BLOCK III: PROBLEM SOLVING WITH PARTIAL DERIVATIVES

13	**30-Mar-09**	**31-Mar-09**	**1-Apr-09**	**2-Apr-09**	**3-Apr-09**
		Lesson 43: Functions of Several Variables	Lesson 44: Partial Derivatives 1	Lesson 45: Partial Derivatives 2	Lesson 46: Directional Derivatives
14	**6-Apr-09**	**7-Apr-09**	**8-Apr-09**	**9-Apr-09**	**10-Apr-09**
	Lesson 47: The Gradient	Lesson 48: PSL 6	**IT Lab: Graded Programming Lab**	**IT Lab: Graded Programming Lab**	Lesson 49: Extrema of Functions of Two Variables 1
15	**13-Apr-09**	**14-Apr-09**	**15-Apr-09**	**16-Apr-09**	**17-Apr-09**
	Lesson 50: Extrema of Functions of Two Variables 2	Lesson 51: Solving Multivariate Optimization Problems 1	**No Class**	Lesson 52: Solving Multivariate Optimization Problems 2	Lesson 53: Project Day
16	**20-Apr-09**	**21-Apr-09**	**22-Apr-09**	**23-Apr-09**	**24-Apr-09**
	Lesson 54: PSL 7	Lesson 55: Solving Multivariate Optimization Problems 3	Lesson 56: PSL 8	**Lesson 57: WPR 3 (Dean's Hour)**	No Class
17	**27-Apr-09**	**28-Apr-09**	**29-Apr-09**	**30-Apr-09**	**1-May-09**
	Lesson 58: WPR 3 AAR **(Project Written Report DUE)**	**IT Lab: Graded Programming Lab**	**IT Lab: Graded Programming Lab**	**Projects/Reading Day**	Lesson 59: Geometric Visualization of Optimization Problems
18	**4-May-09**	**5-May-09**	**6-May-09**	**7-May-09**	**8-May-09**
	Lesson 60: Project Briefs (In Class)	Lesson 61: Project Briefs (In Class)	Lesson 62: Estimations from Data/Functions (MA205 Link)	Lesson 63: Refining Estimates (MA205 Link)	Lesson 64: Review

Block Objective:

Be able to apply the concept of partial derivatives in order to model and solve problems involving multivariate functions.

Supporting Objectives:

1. Understand the graphical, numerical, and algebraic interpretations of functions of two variables.
2. Know how to calculate and interpret rates of change for functions of two or more variables.
3. Know how to calculate and interpret the gradient vector for any multivariate function.
4. Know how to compute and interpret directional derivatives for any multivariate function.
5. Determine the local minima/maxima value of a function of two variables.
6. Determine the absolute minimum/maximum value of a function of two variables on a closed and bounded set.
7. Be able to model and solve constrained multivariate optimization problems.

Technology:

1. Define and plot multivariate functions using *Mathematica*.
2. Differentiate multivariate functions using *Mathematica*.
3. Plot the level curves of a function of two variables in *Mathematica*.

Useful *Mathematica* Commands for Block 3 (*Reminder commands from Block 1and 2 will also be helpful***):**

1. **`Plot3D`**
2. **`ListPointPlot3D`**
3. **`NSolve`**

Think about: -What is the difference between a gradient and a magnitude of the gradient?
-How can you use the **`Solve`** command in *Mathematica* to solve for more than one variable?
-How will trigonometry and geometry help you with Block 3 objectives?
-Why is understanding "what is a unit vector" so important in order to understanding some of the Block 3 objectives? How do you find a unit vector?

Links to Other Pages: Block III: Overview   Block III: Endstate Problems

BLOCK III - ENDSTATE PROBLEMS

By the end of Block III, you should be comfortable solving problems similar to the ones suggested in this section. The problems are labeled as Level 1, Level 2, and Level 3 type problems. The level of difficulty for WPR questions are most similar to the level of difficulty associated with the Level 2 type problems below. Level 1 type problems are most similar to some easier text "Do Problems", and level 3 type problems are comprised of greater difficultly problem solving scenarios. The WPRs ***do*** require you to perform problem solving skills and the questions below are not inclusive of all the Block Objectives.

