose and have approved a topic of their own. Student topics are usually related to their area of interest, and the students' faculty counselors often serve as research advisors as well. This research experience is an opportunity for students to synthesize previous material in tackling a question of their choice, and provides experience in presenting technical results both orally and in a technical report.

More information about the major and field of study programs can be found in the online Redbook at http://www.dean.usma.edu/dean/docs/redbook.htm.

SCHEDULING TEMPLATE FOR OPERATIONS RESEARCH MAJOR

Terms

	4	5	6	7	8
1	PH202	SE387	SE388	SE401	SE402
2	MA206	SE381	SE370	SE481	MA476
3	LX2XX	MA376	SE385	SE485	MA481
4	SS20X	MA381	MA371	*	MA/SE491
5	EV203	SS307	EN302	HI301	HI302
6	MA 391		PL300	LW403	

Shaded blocks indicate Engineering Sequence courses (DO NOT MOVE!)

Paired blocks indicate courses that can be swapped between terms.

***** Choose one of the following:

Courses

MA386	Intro. to Numerical Analysis (fall only)
MA488	Visiting Professor's Course (spring only)
MA492D	Graph Theory and Networks (fall only)
SE384	Stochastic Processes (spring only)
SE420	Production Operations and Systems
SE484	Dynamic Systems Analysis (fall only)
SS382	Microeconomics (fall only)
SS388	Macroeconomics (spring only)
SS469	Econometrics II (spring only)
CS382A	Comp. for the Eng. & Scien. (fall only)
CS383	Management Information Systems
CS384	Data Structures
CS486	Intro to Artificial Intelligence (fall only)

Required

MA 371
MA 381
MA 391
MA 476
MA 481
SE 370
MA 376
SE 385
SE 481
SE 485
* Elective
MA/SE 491

ENGINEERING SEQUENCE COURSES FOR OR MAJOR/FOS

SE 381 ENGINEERING ECONOMY

SCOPE: Engineering Economy presents the economic dimensions in the evaluation of engineering alternatives. The course emphasizes the analytical considerations of money and its impact on the systems design process and engineering management, systems acquisitions, infrastructure renewal, energy management, and current military cost analysis. The course is taught in four lesson blocks. The Time Value of Money (TVM) block provides engineering science models for economic analysis of engineering alternatives by introducing interest concepts, cash flow diagrams and discounting formulas. The Analysis Methods block presents methods for project evaluation and comparison. The After Cash Flow block incorporates the real world effect of taxes, depreciation and inflation into the analysis of alternatives. Finally, the Special Topics block synthesizes topics such as project financing and risk into the economic analysis process. Several lessons will be devoted to a computer assisted design exercise which integrates the four lesson blocks in the context of real world problems typically faced by systems engineers and engineering managers. Communication skills are developed with both written reports and oral presentations

SE 387 DETERMINISTIC MODELS

SCOPE: The first of a two-course sequence that is part of the engineering science foundation for subsequent courses in systems engineering. This course builds on the scientific method and problem solving paradigms embedded in the basic sciences by introducing cadets to the modeling process and many of the classical deterministic models used by systems engineers to analyze decision alternatives. The emphasis is on the creative application of skills learned in the basic sciences to model formulation, solution methods, analysis of results, and interpretation. Models studied include inventory systems, linear programming and its extensions, networks, and project scheduling (PERT/CPM). The course requires cadets to use computer software for model solution and analysis. Communication skills are developed with both written reports and oral presentations.

SE 388 PROBABILISTIC MODELS

SCOPE: The second of a two-course sequence that is part of the engineering science foundation for subsequent courses in systems engineering. This course introduces cadets to many of the classical stochastic models used by systems engineers and continues to emphasize the modeling process studied in SE387. Topics include forecasting techniques, queuing systems, and a brief introduction to decision analysis. The stochastic models extend and build on the deterministic counterparts from SE387. This course includes introducing simulation as an alternative to analytic models. Cadets make use of computer software in solving and analyzing models. A major course exercise involves cadets building a spreadsheet simulation model. Communication skills are developed with both written reports and oral presentations.

