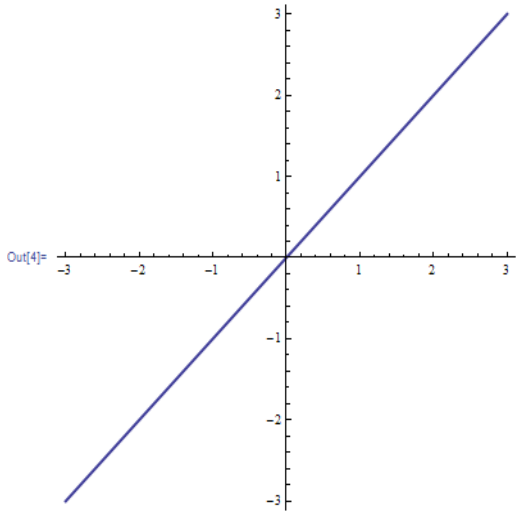
Functions & Lines

Vertical and Horizontal Shifts (Stewart text, Section 1.3, page 37):

Suppose $c > 0$. To obtain the graph of

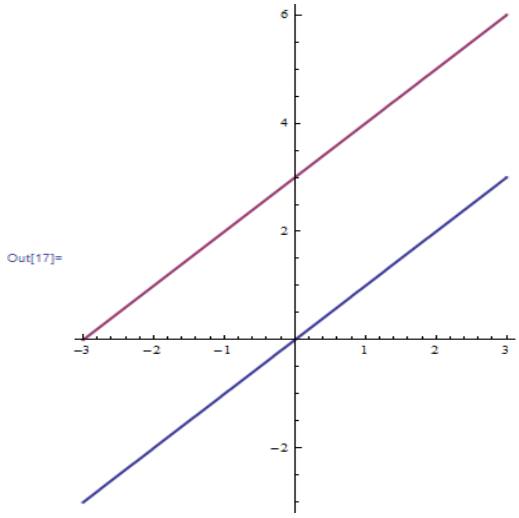
1. $y = f(x) + c$, shift the graph of $y = f(x)$ f a distance of c units upward.
2. $y = f(x) - c$, shift the graph of $y = f(x)$ a distance of c units downward.
3. $y = f(x - c)$, shift the graph of $y = f(x)$ a distance of c units to the right.
4. $y = f(x + c)$, shift the graph of $y = f(x)$ a distance of c units to the left.

1. Graph the function $\boldsymbol{f(x) = x}$.

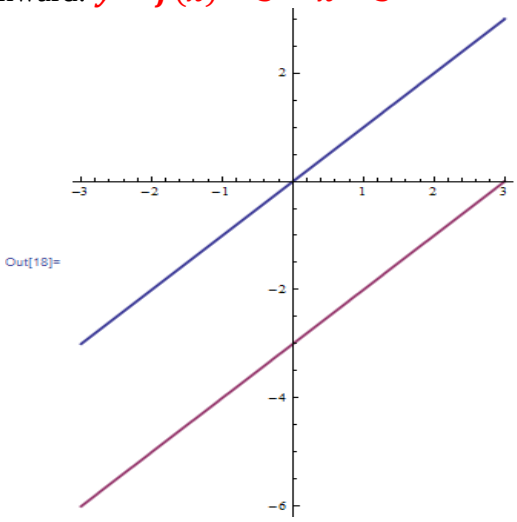


Write the equation that is obtained from the graph of $\boldsymbol{f(x)}$ with the following modification, and then graph the resulting function on the same graph with $\boldsymbol{f(x)}$:

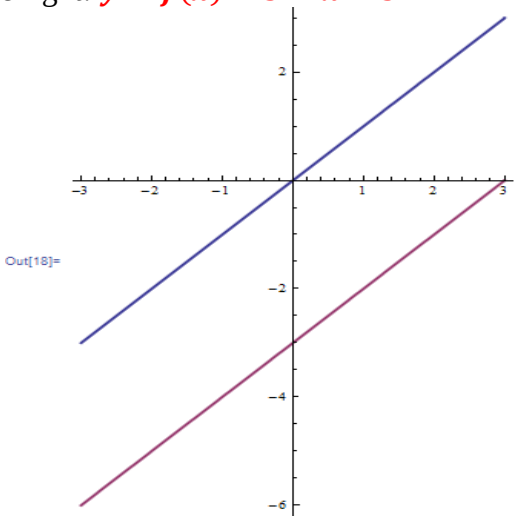
a. Shift 3 units upward: $\boldsymbol{y = f(x) + 3 = x + 3}$