Level 1

1. Find the first partial derivatives of the function $f(x, y) = y^5 - 3xy$.
2. Suppose that Nolan throws a baseball to Ryan and that the baseball leaves Nolan's hand at the same height at which it is caught by Ryan. If we ignore air resistance, the horizontal range *r* of the baseball (given in the table below) is a function of the initial speed *v* of the ball when it leaves Nolan's hand and the angle θ above the horizontal at which it is thrown. Use the table below to find the value when the ball is thrown 80 feet per second at 45 degrees. What are the units to your answer?

		Speed *v* in feet per second			
		75	***80***	***85***	***90***
Angle θ (degrees)	***35***	165	188	212	238
	40	173	197	222	249
	45	176	200	226	253
	50	173	197	222	249

3. Find the gradient of $f(x,y) = \sin(2x+3y)$.
4. Find the rate of change of the function of $f(x,y) = \sin(2x+3y)$ in the direction of $< 2, 5>$.
5. Find the local maximum and local minimum values and saddle point(s) of the function $f(x, y) = x^3 - 12xy + 8y^3$.

Level 2

6. Find the max rate of change of the function of $f(x,y) = \sin(2x+3y)$.
7. Solve the following partial derivatives:

$r(m,n) = 2mn^7$, $\qquad \dfrac{\partial r}{\partial n} = ?$

$f(x, y) = x^2(2y^2)$, $\qquad f_{xxy} = ?$

$g(t,s) = Sin(5s) + \dfrac{\ln(t)}{Cos(6s)}$, $\qquad$ Find all second order partials of $g(t,s)$.

8. Suppose that Nolan throws a baseball to Ryan and that the baseball leaves Nolan's hand at the same height at which it is caught by Ryan. If we ignore air resistance, the horizontal range r of the baseball (given in the table below) is a function of the initial speed v of the ball when it leaves Nolan's hand and the angle θ above the horizontal at which it is thrown. Use the table below to approximate the instantaneous rate of change of r with respect to v, and to approximate the partial derivative of r with respect to θ, when $v = 80\ ft/s$ and $\theta = 40^\circ$.

		Speed *v* in feet per second			
		75	***80***	***85***	***90***
Angle θ (degrees)	***35***	165	188	212	238
	40	173	197	222	249
	45	176	200	226	253
	50	173	197	222	249

9. The temperature (in degrees Celsius) at a point (x, y) on a metal plate in the xy-plane is:

$$T(x,y) = \frac{xy}{1+x^2+y^2}$$

   a. Find the instantaneous rate of change of temperature at (1, 1) in the direction of $\vec{a} = 2\hat{i} - \hat{j}$.
   b. An ant at (1, 1) wants to walk in the direction in which the temperature drops the most rapidly. Find a unit vector in that direction.

10. A cargo container (in the shape of a rectangular solid) must have a volume of 480 cubic feet. Find the dimensions of the container of this size that has minimum cost if the bottom will cost \$5 per square foot to construct and the sides and top will cost \$3 per square foot to construct. *Can you solve this problem using two different constrained optimization techniques?*

Level 3

11. A trough with trapezoidal cross sections is formed by turning up the edges of a 10-inch wide sheet of aluminum. Find the cross section of maximum area.

12. During Camp Buckner this next summer, you and your friends form a small company to make souvenirs for online sale out of used cadet uniform material. You design a stuffed cadet and outfit it with full dress jackets and pants. A .com buys your idea and makes the stuffed cadets. They want you to produce the dress gray jackets and pants for separate sale. You decide you can employ 200 skilled plebes (class of '12!!!) able to make either product. Each plebe puts in 8 hours each Saturday and is paid \$10 an hour. You can get up to 1000 square feet of gray material delivered each week from the laundry plant (they say it is no problem to inadvertently snag a few items of clothing each week) to support production. The laundry plant charges \$2 per square feet. It takes a plebe 50 minutes to complete a jacket and uses 1.2 square feet of material. A plebe can make a pair of gray pants in 30 minutes using 0.9 square feet of material. The buyers will pay \$25 for a

jacket and $20 for a pair of pants. The .com company wants a mix of articles and doesn't want more than three pairs of pants for every jacket produce. How many dress gray jackets and how many pair of pants should you produce each day in order to maximize profits. Model this manufacturing process.