SE 401 INTRODUCTION TO SYSTEMS DESIGN

SCOPE: This course introduces the process of bringing systems, process, and operations into being: systems design. It builds on previous engineering science courses and presents systems engineering design as an iterative, decision-making process, in which the basic sciences, mathematics, and engineering sciences are applied to convert resources optimally to meet the objectives of a client or stakeholder group. Since large-scale systems usually involve people, information, technology, and organization, the design experience also draws on knowledge from the humanities, computer science, physical sciences, and natural sciences taught in the core academic program. Systems engineering design methodology is taught as a top-down, integrated, life cycle approach to formulating, analyzing, and interpreting design alternatives. Emphasis is placed on developing student creativity, using real-world, open-ended problems, learning and using the systems engineering design process, formulating design problem statements, creating alternatives, accounting for feasibility criteria, considering detailed system descriptions, determining system specifications, and including realistic constraints such as economic factors, safety, reliability, aesthetics, social and environmental impact. Communication skills are developed with both written reports and oral presentations.

SE 402 SYSTEMS DESIGN 1

SCOPE: This course integrates the principles, concepts and models explored in earlier core and systems engineering courses by providing cadets the opportunity to design a real-world system or process. The systems design process introduced in SE 401 is reinforced and extended to include many concerns of systems engineering practice. Student creativity is challenged and developed through the use of open-ended problems representative of those found in today's society and an Army being re-engineered for the 21st Century. Specific topics include needs analysis, trade-off analysis, compatibility analysis and systems architecture. This course prepares the Systems Engineering major for the follow-on senior-level capstone design experience, SE 403. Communication skills are developed with both written reports and oral presentations.

FOUNDATION COURSES FOR OR MAJOR/FOS

MA 371 LINEAR ALGEBRA

SCOPE: This course emphasizes both the computational and theoretical aspects of linear algebra one encounters in many subjects ranging from economics to engineering. The course covers solutions of linear systems of equations, the algebra of matrices, and the utility of determinants. The foundational aspects of vector spaces and linear transformations to include linear dependence and independence, subspaces, bases and dimension, inner products, and orthonormalization are developed. This is rounded out with a detailed investigation of eigenvalues and eigenvectors as they relate to diagonalization, quadratic equations, and systems of differential equations. A software package is used to compute solutions to problems. Applications of the course material are included in the form of special problems to illustrate its wide scope.

MA 391 MATHEMATICAL MODELING

SCOPE: This course is designed to give the student the opportunity to develop skills in model construction and model analysis while addressing interesting scenarios with practical applications from a wide variety of fields. The course is divided into nine major blocks of instruction. In the first two blocks, the student is introduced to modeling using graphical methods and the modeling process. During the next block of lessons, the student learns the techniques of modeling using proportionality and constructs models which will be tested and fitted in the next block of lessons, model fitting. Next, the student studies several scenarios that lead to models requiring optimal solutions. The course then addresses empirical model construction. In this block, real data will be used as the basis to construct a model. Next, the student uses dimensional analysis to assist in model building. A short block of lessons on simulation follows. The course concludes with the modeling of dynamic scenarios. The student learns how to use the derivative to model and analyze time dependent situations such as competitive hunter scenarios.

MA 376 APPLIED STATISTICS

SCOPE: This course builds on the foundations presented in the core probability and statistics course to provide a broad introduction to some of the most common models and techniques in applied statistics. The mathematical basis for each of the models and techniques is presented with particular emphasis on the development of the required test statistics and their distributions. The texts are the same as for the core course. Topics covered include hypothesis testing, analysis of variance, categorical data analysis, regression analysis, and nonparametric methods. For the special problem, students can choose an application in their own field.

