UNITED STATES MILITARY ACADEMY

MA153 PROJECT 1
COUGAR H MRAP

MA153 ADVANCED MULTIVARIABLE CALCULUS

SECTION XX

LTC XXXX

BY

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Abstract

Force Protection Incorporated (FPI) is the sole manufacturer of the U.S. Military's best mine and ground explosive defense system, the Mine Resistant Ambush Protected (MRAP) armored vehicle. However, since FPI received its initial contract to build MRAPs, it was poorly equipped and inadequately prepared to produce them with maximum efficiency. In this report we will detail changes to FPI operations and production that will maximize company profit. The following analysis will show that the number of workers required to maximize profit for a monthly production of 180 and 220 MRAPs is 9 and 13 workers respectively on 7 total assembly lines. In order to maximize profit regardless of production rate, we recommend placing 23 workers on each of the 7 open assembly lines. This will produce 287 MRAPs and $5,630,000 of profit monthly.

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Background

Force Protection Incorporated (FPI) is the sole manufacturer of the United States Military's best mine and ground explosive defense system. This epitome of mine defense is the Mine Resistant Ambush Protected (MRAP) armored vehicle. In addition to saving lives in Iraq, FPI's military vehicles are gaining popularity in the popular media, as evidenced in the Buffalo MPCV's (relative of the Cougar MRAP) leading role in Michael Bay's recent movie *Transformers.* FPI is the military's leader in counter Improvised Explosive Device research. Started in 1997, FPI's main purpose is to design, manufacture, and test mine resistant vehicles for use by the United States Military. The Cougar H 4x4 MRAP is capable of holding up to four troops and an Explosive Ordinance Disposal Robot, in comparison to the Cougar H 6x6, capable of holding up to twelve troops. When FPI received its initial contract to build Cougars in 2004 and 2005, it was poorly equipped and inadequately prepared to build Cougars with maximum efficiency. The initial plan consisting of twelve people on one production line brought an end result of one Cougar in five months. As orders continued to come in, it became imperative that FPI find a method to maximize production output and profit in order to keep the company running and the customers satisfied. The purpose of this report is to determine, based on given equations for monthly production output and monthly costs, the number workers and the number of production lines which will maximize profit for 180 MRAPs and 220 MRAPs. Based on the results, a recommendation must be given to the company concerning how many workers and how many lines should be used in order to maximize profit for any number of MRAPs needed.

Facts

The facts influencing our assessment describe the limitations and costs of running the facility. The facility can support up to seven production lines, and each production line can support up to 50 workers. Each worker costs the company $9,000 a month.

Another important fact influencing our assessment is the revenue per MRAP produced. After accounting for all material costs, the revenue per vehicle is expected to be $25,000. This figure does not include production line operating costs or the cost of labor.

Assumptions

In order to assess the profits for the production of 180 and 220 MRAPs per month and to make a recommendation for maximum profit, it is necessary to make the following assumptions to simplify the mathematical model.

1. Each production line in the facility is identical in certain respects: each line is staffed by the same number of workers and produces the same number of MRAPs.
2. Each worker in the facility is also identical in qualities such as skill level, physical ability, work ethic, etc., required to maintain the same productivity level.
3. Force Protection is not able to sell fractions of MRAPs.

4. The facility will be able to shut down after production goals (180 and 220 MRAPs) without incurring additional costs.

5. No external factors will affect the facility. For example, all required labor will be available and disasters such as fires, hurricanes, or tsunamis will not harm the facility. Predicting any of these misfortunes is well beyond the scope of this project.

Analysis

The given equation

$$P(w) = 0.11w^3 - 1.13w^2 + 3.8w \tag{1}$$

describes production rate per line in tens of MRAPs, where w is the number of workers per line. Equation (1) can be modified to show the total production rate of the facility (in tens of MRAPs):

$$\text{Production Rate} = l(0.11w^3 - 1.13w^2 + 3.8w) \tag{2}$$

where l is the number of operating lines and w is tens of workers per.