13. Find the point closest to the origin on the line of intersection of the planes $y+2z=12$ and $x+y=6$.

14. Solve the following partial derivatives:

$r(m,n)=Ln(Cos(2mn^7))$, $\quad \dfrac{\partial r}{\partial n}=?$

$f(x,y)=Tan(Ln(x^2/(2y^2)))$, $\quad f_{xxy}=?$

$g(t,s)=Cos(Sin(5s)+\ln(t)/Cos(6s))$, Find all second order partials of $g(s,t)$.

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 43 – FUNCTIONS OF SEVERAL VARIABLES

OBJECTIVES:

1. Given a function of two variables, determine the domain of the function.
2. Develop a general understanding of the use of level curves and level surfaces to represent functions of two and three variables, respectively.

READ:

1. Stewart: Section 14.1, pages 855-865.
2. Student Notes.

THINK ABOUT: Recall the vertical line test for functions of one variable, what would be an analogous test for functions of more than one variable?

DO: Section 14.1/ 2, 5, 7, 13, 17, 23, 27, 31 (for Problems 23 and 27 use the **`ContourPlot`** command in *Mathematica*).
Following Problem:

$$g(x) = (x^2+5)(4x^3-2x+8)$$
find $g'(x)$

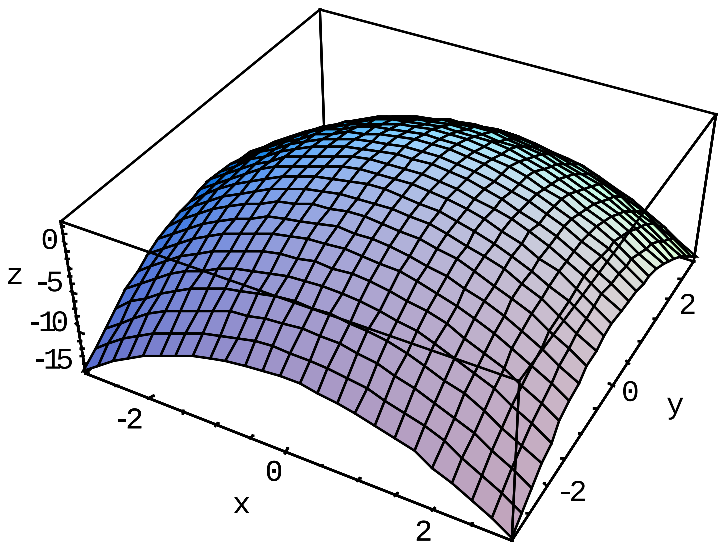
STUDENT NOTES:

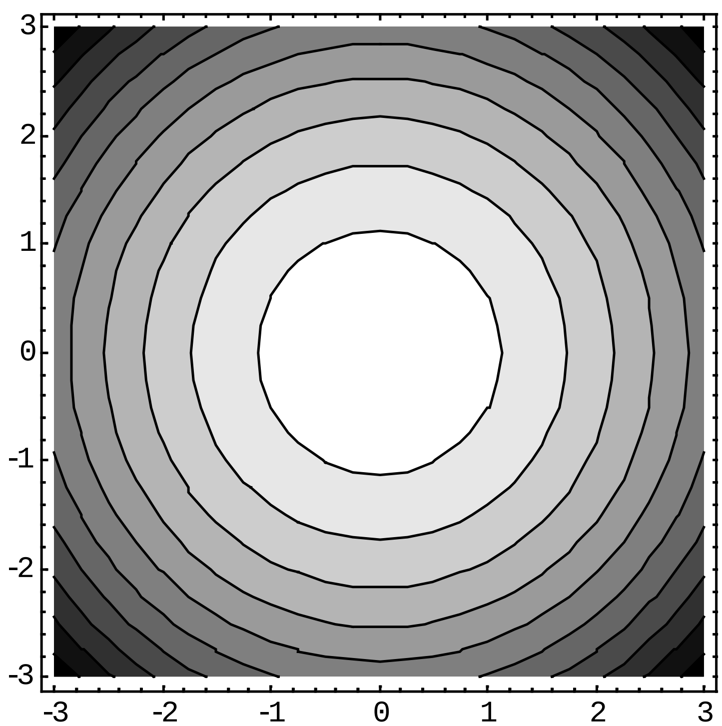
This lesson marks the beginning of our study of multivariable calculus. We start with some basic understanding of how to use a function of more than one variable in this lesson. We will begin to move into doing the same types of operations that we performed with functions of only one variable; like finding derivatives (in the multivariable world they are called partial derivatives), finding maximum and minimum points, and finally optimizing functions of more than one variable. While some of the techniques will change, the ideas and concepts are the same as they were during our single variable blocks.

For this reading assignment there are some important definitions that you need to understand (found on pages 855, 858, and 860). Throughout the reading assignment, there are 15 different examples of multi-variable functions. It is not necessary to study each of these examples in great depth. It is more valuable to skim through the examples, looking for things which are similar to what we have done with single variable functions. Example 7 uses a production function known as the Cobb-Douglas function. While the book uses this as a good example of a multivariable function, there is nothing particularly note-worthy for MA104 in this example. The book continues to use this example in subsequent sections of the chapter. It is a good example, but nothing which you need to be overly concerned about (i.e. we are not expecting you to memorize or be familiar with this particular multivariable function).

There are two ways to graph a function of two variables in *Mathematica*. We can look at a three dimensional plot of the function using the **`Plot3D`** command, or we can look at the level curves using the **`ContourPlot`** command. The process for constructing both of these graphs is very similar to plotting a function of one variable. If we are given the function $f(x,y) = 3 - x^2 - y^2$, our *Mathematica* would look like the following:

```
f[x_, y_] = 3 - x^2 - y^2;
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3}, AxesLabel -> {"x", "y", "z"}];
ContourPlot[f[x, y], {x, -3, 3}, {y, -3, 3}];
```





Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 44 – PARTIAL DERIVATIVES I

OBJECTIVES:

1. Understand what a partial derivative is and describe geometrically what a partial derivative is for a function of two variables.
2. Given a function of several variables, calculate the partial derivatives with respect to each variable by hand and using *Mathematica*.
3. Given a function of several variables, calculate mixed partial derivatives and partial derivatives of a higher order by hand and using *Mathematica*.
4. Approximate the partial derivative of a function of two variables at a point both numerically and graphically.

READ: Stewart: Section 14.3, pages 878-886 (Omit Example 4; Stop at *Partial Differential Equations*).

THINK ABOUT: Could you develop an expression for a plane tangent to a surface using the partial derivatives of a function?

DO: Section 14.3/ 3, 7, 15, 19, 23, 25, 30, 35

Following Problem:

$$h(x) = 3x^3 + 4 + \pi$$

find $\dfrac{dh}{dx}$

Links to Other Pages: Block III: Overview   Block III: Endstate Problems

LESSON 45 – PARTIAL DERIVATIVES II

OBJECTIVES:

Complete lesson problems and objectives from lesson 44.

READ: Stewart: Section 14.3, pages 878-886 (Omit Example 4; Stop at *Partial Differential Equations*).

THINK ABOUT: Why is understanding Clairaut's Theorem useful in the context of problem solving?