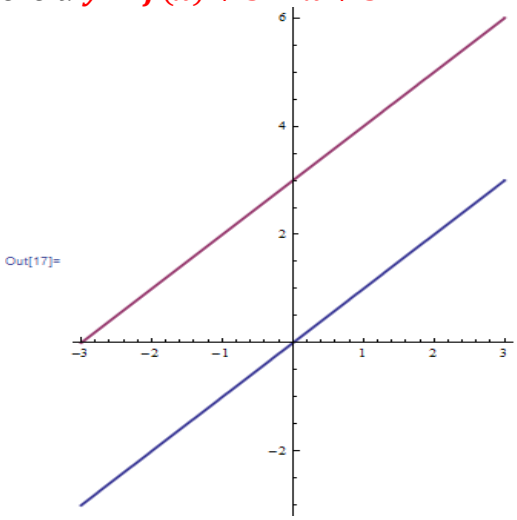
b. Shift 3 units downward: $\boldsymbol{y = f(x) - 3 = x - 3}$



c. Shift 3 units to the right: $\boldsymbol{y = f(x) - 3 = x - 3}$



d. Shift 3 units to the left: $\boldsymbol{y = f(x) + 3 = x + 3}$

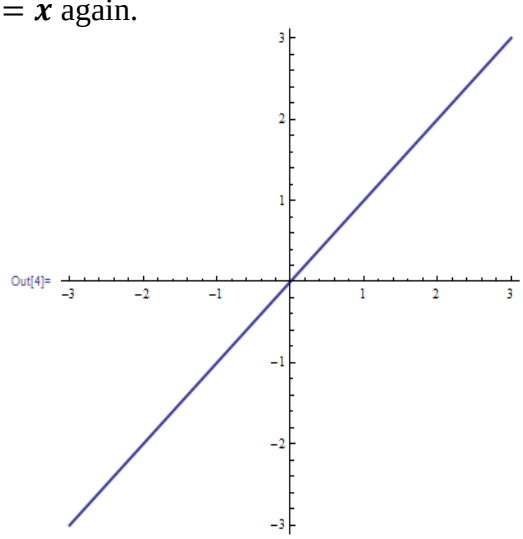


Vertical and Horizontal Stretching and Reflecting (Stewart text, Section 1.3, page 38):

Suppose $c > 1$. To obtain the graph of

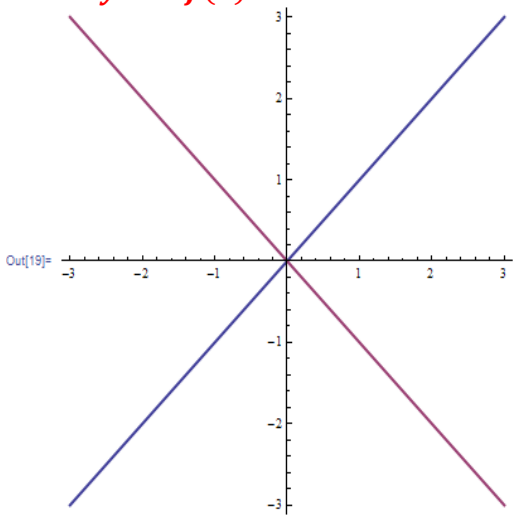
1. $y = c * f(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c.
2. $y = \left(1/c\right) * f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c.
3. $y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c.
4. $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c.
5. $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
5. $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

2. Graph the function $\boldsymbol{f(x) = x}$ again.

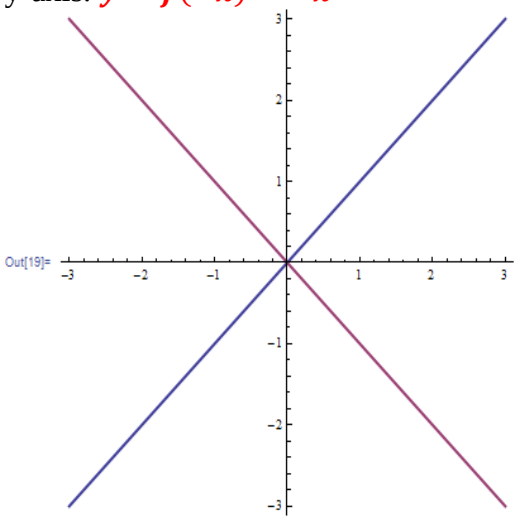


Write the equation that is obtained from the graph of $\boldsymbol{f(x)}$ with the following modification, and then graph the resulting function on the same graph with $\boldsymbol{f(x)}$:

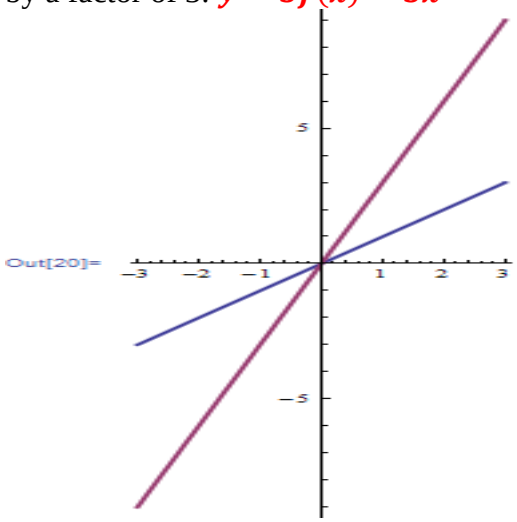
a. Reflect about the x-axis: $\boldsymbol{y = -f(x) = -x}$



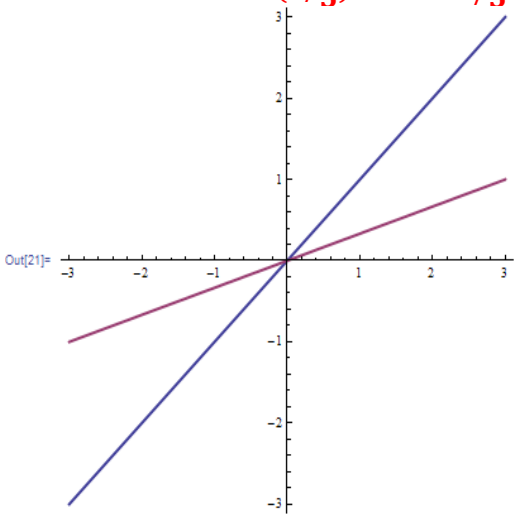
b. Reflect about the y-axis: $\boldsymbol{y = f(-x) = -x}$



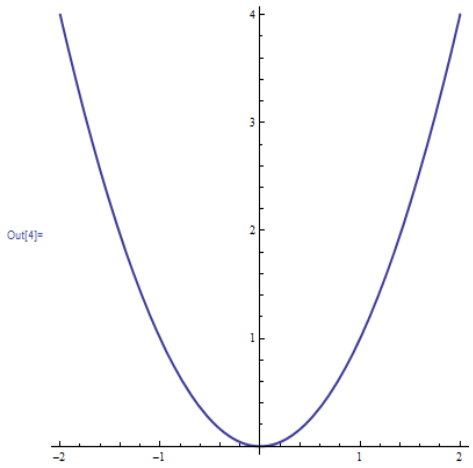
c. Stretch vertically by a factor of 3: $\boldsymbol{y = 3f(x) = 3x}$



