Lesson 27 - Estimations Using Data and Functions II

Objectives

- Use a double Riemann sum to estimate volume under any surface.
- Understand the concept of volume under a surface as the limit of the sum of volumes of rectangular columns.

READ

- Stewart, Chapter 15.1, pages 951-958

THINK ABOUT

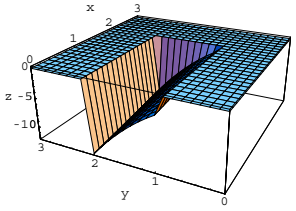
- What is a double Riemann sum?
- How do double Riemann sums relate to single Riemann sums?
- What sample points can you chose when creating rectangular columns under a surface?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

Plotting surfaces on a restricted domain can be done in Mathematica. This allows you to plot only the region you are interested in on a 3 dimensional plot. If you had the function $f(x, y) = x - 3y^2$ entered into Mathematica and you wanted to plot this function restricted to the region $R = [0, 2] \times [1, 2]$ the command to do that would be:

```
Plot3D[If[0 ≤ x ≤ 2 && 1 ≤ y ≤ 2, f[x,y],0], {x,0,3},{y, 0,3}, AxesLabel→{x,y,z}];
```

This produces the graph:



The region sticking down is the region you are interested in. You have set everything else to 0. You can also use Mathematica to compute a double Riemann sum given a function $f(x, y)$. The commands to do this for the function $f(x, y) = x - 3y^2$ on the region $R = [0, 2] \times [1, 2]$ dividing each axis into m and n partitions are:

```
f[x_, y_] = x - 3 * y^2;
a = 0;
b = 2;
c = 1;
d = 2;
deltax = (b - a) / m;
deltay = (d - c) / n;
deltaa = deltay * deltax;
x[i_] = a + i * deltax;
y[j_] = c + j * deltay;
lowerleft[m_, n_] = Sum[Sum[f[x[i - 1], y[j - 1]] * deltaa, {j, 1, n}], {i, 1, m}];
lowerright[m_, n_] = Sum[Sum[f[x[i], y[j - 1]] * deltaa, {j, 1, n}], {i, 1, m}];
```

The command `lowerleft[4,4]` should give you a value of $-165/16$ and the command `lowerright[4,4]` should give you a value of $-149/16$.