SE 370 COMPUTER AIDED SYSTEMS ENGINEERING

SCOPE: This course is an integral part of the Systems Engineering major and field of study and emphasizes the use of modern technology in systems analysis and design. The course includes the use of computer methods as an aid to solving systems engineering problems. Cadets study digital computer techniques in a variety of systems engineering contexts. Topics include data manipulation, analysis, and graphical presentation of

information. Cadets learn and use word processing, spreadsheet, database, graphical drawing, and other specialized engineering software packages. The course places considerable emphasis on the use of the computer as a tool in the systems engineering design process. Communication skills are developed with both written and oral presentations.

SE 385 DECISION ANALYSIS

SCOPE: The course presents basic techniques of modern decision-making concentrating on both theoretical and modeling aspects. The focus is modeling problem structure, uncertainty, and preference in the context of decision-making. Topics include influence diagrams, decision trees, sensitivity analysis, assessing subjective probability, value of information, risk attitudes, utility theory and multiattribute utility model in a real world decision environment for a real world client. The design project provides a key bridge between the mathematical theory and the application of skills to an open-ended problem. This course provides essential engineering science topics, as well as the design experience, necessary for understanding and applying the systems design process. Both systems engineering and engineering managers utilize the concepts presented. Communication skills are developed with both written reports and oral presentations.

DEPTH COURSES FOR OR MAJOR/FOS

MA 381 NONLINEAR OPTIMIZATION

SCOPE: This course provides an undergraduate presentation of nonlinear topics in applied mathematics. These topics include a review of convex functions, minima and maxima of convex functions, Lagrange Multipliers, Kuhn-Tucker optimality conditions, search methods, and an introduction to the calculus of variations.

MA 481 MATHEMATICAL PROGRAMMING

SCOPE: This course emphasizes the applications of optimal solutions to linear algebraic systems using the simplex method of linear programming. This includes an in-depth development of the simplex method, an analysis of the dual problem, parametric programming, sensitivity analysis and the revised simplex procedure. Additional computational techniques that are applicable to specific mathematical models such as the transportation problem, assignment problem and network problems are also studied. Problems illustrating applications are emphasized throughout the course. Use of existing computer software to solve problems is also emphasized.

MA 476 MATHEMATICAL STATISTICS

SCOPE: This course builds on the foundation presented in the core probability and statistics course to provide a mathematical presentation of the important topics in mathematical statistics. The course begins with a review of probability concepts from the core course, adding additional topics such as transformations of random variables and moment generating functions. To provide the mathematical basis for much of statistical practice, certain limit theorems and sampling distributions are proven. The central focus of the course is on decision theory, the theory of estimation and the theory of hypothesis testing.

SE 481 SYSTEMS SIMULATION

SCOPE: This course is the first course of one of two two-course capstone design experiences for the Systems Engineering major. For the engineer and scientist, the simulation of complex systems using the digital computer has become one of the most widely used analytical tools. Simulation modeling can be used to study the effects of changes to existing systems or processes, or evaluate the performance of new systems prior to their implementation. This course emphasizes the concepts and principles of discrete event simulation. In this course, the student will design discrete models of complex systems or processes, implement these models using the ProModel for Windows simulation package, conduct simulation experiments using their models and perform analysis of the results. The student receives specific instructions concerning: modeling; input data analysis; random number generation and testing; random variate generation; verification and validation of simulation models; experimental design; simulation output analysis; and ProModel for Windows. These concepts and tools are applied to such problem areas as personnel distribution systems, inventory systems, scheduling, transportation planning, manufacturing systems design, and general military systems

simulations. Engineering design exercises are an integral part of the instruction. Communication skills are developed with both written reports and oral presentations.