The cost (in hundreds of dollars) of operating the production lines in the facility is described by equation (3), where l is the number of lines in the facility.

$$C(l) = (1500)\ln(\frac{l}{8} + 1) \tag{3}$$

Adjusting for units, multiplying equation (1) by the number of lines to give total production, multiplying Production Rate by revenue per vehicle, and creating an expression for the cost of labor, an equation for total profit (in dollars) can be created, where l is the number of operating production lines and w is tens of workers per line.:

$$\text{Profit} = (\text{Production Rate})(25000)(10) - (1500)(100)\ln(\frac{l}{8} + 1) - (9000)(10)wl \tag{4}$$ [1]

Production Rate in this equation must be rounded down to a whole number because of the assumption that FPI will not be able to sell fractions of MRAPs. Equation (4) gives the total profit by subtracting the cost of labor and the operating costs of the facility from total revenue.

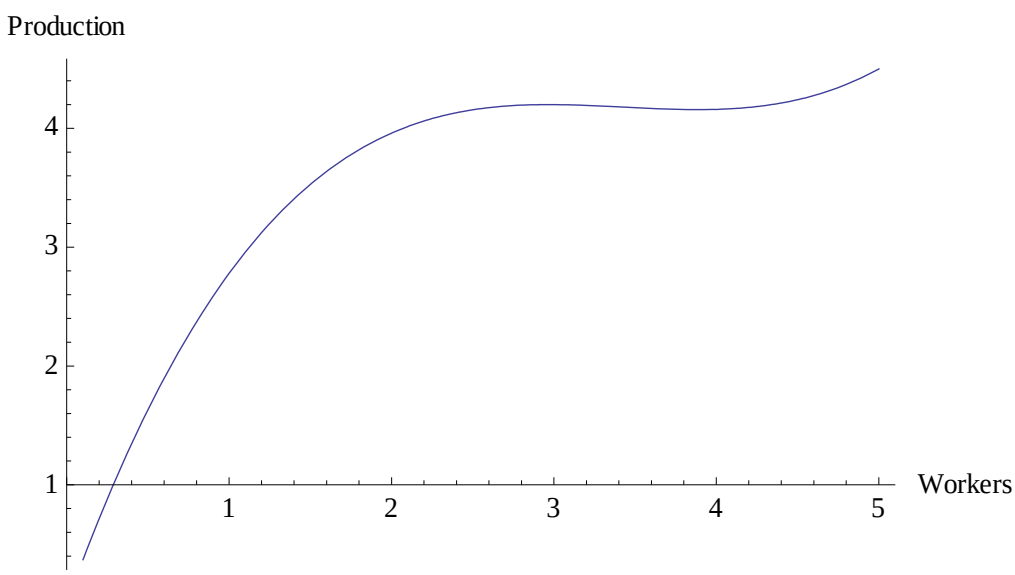

Figure 1: Plot of Production per Line vs. Workers per Line (tens of workers)

Figure 1 shows that as the number of workers increases, production also increases, but only up to a certain point. Somewhere between 20 workers per line and 30 workers per line, production levels off. From this it can be predicted that profit will be maximized by using between 20 and 30 workers per line.

Results

Maximum Profit for Production of 180 and 220 MRAPs:

Using Lagrange multipliers, we found that maximum profit for the production of 180 MRAPs is achieved using 4 production lines and 50 workers per line, and the maximum profit for the production of 220 MRAPs is achieved with 5 production lines and 50 workers on each line. Supporting work for these calculations is found in Appendix C. The exact answer found using this method had to be rounded to the nearest whole numbers within the domain of the problem, as the number of workers was above the maximum limit of 50 workers, and neither fractional parts of production lines or people can be used. Using these values, the predicted profit for the production of 180 MRAPs is $2,640,000 and the predicted profit for the production of 220 MRAPs is $3,180,000.

When using a graphical approach to look at this problem as shown below, it is apparent that the answers we found using Lagrange multipliers are wrong. There are several problems with using the Lagrange multiplier method to solve this problem. First, this method only finds values that are within the exact constraint of the problem. These values are often outside the domain of the problem. For example, if the constraint of the problem limits production to 220 MRAPs, Lagrange multipliers will not find a combination of workers and lines that produces more than 220 MRAPs, even if this combination is more profitable than one that produces exactly 220 MRAPs. In fact, such a combination does not even exist, because the numbers of lines and people must be whole numbers, and there cannot be more than 7 lines or 50 people per line. In short, Lagrange multipliers find maximum and minimum values subject to an exact constraint but not a domain; however, this problem calls for finding maximum values subject to an approximate constraint and a specific domain.

