Logarithms Solution

Laws of logarithms:

1. *if* $\log_a(x) = y$ *then* $a^y = x$	2. $\log_a x = \dfrac{\ln x}{\ln a}$, $\log_e x = \ln x$
3. $\ln(e^x) = x, \ln e = 1$	4. *if* $\ln x = y$, *then* $e^y = x$
5. $\log_a(xy) = \log_a x + \log_a y$	6. $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$
7. $\log_a x^r = r \log_a x$	

1. Using the laws of logarithms, what are the following equivalent to?

a. $\log_b(x) - \log_b(y) =$ **$\log_b(\frac{x}{y})$**

b. $\log_b(u) + \log_b(v) =$ **$\log_b(uv)$**

c. $\log_a y^2 + \log_a x^3 =$ **$\log_a(x^3y^2)$**

d. $\log_a y^3 + \log_a x^4 =$ **$\log_a(x^4y^3)$**

e. $\ln(e) =$ **1**

f. $\ln a + \dfrac{1}{2}\ln b =$ **$\ln(ab^{1/2})$**

g. $2\ln 4 - \ln 2 =$ **$\ln 8$**

h. $\ln x + a\ln y - b\ln z =$ **$\ln(\frac{xy^a}{z^b})$**

2. Solve the following for x using known logarithm properties.

a. $1\ln x = 2$

$x = e^2$

b. $\ln x = -1$

$x = e^{-1} = \frac{1}{e}$

c. $\ln x = 5$

$x = e^5$

d. $13^x = 6$

$x = \log_{13} 6$

e. $7 = 2^x$

$x = \log_2 7$

f. $9^x = 4$

$x = \log_9 4$

g. $e^{5-3x} = 10$

$5 - 3x = \ln 10$

$x = \frac{\ln 10 - 5}{-3}$

h. $(2\text{-}\ln x)\ln x = 0$

$2 - \ln x = 0$; $(\ln x > 0)$

$x = e^2$

i. $2\ln x = 1$

$x = \sqrt{e}$

j. $e^{-x} = 5$

$x = -\ln 5$

k. $e^{2x+3} - 7 = 0$

$\mathbf{2x + 3 = \ln 7}$

$\mathbf{x = \frac{\ln 7 - 3}{2}}$

l. $\ln(5 - 2x) = -3$

$\mathbf{5 - 2x = e^{-3}}$

$\mathbf{x = \frac{e^{-3} - 5}{-2}}$

m. $2^{x-5} = 3$

$\mathbf{x - 5 = 8}$

$\mathbf{x = 13}$

n. $\ln x + \ln(x - 1) = 1$

$\mathbf{x^2 - x = e}$

$\mathbf{x^2 - x - e = 0}$

$\mathbf{x = \frac{1 \pm \sqrt{1 + 4e}}{2}}$

$\mathbf{x = \frac{1 + \sqrt{1 + 4e}}{2}; (x > 0)}$

3. Find the value of each expression.

a. $\log_2 64 =$ $\mathbf{\log_2 2^6 = 6}$

b. $\log_6 \frac{1}{36} =$ $\mathbf{\log_6 6^{-2} = -2}$

c. $\log_8 2 =$ $\mathbf{\log_8 8^{1/3} = \frac{1}{3}}$

d. $\ln e^{\sqrt{2}} =$ $\mathbf{\sqrt{2}}$

e. $\log_{10} 1.25 + \log_{10} 80 =$ $\mathbf{\log_{10} 100 = 2}$

f. $\log_5 10 + \log_5 20 - 3\log_5 2 =$ $\mathbf{\log_5 \frac{200}{8} = \log_5 25 = 2}$

g. $2^{(\log_2 3 + \log_2 5)} =$ $\mathbf{2^{\log_2 3} 2^{\log_2 5} = (3)(5) = 15}$

h. $e^{3\ln 2} =$ $\mathbf{(e^{\ln 2})^3 = 2^3 = 8}$