SE 485 COMBAT MODELING

SCOPE: This is the first course in one of two two-course capstone design experiences for the Systems Engineering major. The course focuses on models and techniques used in state-of-the-art combat simulations and analyses. Major areas include combat attrition models, terrain models, search and detection techniques, war-gaming, and measures of effectiveness associated with current analysis. The combat modeling utilizes JANUS (A), the simulation currently used by Army units, battle labs and research and development organizations. Design of experiments and statistical analysis of combat modeling data assists the cadet in scrutinizing the effectiveness of weapon systems, doctrine, and tactics on the future battlefield

CAPSTONE COURSES FOR OPERATIONS RESEARCH

MA 491 RESEARCH SEMINAR IN APPLIED MATH PROJECTS

SCOPE: The student integrates the mathematical concepts and techniques learned in previous courses with the principles developed in the whole USMA Curriculum to solve a current problem of interest to the Academy or other agencies in the Department of the Army. Students may select problems from a list of suitable projects provided by the Department of Mathematics. Students will be assigned a faculty advisor who has an interest and background in the problem. Students may work individually or in small teams, depending on the nature of the research. Regular workshop sessions will be held and presentations periodically made by the investigators. Course will culminate with a student presentation and a paper prepared to send to the "using" agency.

SE 491 RESEARCH PROJECT IN SYSTEMS ENGINEERING

SCOPE: The cadet integrates the concepts and techniques learned in previous Systems Engineering courses to solve a current problem of interest to the Academy or other agencies in the Department of the Army. Cadets may select project topics compatible with on-going research within the Department of Systems Engineering, the Operations Research Center, or may (subject to approval) select any topic of interest to them. Cadets will be assigned a faculty advisor who has an interest and background in the research area. Cadets may work individually or in small teams, depending on the nature of the research. The course will culminate with a student presentation and a paper prepared to send to the "using" agency.

Service Elective Structure

Other MSE majors require or highly encourage elective mathematics courses in order to develop the mathematical concepts and skills used in the student's major. In addition to the aforementioned elective courses the department of mathematics offers two additional courses specifically to meet this need.

SERVICE ELECTIVE COURSES

MA 364 ENGINEERING MATHEMATICS

SCOPE: This course provides additional mathematical techniques and deepens the understanding of concepts in mathematics to support continued study in science and engineering. Emphasis is placed upon using mathematics to gain insight into natural and man-made phenomena that give rise to problems in differential equations and vector calculus. Calculus topics focus on three-dimensional space curves, vector fields and operations, divergence and curl, and line and surface integrals. Topics in differential equations include systems of ordinary differential equations and Laplace transforms. Introduction is made to classical partial differential equations and their numerical solution.

MA 366 VECTOR CALCULUS AND PARTIAL DIFFERENTIAL EQUATIONS

SCOPE: This course provides additional mathematical techniques and deepens the understanding of concepts in mathematics to support continued study in science and engineering. Emphasis is placed upon using mathematics to gain insight into natural and man-made phenomena that give rise to problems in differential equations and vector calculus. Calculus study focuses on vector fields, differential operators, and the vector integral theorems. This material is then used to derive the diffusion equation. Solution of this equation with Fourier series, separation of variables, and numerical methods is then studied.

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Opportunities in Mathematics and Operations Research

ACTIVITIES

There are many opportunities for interested cadets to become involved with the Math Department and its faculty regardless of the student's major.

Math Forum

This is a student-run organization that meets periodically to listen to or talk about mathematical topics of interest with the faculty, and to plan trips and events. Some of the trips conducted and planned have been to the Army Research Lab, the Air Force Academy, Wall Street, IBM Watson Research Center, etc. Some of the regular activities include tailgates and cookouts with the Math faculty, and competing against the faculty in softball and soccer.

US Service Academy Student Math Conference

The various service academies take turns hosting this. Interested MA491 students attend and talk about their projects; interested Math Forum students compare forum programs and discuss common issues of interest with their peers.

Quarterback Luncheon

In the fall, the department pays the bill for faculty to host a limited number of students every week at the Officer's Club's Quarterback Luncheon. Instructors typically use this as an opportunity to recognize and reward students from their sections and to rub elbows with counselees and other majors.

Coffee Call

Once a month (always on a specified Thursday at 1350), the department holds a coffee call in the department lounge (TH248) to informally recognize matters of interest (performance in competitions, trips, volunteer work, sporting events, birthdays, etc). Cadets are always invited and encouraged to attend.

Athletic Competitions

Every fall and spring, the faculty takes on the cadets in softball, soccer, and any other sports of mutual interest.

