BLOCK II: VECTOR FUNCTIONS AND THE GEOMETRY OF SPACE

7	**16-Feb-09**	**17-Feb-09**	**18-Feb-09**	**19-Feb-09**	**20-Feb-09**
					Lesson 25: Parametric Equations 1
8	**23-Feb-09**	**24-Feb-09**	**25-Feb-09**	**26-Feb-09**	**27-Feb-09**
	Lesson 26: Parametric Equations 2 **(Part 1 of Project DUE)**	Lesson 27: Intro to Vectors and Vector Operations	Lesson 28: Advanced Vector Operations	Lesson 29: The Dot Product and its Applications	**Lesson 30: CME Bridge Building**
9	**2-Mar-09**	**3-Mar-09**	**4-Mar-09**	**5-Mar-09**	**6-Mar-09**
	Lesson 31: The Cross Product and its Applications	Lesson 32: Equations of Lines in Space	**IT Lab: Java Script (NO graded events)**	**IT Lab: Java Script (NO graded events)**	Lesson 33: PSL 3
10	**9-Mar-09**	**10-Mar-09**	**11-Mar-09**	**12-Mar-09**	**13-Mar-09**
	Lesson 34: Equations of Planes in Space	Lesson 35: PSL 4	**No Class**	Lesson 36: Vector Functions and their Derivatives	Lesson 37: Projectile Motion 1
11	**16-Mar-09**	**17-Mar-09**	**18-Mar-09**	**19-Mar-09**	**20-Mar-09**
	Spring Break	**Spring Break**	**Spring Break**	**Spring Break**	**Spring Break**
12	**23-Mar-09**	**24-Mar-09**	**25-Mar-09**	**26-Mar-09**	**27-Mar-09**
	Lesson 38: Technology Lab (Issue Part 2 of Project)	Lesson 39: Projectile Motion 2	Lesson 40: PSL 5	**Lesson 41: WPR 2 (Dean's Hour)**	No Class

Block Objective:

Use vectors and vector functions to model and solve problems involving objects in space.

Supporting Objectives:

1. Develop parametric equations, and understand how to use them to solve problems.
2. Develop vector functions that describe the motion of an object through space.
3. Develop mathematical models used to depict scenarios involving lines and planes in space.
4. Understand how to determine rates of change in a three-dimensional (3D) system.
5. Know the graphical and physical interpretation of the derivatives of vector functions.
6. Know how to calculate and interpret the dot and cross products.
7. Know how to differentiate vector functions.

Technology:

1. Define and plot parametric equations and vector functions using *Mathematica*.
2. Differentiate a vector function in *Mathematica*.
3. Solve a system of equations using *Mathematica*.
4. Find the dot and cross product using *Mathematica*.

Useful *Mathematica* Commands for Block 2 (*Reminder commands from Block 1 will also be helpful):*

1. `ParametricPlot` & `ParametricPlot3D`
2. `FindRoot` (*An alternative to Solve*)

3. `Show`
4. `Dot` *or* `.`
5. `Cross`
6. `Norm`

Think about: -How will your knowledge of trigonometry and geometry help you in Block II?
-What does { } represent in *Mathematica*? Can it be used to mean different things?
-How can you use the **`Solve`** command in *Mathematica* to solve for more than one variable?
-What is the difference between an intersection of two objects along two separate paths and a collision of two objects along two separate paths?
-Does *Mathematica* compute angles in radians or degrees?

Links to Other Pages: Block II: Overview Block II: Endstate Problems

BLOCK II - ENDSTATE PROBLEMS

By the end of Block II, you should be comfortable solving problems similar to the ones suggested in this section. The problems are labeled as Level 1, Level 2, and Level 3 type problems. The level of difficulty for WPR questions are most similar to the level of difficulty associated with the Level 2 type problems below. Level 1 type problems are most similar to some easier text "Do Problems", and level 3 type problems are comprised of greater difficultly problem solving scenarios. The WPRs ***do*** require you to perform problem solving skills and the questions below are not inclusive of all the Block Objectives.

Level 1

1. Find the parametric equations for the line through points (1, 3, 5) and (0, 1, 0).

2. Find the equation of the plane though the points (0, 0, 0), (1, 1, 1) , and (1, 2, 3).

3. Find the Dot Product of the points (1, 3, 5) and (0, 1, 0).

4. A projectile is fired with an initial speed of 500 meters per second at an angle of 30 degrees. Assuming no wind resistance and perfectly flat terrain, what is the range of the projectile? What is the maximum height of the projectile?

Level 2

5. Tom Brady (quarterback for the New England Patriots) releases a pass at a height of 7 feet above the playing field, and the football is caught by a receiver 30 yards directly downfield at a height of 4 feet. The pass is released at an angle of 35 degrees with the horizontal.
    a. Find the speed of the football when it is released.
    b. Find the maximum height of the football.
    c. Find the time the receiver has to reach the proper position after Tom Brady releases the football.

6. The position function $\vec{r}(t)=\left\langle e^t\cos t, e^t\sin t, 4te^{2t}\right\rangle$ describes the path of an object through space. Find the object's speed and its velocity and acceleration vectors after 2 seconds.