DO: Section 14.3/ 4, 16, 36, 51, 77, 82

Following Problem:

$$G(x) = Sin(x) + Cos(2x) + Sin(2x^3)$$

find $G'(x)$

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 46 – DIRECTIONAL DERIVATIVES

OBJECTIVES:

1. Understand what a directional derivative is in terms of a rate of change.
2. Given a function *f* of two variables, find the directional derivative of *f* at a given point in any direction by hand and using *Mathematica*.

READ: Stewart: Section 14.6, pages 910-919.
Student Notes

THINK ABOUT: Under what assumptions does your answer in problem 3 below accurately describe the rate of change?

DO: Section 14.6/ 1, 3, 7, 8, 11, 15
Following Problem:

$$f(x) = 6e^{5x+x^2}$$

find $\dfrac{dx}{dt}$

STUDENT NOTES:

This lesson is directly tied to the following lesson. In this lesson you will need to algebraically compute the gradient in order to solve for a directional derivative. In the following lesson, we will emphasize the significance of the gradient vector through a geometric interpretation of it. The geometric interpretation of the gradient will prove useful when solving three-dimensional optimization problems.

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 47 – THE GRADIENT

OBJECTIVES:

1. Given a function of two variables, find the gradient vector by hand and using *Mathematica* at a specified point.
2. Find the vector that is normal to the level curve $f(x, y) = c$ at a specified point.
3. Understand that the gradient vector gives the direction of the greatest increase in functional value at a given point for a differentiable function.
4. Understand that the magnitude of the gradient vector gives the maximum rate of change of differentiable function at a given point.
5. Understand geometrically how the gradient vector is related to level curves.

READ: Stewart: Section 14.6, pages 910-919.

THINK ABOUT: In what ways could the gradient be helpful in 3D optimization?

DO: Section 14.6/ 21, 23, 28, 32
Following Problem:

$$g(x) = (x^3 + 3)/(4x^5 - x^{-3} + 2)$$

find $g'(x)$

Links to Other Pages: Block III: Overview   Block III: Endstate Problems

LESSON 48 - PROBLEM SOLVING LAB 6

OBJECTIVE:

Solve problems by applying concepts relevant to the current material. Your instructor will provide you guidance for this lesson.

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 49 – EXTREMA OF FUNCTIONS OF TWO VARIABLES I

OBJECTIVES:

1. Given the surface $z = f(x, y)$, defined by a function which has continuous partial derivatives over some region $\mathbb{R}$, examine the level curves for possible maximum or minimum values.
2. Understand the definition of a critical point and be able to find the critical points of a function of two variables.
3. Use the Second Derivatives Test to classify a critical point as a local maximum, local minimum, or saddle point.
4. Understand why the discriminant will always be negative for a saddle point.

READ: Stewart: Section 14.7, pages 922-928 (Stop at *Absolute Max and Min Values*).

THINK ABOUT: Compare the methods of finding extrema for single and multivariate functions. What are the similarities/differences?

DO: Section 14.7/ 3, 5, 7, 9, 11
Following Problem:

$$v(t) = \frac{6t^2}{(2t^6 + 3t)}$$

find $v'(t)$

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 50 – EXTREMA OF FUNCTIONS OF TWO VARIABLES II

OBJECTIVES:

Find the absolute maximum and minimum of a given continuous function $f(x, y)$ on a closed and bounded set using the Extreme Value Theorem and Closed Interval Method for functions of two variables.

READ: Stewart: Section 14.7, pages 928-930 (Start at *Absolute Max and Min Values*).

THINK ABOUT: Why do we sometimes use the Second Derivatives Test and other times we invoke the Extreme Value Theorem for functions of two variables? How will Example 7 in the text on page 929 assist you with the Do Problems?

DO: Section 14.7/ 30, 32, 33

Following Problem:

$f(x) = x\ln(3x+2)$

find $f'(x)$

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 51 – SOLVING MULTIVARIATE OPTIMIZATION PROBLEMS I

OBJECTIVES:

1. Model and solve multivariate optimization problems.
   a. Given a problem, be able to determine a multi-variable objective function.
   b. Given limitations in a problem, write a constraint equation.
   c. Transform a multivariable objective function into a two variable objective function when only one constraint exists.
   d. Interpret the results of a mathematical solution as it applies to a problem.