COMPETITIONS

These are open to cadets of all classes and all majors.

VPI Math Contest

Sponsored by Virginia Polytechnic Institute, this is a competition between students from many eastern institutions. No travel is involved; we administer the exam locally on the competition date, usually on a Saturday morning.

Mathematical Competition in Modeling

The MCM is sponsored by the Consortium for Mathematics and Its Applications (COMAP). It is different in that the entrants are all teams of 3 contestants, teams work on only one problem (they choose one from two that are offered), the problem is applied and complex and usually interdisciplinary, any inanimate resource is allowed, and the exam period is for three continuous days over a weekend. We find that our curriculum is better preparation for this type of test than most other programs, and that teams that include students from other majors often do very well.

AWARDS

Pi Mu Epsilon

National Mathematical honor society. Membership is based on excellence in mathematics courses and contribution to mathematics activities at USMA (such as through the Math Forum).

Top Gun

Awarded by the Department to the cadet who earned the highest grade average in a course at the end of each semester.

First Class "Top Cadet"

Awarded by the Department to the First Classman ranking highest in math classes taken during the current year.

Second Class "Top Cadet"

Awarded by the Department to the Second Classman ranking highest in math classes taken during the current year.

Bradley Award

Awarded annually to the graduating First Classman with the highest grade average in mathematics electives.

Hollis Award

Awarded annually by the Department of Systems Engineering for contributed Operations Research/Systems Analysis work that enriches cadet education, enhances professional development, and links the Academy to the Army in a meaningful way.

Morgan Prize

Awarded annually by the AMS, MAA, and SIAM for outstanding research in mathematics by an undergraduate. There's more information on the prize and past winners at http://www.maa.org/awards/morgan.html.

Hertz/NSF/Marshall Scholarships

Awarded annually by the respective national organizations. We have had several math majors win these scholarships in recent years.

Mathematical and Interdisciplinary Contest in Modeling Award

The Mathematical Contest in Modeling (MCM/ICM) involves three-person teams representing their schools. The teams spend a weekend working on an application problem and produce a written report of their solutions. There are over 600 teams from around the world competing. USMA teams have won the "Outstanding" award four times during the 17-year history of the MCM.

Association for Women in Mathematics

This award is presented to cadets who are nominated by instructors for outstanding achievements in mathematics during the academic year. The recipients receive one-year memberships in the Association for Women in Mathematics.

Laura Winkel Memorial Applied Math Award

This award is presented to the graduating cadet for outstanding performance in the applied mathematics courses in the Math Major program.

"The Problem Of The Week" Mathematics Problem Solving Competition Award

"The Problem of the Week" Mathematics Problem Solving Competition Award is presented each semester to the two cadets with the highest point total accumulated in the Department of Mathematical Sciences' weekly competition.

Robert E. Lee Memorial Award

The Robert E. Lee Memorial Award is presented to the graduating cadet with the highest average in the core mathematics curriculum.

Military Application Society Award

The Military Applications Society Award is presented for excellence in Operations Research.

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MATHEMATICS RESOURCES

ACADEMIC COUNSELING

The Department Academic Counselors (DAC's) run the departmental counseling program on a daily basis. They act as liaison between the junior academic counselors, the senior academic counselors, other departments, and the Operations and Registrar Division (ORD). For AY01-02, these counselors are:

LTC Michael Jaye	x5218	TH250
CPT Marie Hall	x5660	TH229
CPT Scott Gensler	x4012	TH229

The Junior Academic Counselors (JAC's) are the heart of the department's counseling program. They are responsible for all one-on-one coordination with the individual cadet majors. The JAC is the primary link between the cadets and the department. The main areas of responsibility of the JAC are:

a. Initial contact with cadet.
b. Periodic contact to update and correct the 8TAP as required.
c. Monitoring the cadet's academic growth.
d. Coordinating with DAC as required.
e. Being a mentor: Integrating the cadets into department activities: inviting them to social and professional functions as appropriate.