7. The position function of a particle is given by $\vec{r}(t)=t^2\hat{i}+5t\hat{j}+(t^2-16t)\hat{k}$. When is the speed a minimum?

8. Find the parametric equations of the line of intersection of the planes $x+y+z=1$ and $x+2y+2z=1$.

9. A cadet exerts a horizontal force of 35 lbs on a BP cart that he pushes up a ramp that is 12 feet long and inclined at an angle of 20 degrees above the horizontal. Find the work done on the BP cart.

10. Do the paths of the objects with the following space curves intersect: $\langle 2t, 3t+1 \rangle$ and $\left\langle 5-t,\ \frac{t}{2} \right\rangle$? Do they intersect? If they do, show where. Can you tell what the geometric shapes of these curves are? How?

Level 3

11. True/False. If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, then $(\vec{a} \times \vec{b}) \cdot \vec{a} = (\vec{b} \times \vec{a}) \cdot \vec{b}$. Fully justify your answer.

12. A bicycle with a front-wheel brake comes to a sudden stop. The horizontal braking force exerted by the brake on the front wheel is 650 Newtons. The center of mass of the bicycle and its rider is 90 centimeters above the ground and 70 centimeters behind the point at which the front wheel touches the ground.[1]
    a. What is the torque of this force about the center of mass?
    b. What is the direction of torque?

13. The space shuttle is sent to deliver a new crew to the Space Station Mir. The path of the Shuttle is defined by the set of parametric equations $\mathbf{S}(t) = \left\langle \sqrt{t}, 3t+5, -4t \right\rangle$. The set of parametric equations $\mathbf{M}(t) = \left\langle 4\sin(t), -6\cos(t), -t^2 \right\rangle$ describes the path of the Mir. Does the space shuttle make its link up with Mir or did someone used to getting 'follows' credit just waste $20,000,000?

Links to Other Pages: [Block II: Overview] [Block II: Endstate Problems]

[1] Garret Etgen , Einar Hille, *Calculus, One and Several Variables, 8th Edition,* John Wiley & Sons, Inc, 1999, pg 748.

LESSON 25 – PARAMETRIC EQUATIONS I

OBJECTIVES:

1. Understand that parametric equations determine the coordinates of a point on a curve using functions of the same variable.
2. Determine the Cartesian equation of a function from a set of parametric equations and vice-versa.

READ:

1. Stewart: Section 10.1, pages 621-624 (stop after the section on *Graphing Devices*).
2. Student Notes.

THINK ABOUT: What do parametric equations allow us to do that we can't accomplish using regular functions?

DO: Section 10.1/ 1, 3, 5, 7 (instead of sketching use the "`ParametricPlot`" function in *Mathematica* to discover the direction of the curve), 33.
Following Problem:

$f(x) = \ln(x + x^3)$

Find $\dfrac{df}{dx}$.

STUDENT NOTES:

Example 6 on page 624 shows how to use a graphing calculator to look at parametric curves. Since you were issued *Mathematica* and not a graphing calculator, this example is shown below using *Mathematica.*

Example 6: Graph the curve $x = y^4 - 3y^2$.

SOLUTION: If we let the parameter be *t*, then we have the equations:

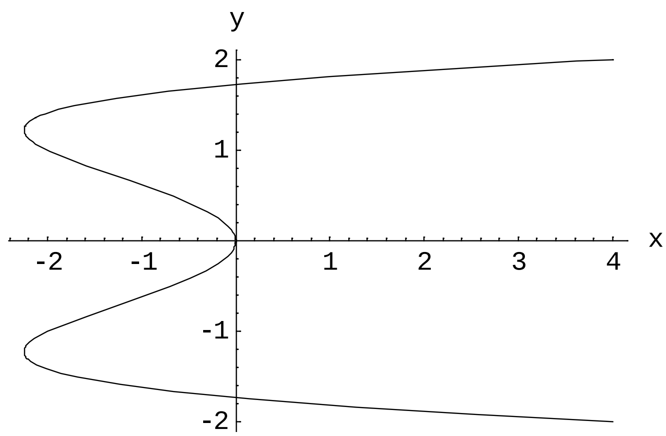
$$x = t^4 - 3t^2$$
$$y = t$$

In *Mathematica*, we can define both *x* and *y* as functions of this new parameter:

x[t_] = t⁴-3t²
y[t_] = t

In order to graph this, the command that we use is "ParametricPlot" as seen below:

```
ParametricPlot[{x[t],y[t]},{t,-2,2},AxesLabel->{"x","y"}]
```



In this example, we graphed this from $t = -2$ to $t = 2$. In many instances (specifically when the parameter *t* is time, then we would not use a negative number, but this range of *t* gives us a graph that looks similar to the one in the textbook on page 624).