The values that we found in the graphical approach were not found using the Lagrange multipliers method because they are not relative maximums; they are absolute maximums within the domain of the problem. The Lagrange multiplier method does not take into account the restrictions of the domain: the number of production lines must be a whole number between 1 and 7 and the number of workers per line must be a whole number between 1 and 50.

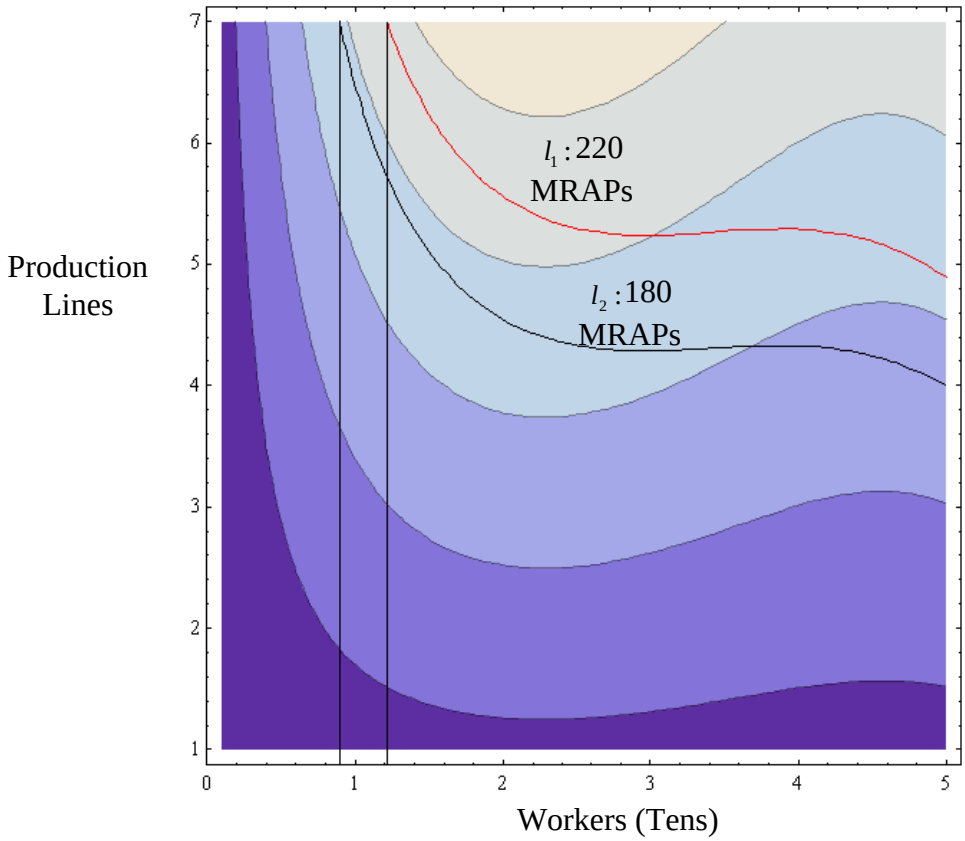

Figure 2: Contours of the Profit Function with Production Constraints

Figure 2 shows a contour plot of profit as a function of production lines and workers per line. The lines l_1 and l_2 are level curves of the production function when the production equals 220 MRAPs and 180 MRAPs, respectively. The gradient of the profit function points in the direction of maximum increase of profit, so the maximum profit for each constraint occurs at a point where the gradient of the curve is parallel to the gradient vector of a level curve representing profits. The gradient is perpendicular to its function, so it follows that where two gradient vectors are parallel, their functions are also parallel.

The points of maximum profits on the constraints were found by identifying the points were the curves are parallel to a level curve of the profit function. Figure 2 shows a vertical line drawn from the point of maximum profit on each constraint to the *w*-axis. These vertical lines make it clear that maximum profit for the production of 180 MRAPs occurs when there are 9 workers per line and the maximum profit for the production of 220 MRAPs occurs when there are 12 workers per line. It is clear that in both cases, maximum profit is reached when there are 7 production lines in operation.