READ:

1. Stewart: Section 14.7, pages 922-930.
2. Student Notes.

DO: Section 14.7/ 39, 43, 46
Following Problem:

$$f(x) = Cos(2x^3)$$

find $f'(x)$

STUDENT NOTES:

The process for solving multivariate optimization problems is very similar to the process used in the single variable case. The only real difference comes during steps four through six of our seven step process. And the big changes are only the difference in the calculus used during Step 6. Below is an excerpt from the table found on Lesson 13 with the changes.

Step	Title	Description	Polya	USMA
4	Define Objective Function and Constraint Equations (if any)	Define Objective Function (what we min/max) in terms of the variables from step 3. If there are more than **two** variables, then we have to reduce it using constraint equations. This is something that is equal to something else in the problem. If an equal sign is not provided look for key words.	Develop A Plan	Transform
5	Simplify Objective Function (May not be required)	If you have more than **two** variables in your objective function, then you must use constraint equations to reduce the number to two.		
6	Solve the problem	This step is the same as before except we are now using the methods from Section 14.7 to do the "calculus". To find the critical points, we still solve for when the slope is zero, that just occurs when the gradient is equal to zero. Also, we still test using either the Second Derivatives Test or Using the Closed Interval Method depending on type of problem.	Carry out the Plan	Solve

Consider the following example:

You own a company that makes two models of speakers, the Ultra Mini and the Big Stack. Each ultra Mini requires 1 square foot of fabric and 3 feet of wire, and each Big Stack requires 5 square feet of fabric and 9 feet of wire. You have ach week 100 square feet of fabric and 270 feet of wire to use. Your profit function is estimated to be $P(x,y) = 10x + 60y + 0.5xy$, where x is the number of Ultra Minis, y is the number of Big Stacks, and P is your profit in dollars. Find the number of each model you should make each week to maximize your profit[1].

- The constraints in this problem come form the limited amount of fabric and wire available.
- The constraint on fabric is $x + 5y \leq 100$
- The constraint on wire is $3x + 9y \leq 270$
- There are also nonnegative constraints such that $x \geq 0$ and $y \geq 0$.

How does this problem look similar to example 7 on page 929 of Stewart? Can you solve it?

<u>Links to Other Pages</u>: Block III: Overview   Block III: Endstate Problems

[1] Stefan Waner and Steven R. Costenoble, *Applied Calculus 3rd Edition*. Thomson Brooks/Cole 2004. pp511.

LESSON 52 – SOLVING MULTIVARIATE OPTIMIZATION PROBLEMS II

OBJECTIVES:

Model and solve multivariate optimization problems.

READ: Stewart: Section 14.7, pages 922-930.

DO: Section 14.7/ 48, 50, 51

Following Problem:

$f(x) = e^{x^2}$

find $\dfrac{df}{dx}$

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 53 – PROJECT DAY

OBJECTIVE:

Provide specified time for students to work together with project partners towards the accomplishment of their group project. Your instructor will provide you with further guidance for this lesson.

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 54 - PROBLEM SOLVING LAB 7

OBJECTIVE:

Solve problems by applying concepts relevant to the current material. Your instructor will provide you guidance for this lesson.

Links to Other Pages: Block III: Overview Block III: Endstate Problems

LESSON 55 – SOLVING MULTIVARIATE OPTIMIZATION PROBLEMS III

OBJECTIVES:

Model and solve multivariate optimization problems.

READ: Stewart: Section 14.7, pages 922-930.

DO: Section 14.7/ 52, 54

Following Problem:

$g(x) = (x^2 + 5)^3 (4x^3 - 2x + 8)$

find $g'(x)$

Links to Other Pages: Block III: Overview   Block III: Endstate Problems

LESSON 56 - PROBLEM SOLVING LAB 8

OBJECTIVE:

Solve problems by applying concepts relevant to the current material. Your instructor will provide you guidance for this lesson.