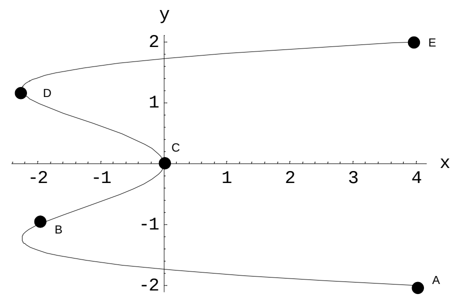
A problem with *Mathematica* is that it draws the graph so quickly, that it is hard to determine which direction an object is going. The axes are the *x* and *y* axes, and *t* is basically *invisible*, as it draws the curve itself. The most direct way of determining the direction is to create a modified T chart with *t, x,* and *y*. Simply substitute in a few *t* values into the parametric equations to discover which direction the object is going. See below:

t	$x = t^4 - 3t^2$	$y = t$
-2	4	2
-1	-2	-1
0	0	0
1	-2	1
2	4	2

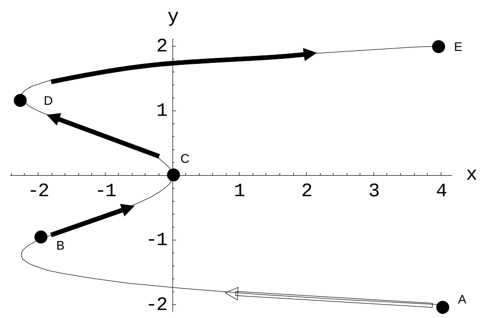
Additionally, Below are the lines of code required to find five points along the curve.

Point A: **`in`** **`a={x[-2],y[-2]}`**
out {4,-2}
Point B: **`in`** **`b={x[-1],y[-1]}`**
out {-2,-1}
Point C: in **`c={x[0],y[0]}`**
out {0,0}
Point D: **`in`** **`d={x[1],y[1]}`**
out {-2,1}
Point E: **`in`** **`e={x[2],y[2]}`**
out {4,2}

If we put these points on the curve we see the following (*using a* **`ListPlot`** *and a* **`Show`** *command*):



Therefore, the direction that the object is traveling is from point *a* to point *e*.



Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 26 – PARAMETRIC EQUATIONS II: INTERSECTION AND COLLISION

OBJECTIVES:

1. Determine if the paths of two objects intersect given parametric equations modeling the paths of the objects in space.
2. Determine whether two objects collide given parametric equations modeling their motion in space.

READ: Student Notes.

THINK ABOUT: What does it mean for objects to intersect or collide in terms of modeling their motion with parametric equations?

DO: Do Problems 1 and 2 – found at the end of this lesson in the Student Notes.
Following Problem:

$f(t) = \cos(52t)$

find $\dfrac{df}{dt}$

STUDENT NOTES:

I. Collisions

Collision Problems in $\mathbb{R}^3$ (3-dimensional space) have applications throughout the scientific world from military problems (does a missile hit its target?) to rocket science (does the spacecraft land on the intended planet?). In order for two objects to collide they must occupy the ***same position*** in space at exactly the ***same time***. This is accomplished by representing the positions of both objects using the same parameter (usually time). Then, set each component (*x, y,* and *z*) of one set of parametric equations equal to the other to form a system of equations. Each set of parametric equations must use the same parameter in order to determine if a collision takes place. This system of equations is solved, if possible, to determine the parameter value at the collision. If no solution exists to the system of equations, then the objects never collide. For a collision to occur, each component of the two objects' positions must be equal at a single parameter value.

Often when solving analytically, the simplest equation in the system of equations is solved. Solving this equality results in the time(s) where both objects have the same position for that component. These times are substituted into the other components and determine if the equalities are true. If a single time value places both objects at the same point, then they collide. If not, no collision occurs.

Example: If the path of particle A is traced by the set of parametric equations[2] $\langle x_A(t), y_A(t), z_A(t)\rangle = \langle t, (t^2+4), 4\rangle$, and path of particle B is traced by the set of parametric

[2] It is useful to write the *x* and *y* components to a set of parametric equations as the *x* and *y* components of a vector function. Vector functions will be covered in detail later in the text during Chapter 13.

equations $\langle x_B(t), y_B(t), z_B(t)\rangle = \left\langle \frac{t^2}{4}, 5t, t \right\rangle$, do the particles collide? We begin by setting each component equal to each other to create our system of equations:

$$t = \frac{t^2}{4} \qquad \text{equation 1}$$

$$t^2 + 4 = 5t \qquad \text{equation 2}$$

$$4 = t \qquad \text{equation 3}$$

We solve the first equation for t which yields $t = 0,\ and\ t = 4$. We then substitute $t = 0$ into each of the components of the two position vectors to see if the particles are at the same location at time $t = 0$.

$$\langle x_A(0), y_A(0), z_A(0)\rangle = \langle 0, (0^2 + 4), 4\rangle = \langle 0,4,4\rangle$$

$$\langle x_B(0), y_B(0), z_B(0)\rangle = \left\langle \frac{0^2}{4}, 5*0, 0 \right\rangle = \langle 0,0,0\rangle$$

Therefore, there is no collision at time $t = 0$ since both particles are not at the same place at $t = 0$. We then substitute $t = 4$ into each of the components of the two position vectors to see if the particles are at the same location at time $t = 4$.

$$\langle x_A(4), y_A(4), z_A(4)\rangle = \langle 4, (4^2 + 4), 4\rangle = \langle 4, 20, 4\rangle$$

$$\langle x_B(4), y_B(4), z_B(4)\rangle = \left\langle \frac{4^2}{4}, 5*4, 4 \right\rangle = \langle 4, 20, 4\rangle$$

Since both particles are at the same location at the time $t = 4$, a collision occurs at the point $(4, 20, 4)$. Note that it would be much easier if we initially set the *z*-components equal. We would have seen immediately that $t = 4$ is the only possible collision time. Therefore, when solving analytically, select the component (*x, y,* or *z*) of the parametric equations that is easiest to solve.