Appendix A shows that 9 workers on 7 lines will in fact be able to produce the goal of 180 MRAPs. Substituting 0.9 for *w*, 7 for *l*, and 180 for Production in the Profit function,

$$\text{Profit} = (\text{Production})(25000)(10) - (1500)(100)\ln(\frac{l}{8}+1) - (9000)(10)wl$$

$$\text{Profit} = (18)(25000)(10) - (1500)(100)\ln(\frac{7}{8}+1) - (9000)(10)(0.9)(7)$$

$$\text{Profit} = \$3{,}840{,}000$$

it is clear that under the constraint of producing 180 MRAPs, a maximum profit of $3,840,000 occurs when there are 7 production lines in operation with 9 workers per line.

Using Figure 2 provided approximate values of 12 workers on 7 lines to achieve maximum profit in producing 220 MRAPs, but as shown in Appendix B, this arrangement would produce only 218 MRAPs. This is less than the desired value of 220, so we rounded *w* up to 1.3. Substituting 1.3 for *w* and 7 for *l* in the Profit function,

$$\text{Profit} = (\text{Production})(25000)(10) - (1500)(100)\ln(\frac{l}{8}+1) - (9000)(10)wl$$

$$\text{Profit} = (22)(25000)(10) - (1500)(100)\ln(\frac{7}{8}+1) - (9000)(10)(01.3)(7)$$

$$\text{Profit} = \$4{,}680{,}000$$

it is clear that under the constraint of producing 180 MRAPs, a maximum profit of $4,680,000 occurs when there are 7 production lines in operation with 12 workers per line.