The following is a timeline of the major recurring actions to be completed by the various members of the department's academic program:

Matrix of Key Events

When	Who	What
September of 3rd Class Yr	DAC	Coordinates Dept. Orientation Briefings
NLT Mid-Oct, 3rd Class Yr	DAC	Assign new majors to JACs
NLT 1 Nov, 3rd Class Yr	JAC	Establish initial contact with new majors
NLT Mid-Feb, 3rd Class Yr	DAC	Enter/ Initial 8TAP worksheet filled out
NLT Spring Break, 3rd Class Yr	DAC	Enter 8TAPs into CORPS
NLT 2nd Semester of 3rd Class Yr	JAC	Review of CDT's 8TAP
NLT 2nd Semester of 2nd Class Yr	JAC	Review of CDT's 8TAP
NLT 1st Semester of 1st Class Yr	JAC	Review of CDT's 8TAP
NLT Spring Break of 1st Class Yr	DAC	Final review of CDT's 8TAP for graduation

MATH AND OR STUDENT LOUNGE

The Student Lounge, located in TH241, is to provide a location for Math and OR Major and FOS students to socialize, study, store their cool Math Mugs (and books too), relax between classes, catch up on the news on CNN, and drink a soda. The lounge features a PIII 733 MHz PC w/cable TV. It's open days, evenings, and weekends. It is available for Math and OR cadets (and their guests) only, any time they are permitted in the academic buildings.

MATH CLINIC

The Math Clinic, located in TH226, is a significant resource for all cadets. It has been designed to provide a place where students can go to join other students to study mathematics. For many students, group study is a valuable form of collaborative learning. The Math Clinic has been resourced with equipment and materials to assist students in their study. Some of these resources include:

- Carrels for individual study, and tables and blackboards for group study.
- Course solution books (for courses that feel a need to maintain such a book).
- WPR solutions (posted for some finite period of time after the WPR).
- PC work stations with network access and TI-89 Graphlink capability.

If you, as an instructor in another MSE Department, have a student with a mathematical problem (either remedial or advanced) that you think can be best addressed by a math instructor, please refer the student to the Math Clinic for assistance. This referral process often works best if you close the loop by contacting your Departmental Liaison Professor to discuss the nature of the need and its resolution.

MATH LIBRARY

Located in TH256, the Math Library is another valuable resource for students. It provides a place for individual or quiet small group study. The library contains all departmental library holdings (including over 5500 volumes and numerous periodicals) as well as a reserved book section in the Departmental library holdings. PC work stations with network access and TI-89 Graphlink capability are also available.

LABS

The department maintains a PC lab and a combined PC/Unix lab for use by cadets. Each lab contains common course software as well as more specialized mathematical software. Cadets can gain access to these facilities by seeing either their instructor or their counselor.

History of the Department of Mathematical Sciences at USMA

A teacher affects eternity; he can never tell where his influence stops. -- Henry Adams

The Department of Mathematical Sciences, USMA, has had a rich history of contributing to the education of cadets as confident problem solvers and of developing its faculty as effective teachers, leaders, and researchers. The story of mathematical education at West Point is full of interest: faculty curriculum developments, teaching methods and tools, and technological equipment. Many of the Department's advances have been exported outside the Academy to be utilized by other civilian and military educational institutions.

EARLY BEGINNINGS The actual teaching of mathematics at West Point dates from even before the Academy was established. In 1801, George Baron taught a few Cadets of Artillery and Engineers some of the fundamentals and applications of algebra. The Academy at West Point was instituted by act of Congress and signed into law by President Thomas Jefferson on 16 March 1802. The first acting Professors of Mathematics were Captains Jared Mansfield and William Barron. They taught the first few cadets algebra, geometry, and surveying.