II. Intersections

Determining if the paths of two objects intersect is very similar to collisions except that the parameter values no longer need to be exactly the same at the point of intersection. The position of the two objects must be represented using two different parameters. Each position is represented by its own parametric variable. If an intersection occurs, each component of the two objects' positions must be equal given a single parameter value for each object. This is accomplished by setting each component (*x*, *y*, and *z*) of one position equal to the same component of the other to form a system of equations. This system of equations is solved, if possible, to determine the parameter value for each object where the paths intersect. A solution to the system may not exist in which case the objects never intersect.

Example: Do the paths of the objects with the following space curves intersect: $\overrightarrow{curve_1} = \langle 2t, 3t+1 \rangle$ and $\overrightarrow{curve_2} = \left\langle 5-t, \quad \frac{t}{2} \right\rangle$? Do they intersect? If they do, show where. Can you tell what the geometric shapes of these curves are? How?

In order to see if they intersect, we set each component equal to each as we did before to form the system of equations:

$$2t = 5 - t \qquad \text{equation 4}$$

$$3t + 1 = \frac{t}{2} \qquad \text{equation 5}$$

Mathematically, solving for the value of *t* with equation 4 does not satisfy equation 5 nor vise versa. As such, it is helpful in mathematics at times to rename the parameter *t* to create two equations with two unknowns. As such, subscripts are added to equations 4 and 5 to gain equations 6 and 7 below: (This technique will be useful when solving in *Mathematica* too.)

$$2t_{curve1} = 5 - t_{curve2} \qquad \text{equation 6}$$

$$3t_{curve1} + 1 = \frac{t_{curve2}}{2} \qquad \text{equation 7}$$

This system of equations has a solution at $t_{curve1} = \frac{3}{8},\ and\ t_{curve2} = \frac{17}{4}$. When $t_{curve1} = \frac{3}{8}$, our first object is located at the point $\left\langle 2\left(\frac{3}{8}\right), 3\left(\frac{3}{8}\right)+1 \right\rangle = \left\langle \frac{3}{4}, \frac{17}{8} \right\rangle$, and when $t_{curve2} = \frac{17}{4}$, our second object is located at $\left\langle 5 - \frac{17}{4}, \frac{\frac{17}{4}}{2} \right\rangle = \left\langle \frac{3}{4}, \frac{17}{8} \right\rangle$. Therefore, we can see that the point of intersection of the two objects is at $\left\langle \frac{3}{4}, \frac{17}{8} \right\rangle$ since they both reach the same point on their individual space curves but at *different* times.

We can also determine of the two objects collide using the analysis above. We use one parameter and set each component equal to each other to form the system of equations:

$$2t = 5 - t$$

$$3t + 1 = \frac{t}{2}$$

As noted earlier, this system of equations has no solution therefore they do not collide.

DO PROBLEMS:

1. The position of a bomb is parameterized by the set of parametric equations $\mathbf{B}(t) = \langle t, 20t - t^2 \rangle$. Anticipating that this bomb will be launched, an alert crew attempts to intercept it with a missile whose path is given by the set of parametric equations $\mathbf{M}(t) = \langle 30 - 5t, 15t \rangle$. Does the missile hit the bomb? If so, when and where? Does the path of the missile intersect the path of the bomb? If so, when and where?

2. You are an Infantry platoon leader on a defensive mission at the National Training Center. You are digging two tunnels with two teams, TM A and TM B, each with one 16-inch router. You model the underground dig pattern of TM A as $x = t + 1$ and $y = t$, and that of TM B as $x = 6 - 2t$ and $y = 2t$, where t is in hours and x and y are in meters for both models. Both of the models hold for $0 \le t \le 3$. (The router teams start digging at t = 0 and must finish three hours later). Will the routers collide? Show why or why not. Will the paths of the routers intersect? If so, where? Fully justify your answers.

3. Assume the path an African Swallow travels along the curve drawn by the parametric equations: $x(t) = 3 + (3 - 0.5t)^2$ and $y(t) = t$.
   And the path of an European Swallow travels along the curve drawn by the parametric equations: $x(t) = 8 - t$ and $y(t) = 1 + t$.
   Can you infer the answers to questions a. and b. below first without using technology?
   a. What is the geometric shape of the parametric curve describing the African Swallow's path?
   b. What is the geometric shape of the parametric curve describing the African Swallow's path?
   c. How many occasions do the flight paths of the two Swallow's intersect? Do they ever collide? List all the coordinates of intersection and/or collisions --- and include the flight time for each bird to get to that coordinate. Did you make any assumptions, if so, what where they?
   d. Assuming that there was no collision, the European Swallow gets upset, changes direction and then spits at the African Swallow. The African swallow, follows the same course as before $x(t) = 3 + (3 - 0.5t)^2$ and $y(t) = t$ but the spit from the European Swallow travels along the curve drawn by the $x(t) = 4 + (2 + t)^2$ and $y(t) = t$. Does the spit from the European swallow hit the African Swallow for any values of t > 0. If so, when and where? [3]

Links to Other Pages: Block II: Overview Block II: Endstate Problems

[3] Problem developed by MAJ Charles Sulewski, Assistant Professor Department of Mathematical Sciences, United States Military Academy, June 11, 2008.