Links to Other Pages: Block III: Overview Block III: Endstate Problems

<u>LESSON 57 – WRITTEN PARTIAL REVIEW III</u>

<u>NOTE:</u> Attendance at WPRs is mandatory. Cadets will not make any appointments, except in emergencies, that would preclude attendance. **<u>CADETS ARE REQUIRED TO NOTIFY THEIR INSTRUCTOR IN ADVANCE OF ANY WPR ABSENCE</u>**.

WPR IS DURING DEAN'S HOUR

<u>OBJECTIVE:</u>

WPR III will assess your knowledge of Lessons 43-56 and will be given during Dean's hour on 23 April 2009. This WPR will be given in two parts: a non-technology part and a technology part. Initially, you will be given the entire exam but will only be authorized your issued calculator. Upon completion of the non-technology portion of the exam, you will turn it in. Once the non-technology portion is submitted, you will be authorized the use of your laptop computer (wireless disconnected) and a sheet of hand written notes to complete the technology portion. Additional instructions will be provided by your instructor.

<u>Links to Other Pages</u>: Block III: Overview Block III: Endstate Problems

LESSON 58 – WRITTEN PARTIAL REVIEW III AAR (PROJECT QUIZ)

OBJECTIVE:

The group project is due at the beginning of this period,. Immediately after turning in the project you will take a project quiz worth 50 points. This quiz is designed to test your basic understanding of the project and may be tailored to have questions that are specific about individual projects.

Additionally, this lesson is set aside for conducting an extensive After Action Review of your performance on WPR III. After this lesson you should go back and resolve all problems that you had on the test. This will help you start studying for the Term End Exam.

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LESSON 59 – GEOMETRIC VISUALIZATION OF OPTIMIZATION PROBLEMS

OBJECTIVE:

1. Understand why geometry identifies potential minimum and maximum function values.
2. Solve optimization problems for functions of several variables subject to a single constraint using a ratio to relate the gradient of the objective function and the gradient of the constraint function.
3. Understand the geometric interpretation of the gradient vector with respect to level curves.

READ: Stewart: Section 14.8, pages 934-938 (Stop at *Two Constraints*).
Student Notes

THINK ABOUT: Given a function and a single constraint on the function, why is the Lagrange multiplier (the symbol λ) needed if the gradient of the function and the vector comprised of the partial derivatives of the constraint lie on the same line or in the same hyperplane?

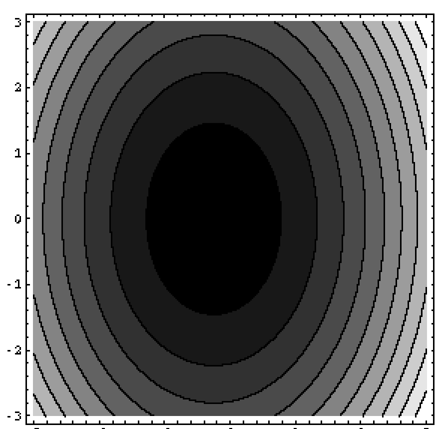
DO: Section 14.8/ 1, 3, 5, 7, 8
Following Problem:

$$v(t) = \frac{2t^3}{(t^4 + t)}$$

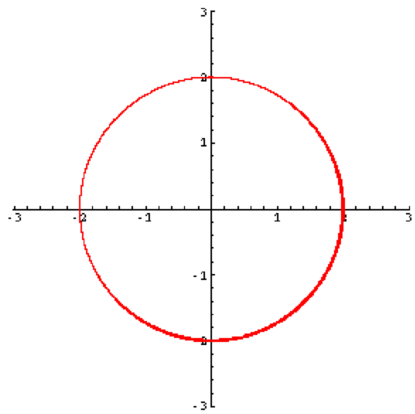
find $v''(t)$

STUDENT NOTES:

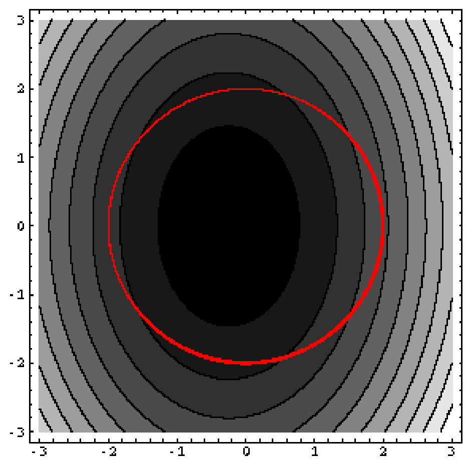
A way to study the method of Lagrange multipliers is through the objective function's level curves (i.e., the curves in the contour diagram). Here *Mathematica* uses shading to indicate height (darker represents lower on the z-axis).



In our example, the constraint equation can also be represented in two dimensions.



So here we look at the relationship between the level curves of the surface and the constraint equation.



Trace with your finger along the constraint curve. Note how the objective function's values change as you do that. For example, traveling along the constraint from the point (2,0), in either direction, the objective function's values decrease (remember that darker shading represents values farther down the z-axis). So (2,0) yields a local maximum with respect to travel on the constraint curve.

Thus, it turns out that an extreme point of our 3D curve occurs when the constraint equation (circle) is tangent to a level curve. This observation leads us to note that the gradient vector of the objective function is parallel to the gradient of the constraint (since both are perpendicular to the tangent vector at the point of tangency between the constraint curve and the level curve). [2]

Links to Other Pages: Block III: Overview   Block III: Endstate Problems

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[2] Michael Hofer, *The Geometry of Lagrange Multipliers*, University of Graz, retrieved 12 June 2008 from the World Wide Web at http://www.math.ou.edu/~tjmurphy/Teaching/2443/Lagrange/lagrange.html .

LESSONS 60 and 61 – STUDENT PROJECT BRIEFS

OBJECTIVE:

During these two lessons, students will provide an oral presentation to their instructor and to their class on their semester project. Additional guidance will be given by your instructor.

Links to Other Pages: Block III: Overview

LESSON 62 – ESTIMATIONS FROM DATA AND FUNCTIONS

OBJECTIVE:

1. Estimate the area under a curve given a set of data.
2. Estimate the area between a curve and the *x*-axis when given a function.
3. Estimate total distance travelled and total displacement.

READ:

1. Stewart: Section 5.1, pages 355-363.
2. Student Notes.

DO: Section 5.1/ 3, 5, 11, 14

STUDENT NOTES:

The next two lessons are an introduction to MA205, Integral Calculus, which you will be taking next semester. Whereas MA104 primarily deals with the derivative and rates of change, MA205 will answer the question of "how much change". For example, if we know the velocity of an object (the rate of change of the displacement with respect to time) from point *a* to point *b* and the time it took to get there, we can calculate how far the object traveled, or in other words, how much change occurred.

We will begin by estimating the area under a curve and distances travelled using a small number of rectangles, and then refine our estimates using more and more rectangles (an infinite number in fact!) until we can calculate the exact area under a curve using what is known as the definite integral. By the end of these two lessons, you should understand the definition of the definite integral, and be better prepared for MA205 in the fall.

Links to Other Pages: Block III: Overview   Block III: Endstate Problems

LESSON 63 – REFINING ESTIMATES

OBJECTIVE:

1. Improve all estimations by increasing the number of subintervals.
2. Understand what is meant by a Riemann Sum and the definition of the Definite Integral.

READ:

Stewart: Section 5.2, pages 366-373 (Stop at *Properties of the Definite Integral*).

DO: Section 5.2/ 1, 15, 21, 33

Links to Other Pages: Block III: Overview

LESSON 64 – REVIEW

OBJECTIVE:

This lesson will be an opportunity for a student led review of the semester. Your instructor will provide you with specific guidance for this lesson.

Links to Other Pages: Block III: Overview