CONTRIBUTIONS TO THE NATION Since the Academy was the first scientific and technical school in America, the early mathematics professors at USMA had the opportunity to make significant contributions not only to the Academy, but also to other American colleges. Perhaps the most prominent contributors were the early 19th century department heads Charles Davies and Albert E. Church. The work of these two professors had a significant impact on elementary schools, high schools, and colleges across the country. Davies became the Professor of Mathematics in 1823. He was one of the most prolific textbook authors of his day, writing over 30 books from elementary arithmetic to advanced college mathematics. His books were used in schools throughout the country from grade school to college. He had tremendous influence on the educational system of America throughout the 19th century. Albert Church succeeded Davies in 1837, and served as Department Head for the next 41 years. Another influential author, he published seven college mathematics textbooks.

PRODUCING LEADERS FOR THE NATION Faculty members from the Department have been notable military leaders for the country. Robert E. Lee was a standout student-instructor in the Department; Omar Bradley served as an Instructor for four years; Harris Jones and William Bessell were Deans of the Academic Board at USMA for a total of 15 years; and Department Heads Harris Jones, William Bessell, Charles Nicholas, John Dick, and Jack Pollin served impressively during two world wars. The unique technical curriculum in place at the Academy during the middle of the 19th century produced many successful mathematicians and scientists for the country at large. West Point graduates Horace Webster, Edward Courtenay, Alexander Bache, James

Clark, Francis Smith, Richard Smith, Henry Lockwood, Henry Eustis, Alexander Stewart, and William Peck filled positions as professors of mathematics or college presidents at other schools such as the U. S. Naval Academy, Geneva College, University of Virginia, University of Pennsylvania, University of Mississippi, Yale, Brown, Harvard, Columbia, Virginia Military Institute, Cooper Institute, and Brooklyn Polytechnic Institute. Two mathematics department heads became college presidents after leaving USMA: Alden Partridge founded and became the first president of Norwich University, and David Douglass served as president of Kenyon College in Ohio for four years. Jared Mansfield was appointed surveyor-general of the Northwest Territory, and Ferdinand Hassler became superintendent of the United States Coastal Survey. The West Point model of undergraduate mathematics education was exported throughout the nation by capable individuals such as these.

While the faculty at USMA has been primarily military, the Department has benefited from civilian visiting professors since 1976. As part of the goal for civilianization of 25% of the faculty by 2002, begun in 1992, the Department established in 1994 a Center for Faculty Development in Mathematics. This Center establishes faculty development models and curricula and provides for the development of the "Davies Fellows", who serve as rotating civilian faculty members.

HISTORICAL AND LIBRARY HOLDINGS Sylvanus Thayer's first task before assuming the Superintendancy in 1817 was to tour the technical institutions of Europe and assess what features USMA could use to advantage. One of Thayer's many accomplishments was to obtain numerous mathematics and science books from Europe. Thayer's book collection included many of the finest books available at that time. His books provided a solid foundation for the USMA library to build upon. Today, the West Point Library has one of the finest collections of pre-20th century mathematics books in the world. Also during the middle of the 19th century, the Academy instructors used elaborate physical models made by Theodore Oliver to explain the structures and concepts of geometry. This magnificent collection of string models is still in the Department today.

CURRICULAR DEVELOPMENT After Thayer studied the military and educational systems of Europe, he reorganized the Academy according to the French system of the Ecole Polytechnic. The Department of Mathematics faculty (which included as Professor the distinguished scientist and surveyor Andrew Ellicott, and the famous French mathematician Claude Crozet, whom Thayer recruited during his European trip to bring to USMA and America his expertise in Descriptive Geometry, advanced mathematics, and fortifications engineering) combined the French theoretical mathematics program with the practical methods of the English to establish a new model for America's program of undergraduate mathematics. This program of instruction in Mathematics grew over several decades and was emulated by many other schools in the country. The initial purpose of the Military Academy was to educate and train military engineers. Sylvanus Thayer, the "Father of the Military Academy" and Superintendent from 1817-1833, instituted a four-year curriculum with supporting pedagogy to fulfill this purpose. Thayer's curriculum was very heavy in mathematics; from Thayer's time to the