LESSON 27 – INTRODUCTION TO VECTORS AND VECTOR OPERATIONS

OBJECTIVES:

1. Understand what a vector is and the properties of vectors given on page 774.
2. Understand vector addition, subtraction, and scalar multiplication algebraically, physically, and graphically.
3. Understand what a unit vector is and how to calculate the unit vector for any given vector.
4. Understand how vectors can be used to describe several forces acting on an object, and how the resultant force is the sum of these vectors.
5. Determine a vector between two points.

READ: Stewart: Section 12.2, pages 770-776.

THINK ABOUT:

1. What is a *unit vector*?
2. What information can a *unit vector* relate?

DO: Section 12.2/ 4, 7, 9, 15, 19, 22, 24, 26
Following Problem:

$$h(x) = e^5 + x^2 + z^2$$

find $\frac{dh}{dx}$

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 28 – ADVANCED VECTOR OPERATIONS

OBJECTIVES:

1. Understand how vectors can be used to describe several forces acting on an object, and how the resultant force is the sum of these vectors.
2. Model and solve application problems using vectors and vector operations.

READ: Stewart: Section 12.2, pages 770-776.

THINK ABOUT:

What is a *unit vector*?
What information can a *unit vector* relate?

DO: Section 12.2/ 3, 30, 31, 33, 35
Following Problem:

$$f(x) = \ln\left(x^4 + 2x\right)$$

Find $f'(x)$.

Links to Other Pages: Block II: Overview   Block II: Endstate Problems

LESSON 29 – THE DOT PRODUCT AND ITS APPLICATIONS

OBJECTIVES:

1. Understand the definition of the dot product and how it can be computed.
2. Understand what it means for two vectors to be orthogonal.
3. Understand what a vector and scalar projection is and how they are computed. (This lesson objective will not be covered in class. Students are encouraged to learn this lesson objective on their own as they will see an application requiring knowledge of this objective in the future. Students will also be assessed on their understanding of this objective in future classroom evaluations.)
4. Apply the dot product to find the work done by a force.

READ: Stewart: Section 12.3, pages 779-784 (Skip *Direction Angles and Direction Cosines*).

THINK ABOUT: Interpret what the dot product of two vectors means.

DO: Section 12.3/ 5, 7, 9, 14, 18, 23, 35, 46, 48
Following Problem:

$f(x) = e^{5x}$

find $f'(x)$

STUDENT NOTES:

1. Understand when the vectors **a** and **b** are perpendicular (Definition 7, page 781).
2. Understand how the angle θ between two vectors is calculated using the dot products (Theorem 3 on page 780). This theorem will assist when applying the dot product to find the amount of work done upon an object.

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 30 – CIVIL AND MECHANICIAL ENGINEERING BRIDGE DESIGN

OBJECTIVES:

1. Apply vectors and vector operations to solve problems in civil engineering.
2. Become familiar with the Civil and Mechanical Engineering Department.

READ: None.

DO: Load the West Point Bridge Design software on your computer (Instructions will be sent out by your instructor)

Following Problem:

$$f(x) = 5\ln(3x+2)$$

find $f'(x)$

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 31 – THE CROSS PRODUCT AND ITS APPLICATIONS

OBJECTIVES:

1. Understand the definition of the cross product and what it gives us.
2. Know how to calculate the cross product using *Mathematica*.
3. Apply the cross product to model and solve problems.

READ:

1. Stewart: Section 12.4, the first two paragraphs on page 786 and pages 788-789 (Read and understand Theorems 5 and 6, and Corollary 7).
2. Student Notes.

THINK ABOUT: Interpret what the cross product of two vectors means.

DO: Section 12.4/ 4, 5, 7, 16, 19, 40, 41
Following Problem:

$$f(x) = e^{5x+x^2}$$

Find $\dfrac{df}{dt}$.

STUDENT NOTES:

There are a few things that you are required to know about the cross product as well. In MA104, you are not responsible for being able to calculate the cross product by hand (so you may ignore Definition 1 on page 786). You are responsible for understanding the following:

1. The cross product of two vectors, results in a third vector which is orthogonal to the two vectors (Theorem 5, page 788).
2. Understand how the angle θ between two vectors is calculated using the cross products (Theorem 6 on page 788).
3. Understand when the vectors **a** and **b** are parallel (Corollary 7, page 789).