d. Shrink vertically by a factor of 3: $\boldsymbol{y = (1/3)f(x) = x/3}$

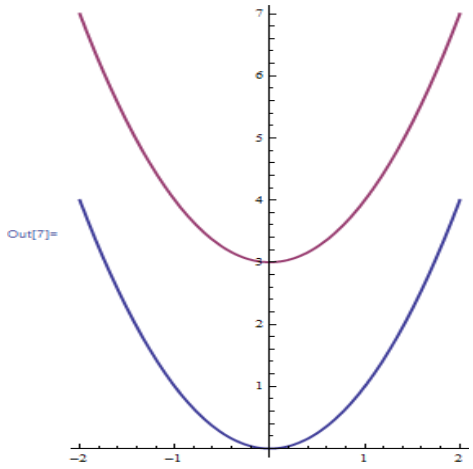


3. Graph the function $\boldsymbol{g(x) = x^2}$.

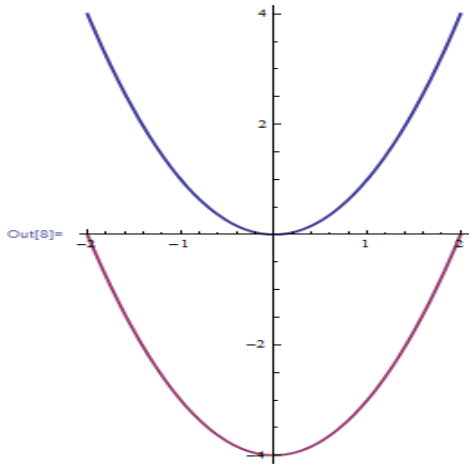


Graph the following functions on the same graph with $\boldsymbol{g(x)}$:

a. $y = x^2 + 3$

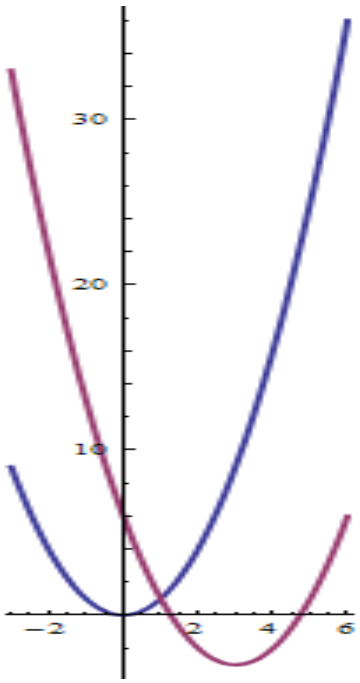


b. $y = x^2 - 4$



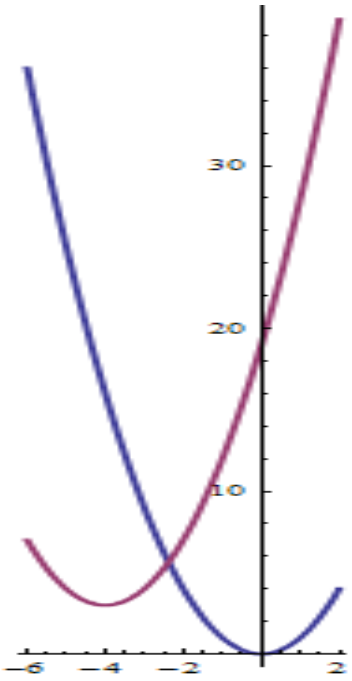
c. $y = (x-3)^2 - 3$

Out[15]=

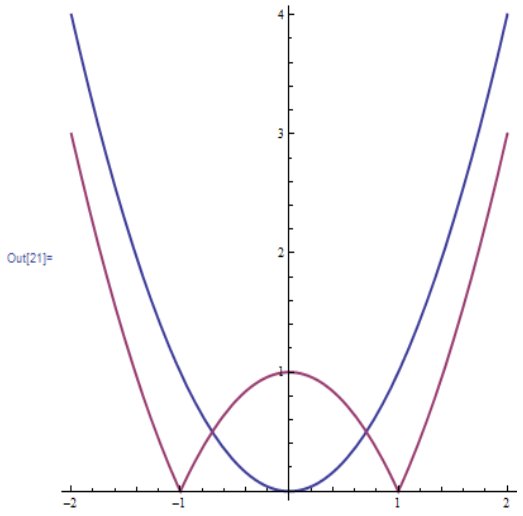


d. $y = (x+4)^2 + 3$

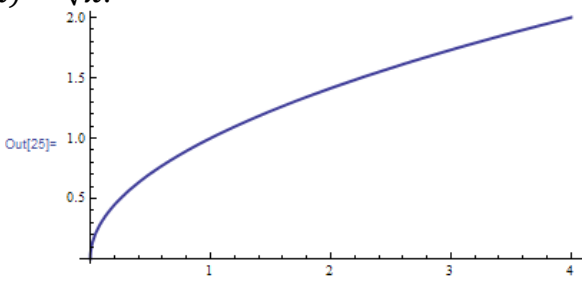
Out[19]=



e. $y = |x^2 - 1|$

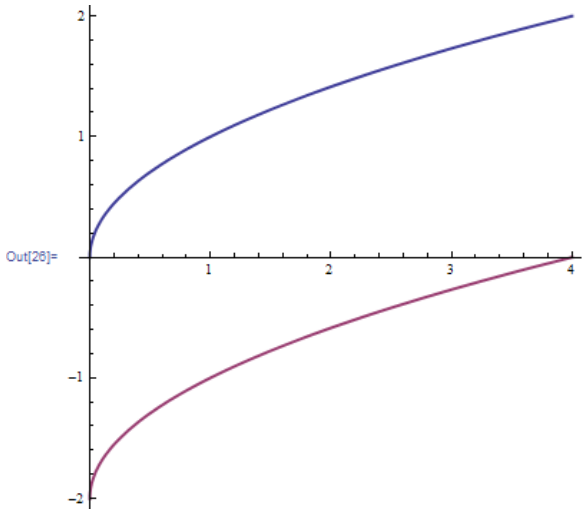


4. Graph the function $\boldsymbol{h(x)} = \sqrt{\boldsymbol{x}}$.

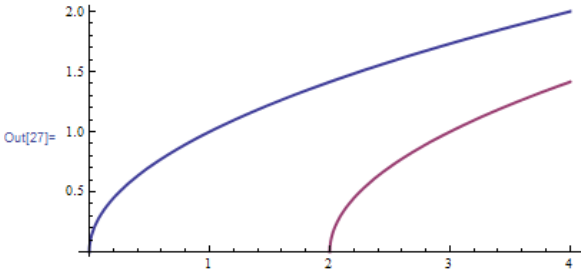


Graph the following functions on the same graph with $\boldsymbol{h(x)}$:

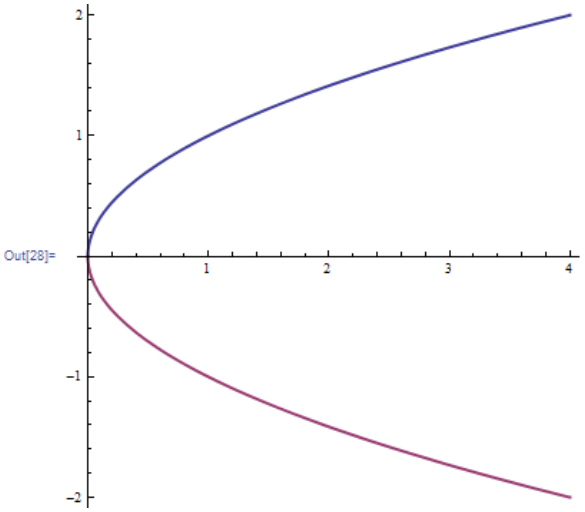
a. $y = \sqrt{x} - 2$



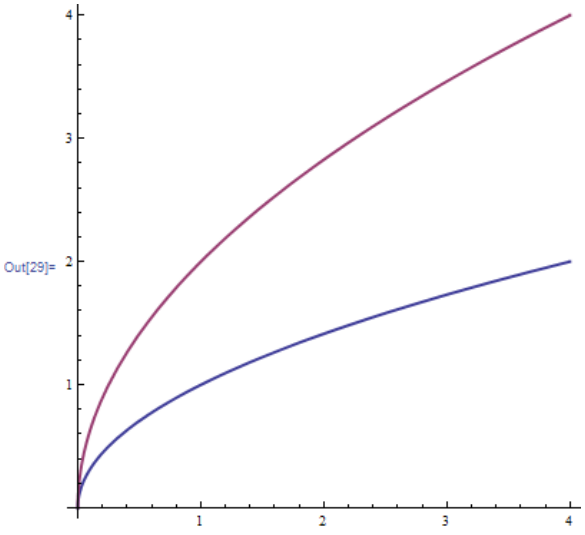
b. $y = \sqrt{x-2}$



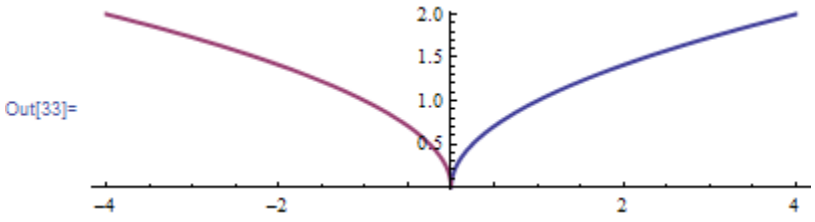
c. $y = -\sqrt{x}$



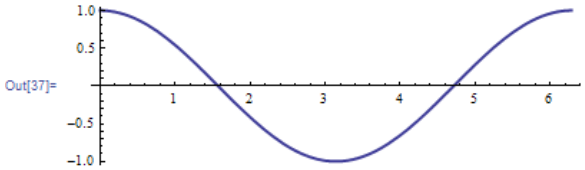
d. $y = 2\sqrt{x}$



e. $y = \sqrt{-x}$

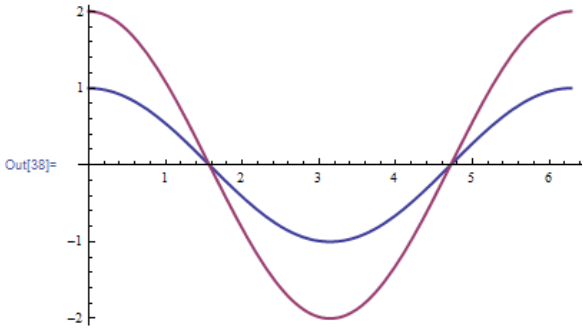


5. Graph the function $\boldsymbol{k(x) = \cos x}$.

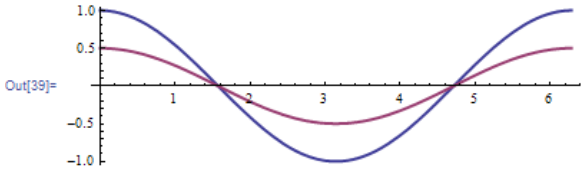


Graph the following functions on the same graph with $\boldsymbol{k(x)}$:

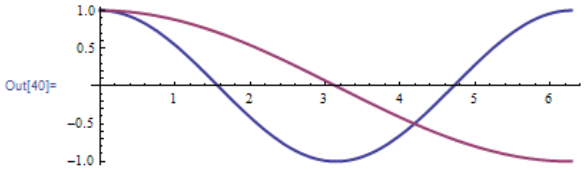
a. $y = 2\cos x$



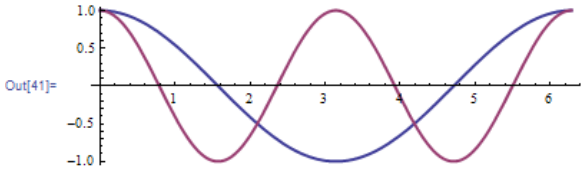
b. $y = (1/2)\cos x$



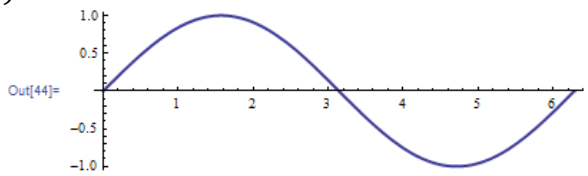
c. $y = \cos(x/2)$



d. $y = \cos(2x)$

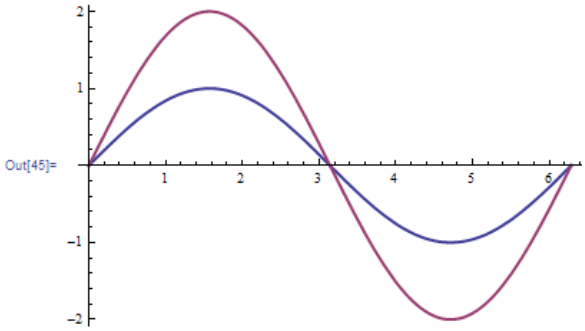


6. Graph the function $\boldsymbol{l(x) = \sin x}$.

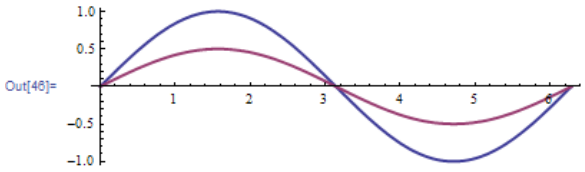


Graph the following functions on the same graph with $\boldsymbol{l(x)}$:

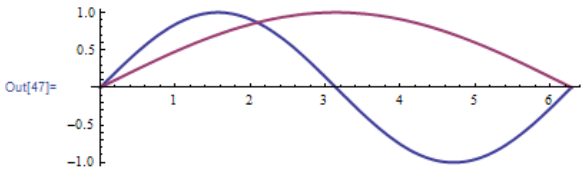
a. $y = 2 \sin x$



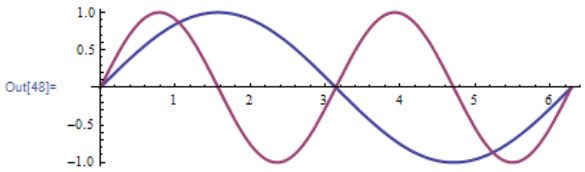
b. $y = (1/2) \sin x$



c. $y = \sin(x/2)$

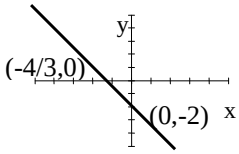


d. $y = \sin(2x)$

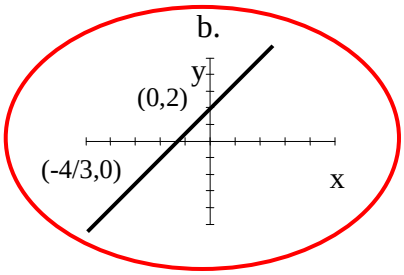


7. Which of the following is the graph of the function $2y - 3x = 4$?

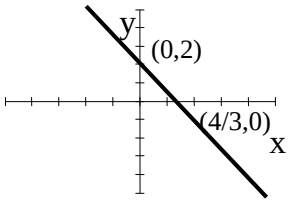
a.



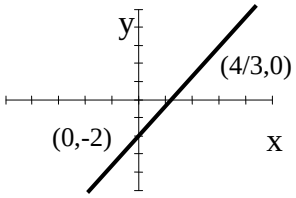
b.



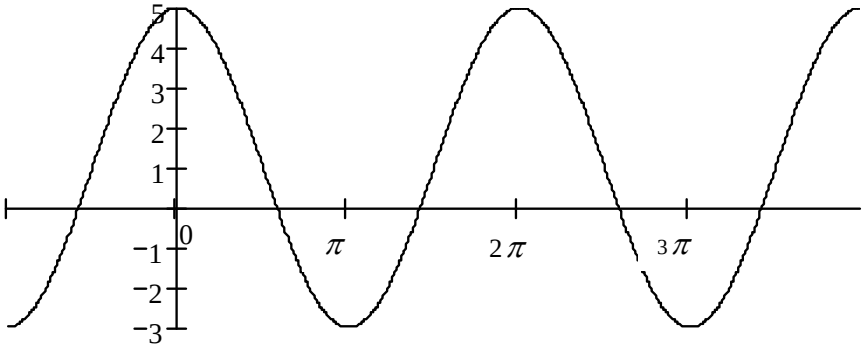
c.



d.



8. What is an equation of the following graph?



a. $4 \sin x + 1$

b. $4 \cos x + 1$

c. $5 \sin x$

d. $5 \cos x$

9. Given $f(x) = \sin x$, and $g(x) = 1 - \sqrt{x}$, find the functions $f \circ g, g \circ f, f \circ f, g \circ g$ and state their domains.

$f \circ g = \sin(1 - \sqrt{x})$; Domain $= \{x \mid 0 \leq x \leq 1\}$

$g \circ f = 1 - \sqrt{\sin x}$; Domain $= \{x \mid 0 \leq x \leq \pi\}$

$f \circ f = \sin(\sin x)$; Domain $= \{x \mid x \in \mathbb{R}\}$

$g \circ g = 1 - \sqrt{1 - \sqrt{x}}$; Domain $= \{x \mid 0 \leq x \leq 1\}$

10. Given $f(x) = 1 - 3x$, and $g(x) = 5x^2 + 3x + 2$, find the functions $f \circ g$, and $g \circ f$, and state their domains.

$f \circ g = 1 - 3(5x^2 + 3x + 2)$; Domain $= \{x \mid x \in \mathbb{R}\}$

$g \circ f = 5(1 - 3x)^2 + 3(1 - 3x) + 2$; Domain $= \{x \mid x \in \mathbb{R}\}$

11. Given $(x) = \sqrt{x - 1}$, $g(x) = x^2 + 2$, and $h(x) = x + 3$, find the function $f \circ g \circ h$, and state its domain.

$f \circ g \circ h = \sqrt{((x + 3)^2 + 2) - 1}$; Domain $= \{x \mid x \in \mathbb{R}\}$