late 1800's, cadets took the equivalent of 54 credit hours of mathematics courses. The topics covered in these courses were algebra, trigonometry, geometry, descriptive geometry (engineering drawing), analytic geometry, and calculus. Over the years, the entering cadets became better prepared and fewer of the elementary subjects were needed. During Davies' tenure (1823-37), calculus was introduced as a requirement for all cadets, and was used in the development of science and engineering courses. The time allotted for the mathematics curriculum decreased to 48 credit hours by 1940, and to 30 credit hours by 1950. During the 1940's, courses in probability & statistics and in differential equations were introduced into the core curriculum and a limited electives program was started for advanced students. In the 1960's, department head Charles Nicholas (previously one of the organizers of the Central Intelligence Agency) wrote a rigorous and comprehensive mathematics textbook (the "Green Death") that cadets used during their entire core mathematics program. With this text, he was able to adapt the mathematics program to keep up with the increasing demands of modern science and engineering. In AY61-62, electives were first offered in the department (beginning with MA482, Advanced Calculus). In the 1970's and 1980's, Academy-wide curricular changes provided opportunities for cadets to major in mathematics. The first mathematics majors graduated in 1986 and the first operations research majors graduated in 1989.

During the 1980's, a mathematical sciences consulting element was established that allowed faculty members and cadets to support the research needs of the Army. This type of research activity continues today in the Army Research Laboratory (ARL) Mathematical Sciences Center of Excellence and in the Operations Research Center (ORCEN). In 1990 the Department introduced a new core mathematics curriculum that included a course in discrete dynamical systems, with embedded linear algebra. In that same year, the department changed its name to the Department of Mathematical Sciences to reflect broader interests in applied mathematics, operations research, and computation.

TECHNOLOGICAL DEVELOPMENT USMA has a long history of technological innovation in the classroom. It was Crozet and other professors at USMA in the 1820's who were the first professors in the nation to use the blackboard as the primary tool of instruction. In 1944, the slide rule was issued to all cadets and was used in all plebe mathematics classes. During William Bessell's tenure (1947-1959), the mathematics classrooms in Thayer Hall (a converted riding stable) were modernized with overhead projectors and mechanical computers. Bessell was also instrumental in establishing a computer center at West Point. The hand held calculator was issued to all cadets beginning in 1975, and pre-configured computers were issued to all cadets beginning in 1986. In recent years, the department has integrated computer algebra systems and statistical analysis packages into classroom instruction. We are currently implementing web-based instructional tools and preparing for the transition to a mobile computing environment.

NATIONAL COMPETITION In the spring of 1933 West Point entered an interesting competition in mathematics. After USMA defeated Harvard at a football game the previous fall, a chance remark from President Lowell of Harvard to

Superintendent Jones of USMA led the two schools to arrange a mathematics challenge match between the two schools with the two competing teams each composed of 12 second-year students. Army was the home team, so the Harvard competitors traveled by train to West Point. All the competitors took a test written by the president of the Mathematics Association of America. The **New York Times**, which had promoted the event with a series of articles in its sports section, reported the results. The West Point "mathletes" defeated Harvard in the competition that was the precursor to the national Putnam Competition. Since its inception in 1984, the Academy has entered two three-person teams in the International Mathematics Competition in Modeling. USMA won the top prize in 1988, 1993, 1999, 2000 and 2001.

Under the leadership and guidance of Frank R. Giordano and Chris Arney during the first half of the 1990's, the Department of Mathematical Sciences became recognized as one of the more progressive mathematics programs in the country. The Department has developed a strong "7 into 4" program that is exciting as well as innovative. A similar initiative in using Interdisciplinary Lively Applications Projects (ILAP's) in mathematics courses has broadened the cadet perspective and enhanced the development of cadet's quantitative problem solving skills. USMA is now helping others adapt this model nationally through its administration of a multimillion dollar NSF grant. The USMA mathematics program has had great influence on mathematics education in America throughout its history and strives to continue contributing to its improvement.

For a more extensive exposition on the history of the Department, refer to http://www.dean.usma.edu/math/about/history/dept.htm.