Calculating the Cross Product in *Mathematica:*

Given two vectors: $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$

```
a = {a1, a2, a3}
b = {b1, b2, b3}
Cross[a,b]

{-a3b2 + a2b3, a3b1 – a1b3, -a2b1 + a1b2}
```

You can easily verify that this is the same answer as Definition 1 on page 786. You can also verify the answer to Example 1, found on page 787:

Example 1

If $\mathbf{a} = \langle 1, 3, 4 \rangle$ and $\mathbf{b} = \langle 2, 7, -5 \rangle$ find $\mathbf{a} \times \mathbf{b}$. The following represents possible *Mathematica* input and output

```
a = {1, 3, 4};
b = {2, 7, -5};
Cross[a,b]

{-43, 13, 1}
```

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 32 – EQUATIONS OF LINES IN SPACE

OBJECTIVES:

1. Understand how to develop the vector and parametric equations of a line in space using a vector.
2. Understand what two pieces of information are necessary to find the equation of a line in three dimensional space.

READ: Stewart: Section 12.5, pages 794-797 (Stop at Planes).

THINK ABOUT: How is finding the equation of a line in two dimensions similar to finding the equation of a line in three dimensions? How are they different?

DO: Section 12.5/ 3, 4, 13, 17, 18

Following Problem:

$$g(x) = (x^3 + 3)(4x^5 - x^{-3} + 2)^2$$

find $g'(x)$

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 33 - PROBLEM SOLVING LAB 3

OBJECTIVE:

Solve problems by applying concepts relevant to the current material. Your instructor will provide you guidance for this lesson.

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 34 – EQUATIONS OF PLANES IN SPACE

OBJECTIVES:

1. Understand how to develop the vector and scalar equations of a plane in space using vectors.
2. Develop an equation of a plane in space given either a point on the plane and a vector normal to the plane, or three points on the plane that do not lie on the same line.
3. Find the distance from a point to a given plane. (This lesson objective will not be covered in class. Students are encouraged to learn this lesson objective on their own as they will be assessed on their understanding of this objective in future classroom evaluations.)

READ: Stewart: Section 12.5, pages 797-801 (Start at Planes).

THINK ABOUT:

1. What are the possible geometries of the intersection between two planes in space?
2. How will the red box on page 783 be useful for this lesson?

DO: Section 12.5/ 5, 23, 29, 31, 35, 69

Following Problem:

$$v(t) = \frac{7t^3}{(2t^6 + 2)}$$

find $v'(t)$

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 35 - PROBLEM SOLVING LAB 4

OBJECTIVE:

Solve problems by applying concepts relevant to the current material. Your instructor will provide you guidance for this lesson.

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 36 – VECTOR FUNCTIONS AND THEIR DERIVATIVES

OBJECTIVES:

1. Understand the definition of a vector function and be able to find the domain of a vector function.
2. Understand how to calculate rates of change for vector functions.
3. Understand the physical interpretation of the derivative of a vector function.

READ:

1. Stewart: Section 13.1, pages 817-822.
2. Stewart: Section 13.2, pages 824-827 (Stop at *Integrals*).
3. Student Notes.

THINK ABOUT: How are vector functions related to parametric equations?

DO: Section 13.1/ 2, 9, 15, 19, 20, 24
Section 13.2/ 3, 8, 9, 13, 23
Following Problem:

$$r(t) = \langle t^3 \sin(5t), e^{-2t} t \rangle$$

If $\boldsymbol{r}(t)$ is a position vector, find the derivative of $\boldsymbol{r}(t)$ with respect to t.

STUDENT NOTES:

If you refer back to Lesson 25 on parametric equations and take a look at how we graphed these parametric equations in *Mathematica*, you'll notice that in the **`ParametricPlot`** command, each parametric equation was a separate component contained within a set of braces that looked like **`{x[t],y[t]}`**. These braces are *Mathematica*'s syntax for a vector, and since the components of this particular vector are functions which we call parametric equations (these functions are also called component functions in your text), **`{x[t],y[t]}`** is in reality a vector function. This should help you in finding a relationship between vector functions and parametric equations.

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 37 – PROJECTILE MOTION I

OBJECTIVES:

1. Understand the relationship between the position, velocity, and acceleration vectors for an object's motion in space.
2. Find the velocity and acceleration vectors when given the position vector and an initial condition.
3. Be able to use the parametric equations of motion in order to solve projectile motion problems.

READ:

1. Stewart: Section 13.4, pages 838-842 (Stop at *Tangential and Normal Components of Acceleration*).
2. Student Notes.

THINK ABOUT:

1. What does the magnitude of the velocity vector represent?
2. What assumptions are made in the book when solving projectile motion problems?

DO: Section 13.4/ 3, 5, 10, 13, 23
Following Problem:

$f(x) = \tan(2x)$

Find $f'(x)$.

STUDENTS NOTES:

In your reading tonight you will encounter the symbol $\int$, which, as some of you may know, is an integral. Another name for an integral, and a more helpful one to us, is an anti-derivative. What does it mean to find the anti-derivative of a function? Well, let's take a function and take its anti-derivative. Then, if we take the derivative of the anti-derivative, what are we left with? We are led back to where we started, the original function.

