Lesson 36 - Polar Regions I

Objectives

- Understand the polar rectangle and its relationship to: $\int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} f(r, \theta) r \, dr \, d\theta$.
- Understand when polar coordinates make setting up an iterated integral easier.
- Use polar coordinates to solve iterated integrals.

READ

- Stewart, Chapter 15.4, pages 974-978.

THINK ABOUT

- When should you consider changing to polar coordinates?
- Can you explain the extra r in the polar integration form?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

Plotting circles:
Since the equation of a circle is not a function you must plot both halves of the circle in order to get the correct picture in cartesian coordinates. To plot the equation $x^2 + y^2 = 4$ the command would look like:

$$\texttt{Plot[}\{\sqrt{4-x^2}, -\sqrt{4-x^2}\}\texttt{,}\{\texttt{x,-2,2}\}\texttt{]}$$

Can you see the advantage to using polar coordinates in this example? Here the equivalent polar equation is $r = 2$ so you have the function $r = f(\theta)$ with $0 \leq \theta \leq 2\pi$. To plot the same equation using polar coordinates can use the command:

PolarPlot[2,$\{\theta, 0, 2\pi\}$]

Try polar plotting the following functions:

Log$[\theta + 1]$ for $0 \leq \theta \leq \pi$ and Sin$[5\theta]$ for $0 \leq \theta \leq \pi$ and $0 \leq \theta \leq 2\pi$

Converting cartesian coordinates to polar coordinates:
Given the equation $f(x, y) = 2x^2y^2 + 5xy + 6$ you can easily convert it to polar coordinates by entering the following:

$$f[x\_, y\_] := 2x^2y^2 + 5xy + 6$$

Then, using the defined function, convert to an equivalent polar function by executing the command:

f[r*Cos[θ],r*Sin[θ]]

The result is the converted equation.