<u>Maximizing Profit Regardless of Production Rate:</u>

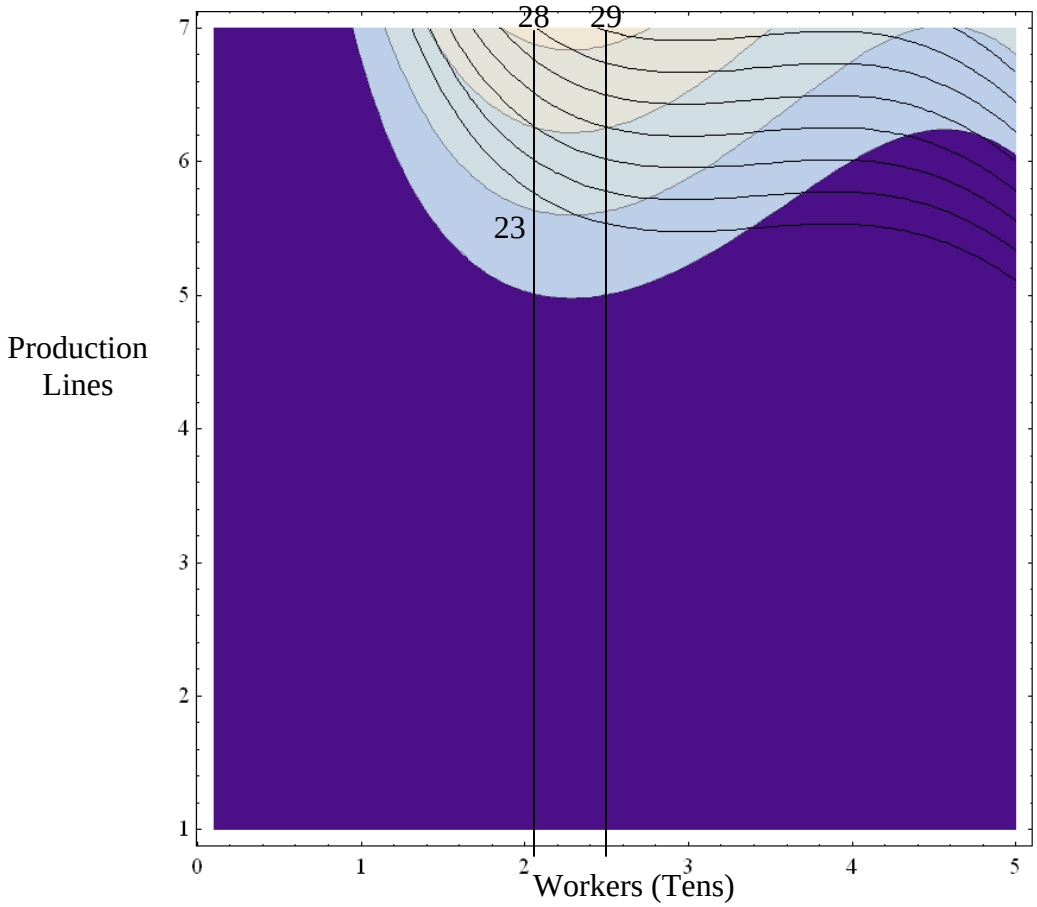

Figure 3: Selected Contours of the Profit Function and Production Function.

Figure 3 shows contours of the profit and production functions near the area of maximum profit. Contour lines showing 230, 280, and 290 MRAPs are labeled. The contour of maximum profit is not shown, but it is located somewhere above the contour representing highest value, $5,500,000, on the graph. It is clear that maximum profit will occur somewhere between the production of 280 and 290 MRAPs because these are the two curves closest two being parallel to the theoretical contour of maximum profit. Vertical lines have been drawn from these points to the *w* axis to show that between 21 and 26 are workers needed to produce the number of MRAPs that will lead to maximum profit. The point at which a production contour will be parallel to a profit contour occurs at a point where the number of production lines in operation is 7 or very close to 7. The number of production lines must be a whole number, so obtaining maximum profit will require the use of all 7 production lines.

After determining that all seven lines must be open and in use and that the number of workers must be a whole number between 21 and 26, all that remained to be done was to test each of these six values. Holding the number of lines (*l*) at seven, the value of *w* was changed and the profit for each possibility was found using the profit equation. For 22 workers the profit was approximately $5,620,000. For 23 workers it was $5,630,000, and for 24 workers it fell back to $5,620,000. This shows that at seven open lines of production, a maximum profit of $5,630,000 can be gained by using 23 people per line on 7 lines. This combination will produce 41 MRAPs per line, or 287 total MRAPs monthly.

Discussion

Observing the Figure 2, the area of the highest contour region for profit that the constraint lines of 180 and 220 MRAPs run through can be easily seen. It can be established that the only way to maximize profit is to run all seven lines, because the highest contour region for profit lies significantly above 6 production lines and cannot go above 7 lines. The number of workers needed for each constraint was found by identifying the corresponding values on the *w* axis for the points of maximum profit.

Figure 3 displays the contour lines of the production function superimposed on the contour lines of the profit equation. The task of the problem is to recommend the number of workers and the number of lines necessary to maximize the profit for Force Protection Inc. The number of workers, found on the *w*-axis, must lie between the *w* values found between the production contours of 280 and 290 at 7 lines (somewhere between 21 and 26 workers). In order to obtain an acceptable answer, it was necessary to round the number of lines up to 7 and to round the number of workers to the nearest number that would still produce the desired number of MRAPs. The fact that fractional workers cannot man the lines is detrimental to profit, as rounding the number of workers up incurs additional cost without making any appreciable increase in profit.

Conclusion and Recommendation

We found that profit for the production of 180 MRAPs reaches a maximum of $3,840,000 when there are 7 lines in operation and 9 workers per line, and the profit for the production of 220 MRAPs reaches a maximum of $4,680,000 when there are 7 lines in operation and 12 workers per line. We recommend the use of all 7 production lines with 23 people per line. This combination produces 287 total MRAPs as well as the largest possible profit, $5,630,000, within the constraints of the facility.

Although we believe our model is a good one, it does contain error. Different results may be found if some production lines contained different amounts of workers. It was often necessary to treat the production and profit functions as continuous functions when they are, in fact, not continuous, due to the peculiar nature of human beings and production lines (they do not work in parts).