Example: Let $f(x) = 2x$. The anti-derivative of this function is $x^2 + C$, where C is a constant (do not worry about how we calculated that. You will study this in MA205). Now, if we take the derivative of $x^2 + C$, we get $2x$, which was our original function.

We can also understand this concept in the context of position, velocity, and acceleration. Velocity is the derivative of position (with respect to time), so position is the anti-derivative of velocity. Acceleration is the derivative of velocity, and so velocity is the anti-derivative of acceleration.

This idea of an anti-derivative is used in the derivation of the parametric equations of motion, and while we will never test you on this derivation (only on the uses of the result), you should have a general understanding of where these equations of motion come from.

WARNING. On page 841, Equations 4 omit initial conditions in either the horizontal (x_o) or vertical (y_o) parametric equations by assuming $\boldsymbol{r}(0) = \langle 0, 0 \rangle$. By omitting these, the authors continue to develop an equation for range $d = \frac{v_0^2 \sin 2\alpha}{g}$. ***NOTE: This formula for range is only valid when your initial vertical starting (y_o) position is equivalent to the final vertical position (y_{final}).***

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 38 – TECHNOLOGY LAB

OBJECTIVES:

1. Familiarize yourself with some of the additional capabilities of Mathematica in terms of graphing and solving problems.
2. Use a technology example to review some of the learning objectives of Block 2, namely parametric equations, intersection and collision, and projectile motion.

READ: Student Notes.

DO: Download the *Mathematica* file *golf.nb* from the MA104 Homepage link.
Following Problem:

$$r(t) = \left\langle t^3 e^t, \ln(t)\cos(3t) \right\rangle$$

If $\boldsymbol{r}(t)$ is a position vector, find $\boldsymbol{v}(t)$.

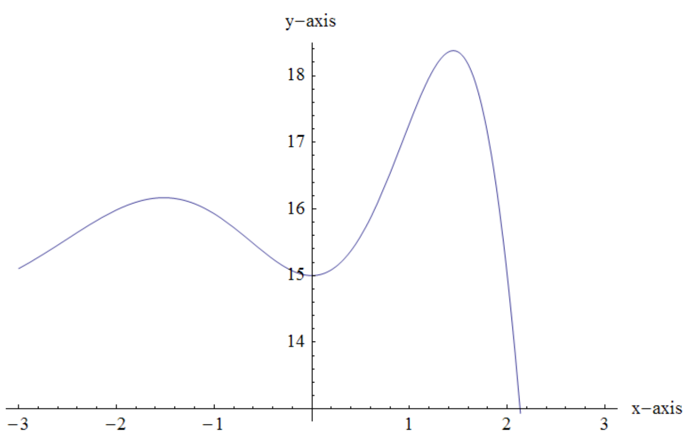
STUDENTS NOTES:

This lesson is designed to help you better utilize *Mathematica* when creating plots and solving problems, as well as prepare you for using some of the commands that will be very helpful during Block 3. In order to make the in class experience as beneficial as possible, we will work through a couple of small examples and show some of the capabilities of *Mathematica*.

First, we will learn some of the options available for the **`Plot`** command using the function $f(x) = e^x \sin(x)\left(2x - x^2\right) + 15$. The following is some code and output that should look familiar.

```
In[1]:= f[x_] = E^x Sin[x] (2 x - x^2) + 15;
        Plot[f[x], {x, -3, 3}, AxesLabel → {"x-axis", "y-axis"}]
```

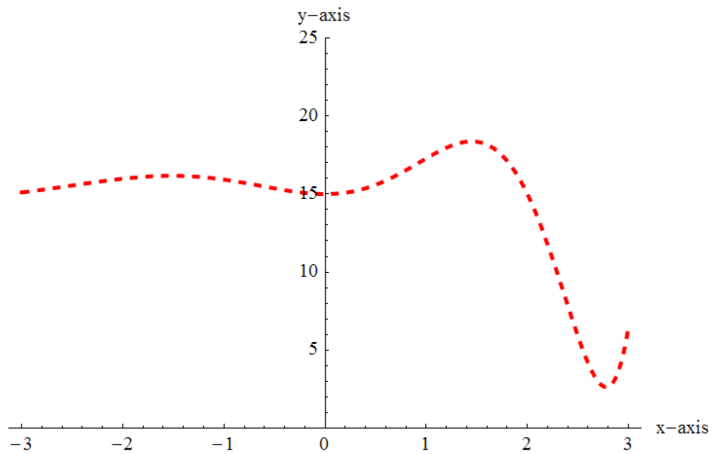
Out[2]=



Mathematica naturally selects the output range to display. In this case, it is only showing the function values for about 13 to 19. If we wanted to specify a specific range, then we could use the option command **`PlotRange`**. Also, if we wanted to change the look of the function then we could use the option **PlotStyle**. **PlotStyle** allows us to change not only the color, but the thickness, and type of line. What follows is our modified code and the graph it created.

```
In[3]:= Plot[f[x], {x, -3, 3}, AxesLabel → {"x-axis", "y-axis"}, PlotRange → {0, 25},
         PlotStyle → {Red, Thick, Dashed}]
```

Out[3]=

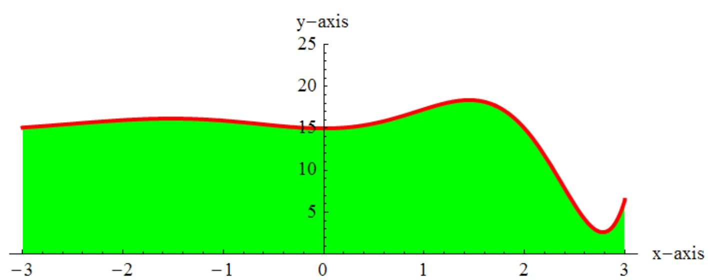


We know see the complete graph over our domain (-3, 3) and it appears that it has a local minimum of about 3 at x = 2.8. If we had chosen different values for our **PlotRange**, then we might not have seen the entire function. Additionally, the color of graph is now red, much thicker, and a dashed line. We can actually control these in more detail. If you did not want a dashed line, then you could simply omit it. The command **PlotStyle** requires at least one option (color, thickness, style, etc.).

Finally, for this graph we are going to change the ratio of the axes and shade below the graph. The first command is **AspectRatio**. It has a default of about 1 to 1.6 for the y to x ratios. The second command for the shading is **Filling**. There are several default options, but the three most important are **Top**, **Bottom**, and **Axis**. They will fill exactly as you expect. The **Axis** option will fill both above and below and to whatever horizontal axis is displayed (it does not have to be equal to zero). As with the **Plot**, you can change how you fill using **FillingStyle**. What follows is our final modifications on our plot.

```
In[5]:= Plot[f[x], {x, -3, 3}, AxesLabel → {"x-axis", "y-axis"}, PlotRange → {0, 25},
         PlotStyle → {Red, Thick}, AspectRatio → 1 / 3, Filling → Bottom, FillingStyle → Green]
```

Out[5]=



From this graph, we removed the **`Dashed`** option and filled below the curve in Green. The normal colors are all defaults, but you can choose any color by using different commands. You will see that in the *golf.nb* file.

The final part for this reading will want you to further explore using **`FindRoot`** when **`Solve`** does not work. For our function, there is a sine component and consequently it has an infinite number of local minimums and local maximums. If we use only the **`Solve`** command to find them, this is the result we get.

```
In[6]:= Solve[f'[x] == 0, x]
        Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way. >>
Out[6]= Solve[e^x (2 x - x^2) Cos[x] + e^x (2 - 2 x) Sin[x] + e^x (2 x - x^2) Sin[x] == 0, x]
```

This does not help us at all. In some instants, the **`Solve`** command will return only the solutions from $-\pi$ to π. This might not be what you are looking for, so you will have to use the **`FindRoot`** command. This command requires that you use an initial guess. So for this problem, let us try to find the absolute minimum. From the graph, it appears to about $x = 2.8$. The following code shows us how to use **`FindRoot.`**

```
In[7]:= FindRoot[f'[x] == 0, {x, 2.8}]
Out[7]= {x -> 2.77978}
```

From this we can see that the minimum in our picture occurs at $x = 2.77978$. Again, download the file and scan through the code before you come to class so you can see what has been done. We will walk through this during class, but if you have questions that are not answered during class, then ask.

<u>Links to Other Pages:</u> [Block II: Overview] [Block II: Endstate Problems]

LESSON 39 – PROJECTILE MOTION II

OBJECTIVES:

Model and solve projectile motion problems.

READ: Stewart: Section 13.4, pages 838-842 (Stop at "*Tangential and Normal Components of Acceleration*").

DO: Section 13.4/ 19, 24, 25, 26, 28

Following Problem:

$f(x) = e^{3x+2}$

find $\frac{df}{dx}$

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 40 – PROBLEM SOLVING LAB 5

OBJECTIVE:

Solve problems by applying concepts relevant to the current material. Your instructor will provide you guidance for this lesson.

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 41 – WRITTEN PARTIAL REVIEW II

NOTE: Attendance at WPRs is mandatory. Cadets will not make any appointments, except in emergencies, that would preclude attendance. **CADETS ARE REQUIRED TO NOTIFY THEIR INSTRUCTOR IN ADVANCE OF ANY WPR ABSENCE**.

WPR IS DURING DEAN'S HOUR

OBJECTIVE:

WPR II will assess your knowledge of Lessons 25-40 and will be given during Dean's hour on 26 March 2009. This WPR will be given in two parts: a non-technology part and a technology part. Initially, you will be given the entire exam but will only be authorized your issued calculator. Upon completion of the non-technology portion of the exam, you will turn it in. Once the non-technology portion is submitted, you will be authorized the use of your laptop computer (wireless disconnected) and a sheet of hand written notes to complete the technology portion. Additional instructions will be provided by your instructor.

Links to Other Pages: Block II: Overview Block II: Endstate Problems

LESSON 42 – WRITTEN PARTIAL REVIEW II AAR

OBJECTIVE:

This lesson is set aside for conducting an extensive After Action Review of your performance on WPR II. After this lesson you should go back and resolve all problems that you had on the test. This will help you start studying for the Term End Exam.

Links to Other Pages: Block II: Overview   Block II: Endstate Problems