Appendix A Production Rate for 180 MRAPs

$$\text{Production Rate} = l(0.11w^3 - 1.13w^2 + 3.8w)$$
$$\text{Production Rate} = 7[(0.11)(0.9)^3 - (1.13)(0.9)^2 + (3.8)(0.9)]$$
$$\text{Production Rate} = 18.1$$

Appendix B Production Rate for 220 MRAPs

$$\text{Production Rate} = l(0.11w^3 - 1.13w^2 + 3.8w)$$

$$\text{Production Rate} = 7[(0.11)(1.3)^3 - (1.13)(1.3)^2 + (3.8)(1.3)]$$

$$\text{Production Rate} = 22.9$$

Appendix C Lagrange Multipliers

$\vec{\nabla} f(w,l) = \lambda \vec{\nabla} g(w,l)$, such that $g(w,l) = 180$, where

$f(w,l) = l(0.11w^3 - 1.13w^2 + 3.8w)\ (25000)(10) - (1500)(100)\ln(\frac{l}{8}+1) - (9000)(10)wl$

$g(w,l) = l(0.11w^3 - 1.13w^2 + 3.8w)$.

Gradients:

$\vec{\nabla} f(w,l) = <-(90000)(l) + (250000)(l)(3.8 - 2.26w + 0.331.13\ w^2),\ -\frac{18750}{1+l/8} -$
$(90000)(w) \quad + (250000)(3.8w - 1.13\ w^2 + 0.11\ w^3)>$

$\vec{\nabla} g(w,l) = <(l)\ (3.8 - 2.26w + 0.331.13\ w^2),\ + (3.8w - 1.13\ w^2 + 0.11\ w^3)>$

Lagrange multiplier equations:

$-(90000)(l) + (250000)(l)(3.8 - 2.26w + 0.331.13\ w^2) =$

$\lambda\ (l)\ (3.8 - 2.26w + 0.331.13\ w^2)$

$-\frac{18750}{1+l/8} - (90000)(w) + (250000)(3.8w - 1.13\ w^2 + 0.11\ w^3) =$

$\lambda\ (3.8w - 1.13\ w^2 + 0.11\ w^3)$

$l(0.11w^3 - 1.13w^2 + 3.8w) = 18$

$l = 3.92$
$w = 5.12$

$-(90000)(l) + (250000)(l)(3.8 - 2.26w + 0.331.13\ w^2) =$

$\lambda\ (l)\ (3.8 - 2.26w + 0.331.13\ w^2)$

$-\frac{18750}{1+l/8} - (90000)(w) + (250000)(3.8w - 1.13\ w^2 + 0.11\ w^3) =$

$\lambda\ (3.8w - 1.13\ w^2 + 0.11\ w^3)$

$l(0.11w^3 - 1.13w^2 + 3.8w) = 22$

$l = 4.79$
$w = 5.12$

Appendix D Maximizing Profit

$$\text{Production Rate} = l(0.11w^3 - 1.13w^2 + 3.8w)$$
$$\text{Production Rate} = 7[(0.11)(2.3)^3 - (1.13)(2.3)^2 + (3.8)(2.3]$$
$$\text{Production Rate} = 28.7$$

$$\text{Profit} = (\text{Production})(25000)(10) - (1500)(100)\ln(\frac{l}{8} + 1) - (9000)(10)wl$$

$$\text{Profit} = (28.7)(25000)(10) - (1500)(100)\ln(\frac{7}{8} + 1) - (9000)(10)(2.3)(7)$$

$$\text{Profit} = \$5{,}630{,}000$$

Appendix E Profit for Lagrange Multiplier Results

<u>180 MRAPs:</u>

$$\text{Profit} = (\text{Production})\,(25000)(10) - (1500)(100)\ln(\frac{l}{8}+1) - (9000)(10)wl$$

$$\text{Profit} = (18)\,(25000)(10) - (1500)(100)\ln(\frac{4}{8}+1) - (9000)(10)(5)(4)$$

$$\text{Profit} = \$2{,}640{,}000$$

<u>220 MRAPs:</u>

$$\text{Profit} = (\text{Production})\,(25000)(10) - (1500)(100)\ln(\frac{l}{8}+1) - (9000)(10)wl$$

$$\text{Profit} = (22)\,(25000)(10) - (1500)(100)\ln(\frac{5}{8}+1) - (9000)(10)(5)(5)$$

$$\text{Profit} = \$3{,}180{,}000$$

Endnotes

[1] CDT Xxxxx Xxxxxx, XX, 11', assistance given to CDT Xxxxxxxxx, verbal discussion, West Point, NY 8 October 2007. CDT Xxxxxx pointed out that in order to find the exact profit, I had to replace the production function in the profit equation with a whole number.

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