

MA205 - Integral Calculus
Lesson 25: Functions of Several Variables

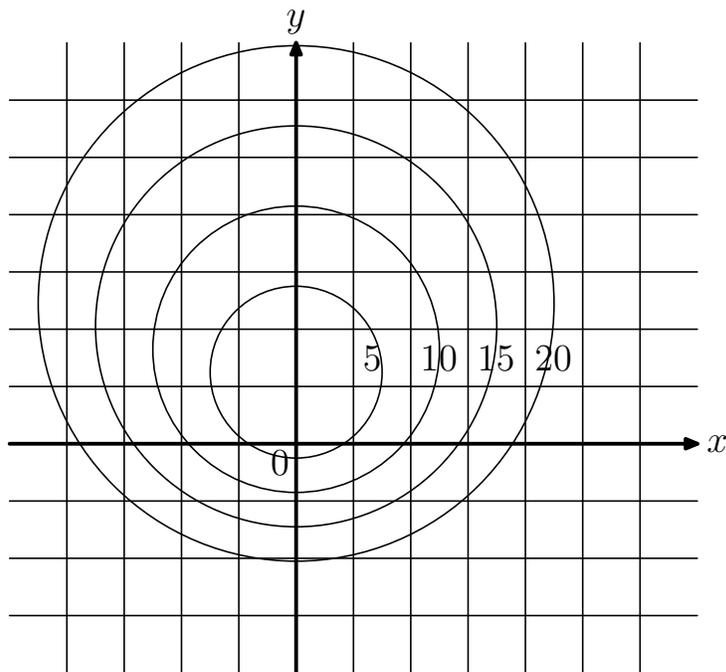
Mechanic Based Problems

1. Let $f(x, y) = \ln(x + y - 1)$.
 - (a) Evaluate $f(1, 1)$.
 - (b) Evaluate $f(e, 1)$.
 - (c) Find and sketch the domain of f .
 - (d) Find and sketch the range of f .

2. Find and sketch the domain of the function:

$$f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$$

3. A contour map for a function f is shown below. Use it to estimate the values of $f(-3, 3)$ and $f(3, -2)$. What can you say about the shape of the graph?



Problem Solving Problems

1. The table below depicts data given for the function $I = f(T, v)$, where I is the wind-chill index, T is the actual temperature, and v is the wind speed.

- (a) What is the value of $f(8, 60)$? What is its meaning?
- (b) Describe in words the meaning of the question “For what value of v is $f(-12, v) = -26$ ”? Then answer the question.
- (c) Describe in words the meaning of the question “For what value of T is $f(T, 80) = -14$ ”? Then answer the question.
- (d) What is the meaning of the function $I = f(-4, v)$? Describe the behavior of this function.
- (e) What is the meaning of the function $I = f(T, 50)$? Describe the behavior of this function.

$T \setminus v$	6	10	20	30	40	50	60	70	80	90	100
20	20	18	16	14	13	13	12	12	12	12	12
16	16	14	11	9	7	7	6	6	5	5	5
12	12	9	5	3	1	0	0	-1	-1	-1	-1
8	8	5	0	-3	-5	-6	-7	-7	-8	-8	-8
4	4	0	-5	-8	-11	-12	-13	-14	-14	-14	-14
0	0	-4	-10	-14	-17	-18	-19	-20	-21	-21	-21
-4	-4	-8	-15	-20	-23	-25	-26	-27	-27	-27	-27
-8	-8	-13	-21	-25	-29	-31	-32	-33	-34	-34	-34
-12	-12	-17	-26	-31	-35	-37	-39	-40	-40	-40	-40
-16	-16	-22	-31	-37	-41	-43	-45	-46	-47	-47	-47
-20	-20	-26	-36	-43	-47	-49	-51	-52	-53	-53	-53

2. The temperature-humidity index I is the perceived air temperature when the actual temperature in degrees Fahrenheit is T and the relative humidity is h , so we can write $I = f(T, h)$. The table below for values of I is an excerpt from a table compiled by the National Oceanic and Atmospheric Administration.

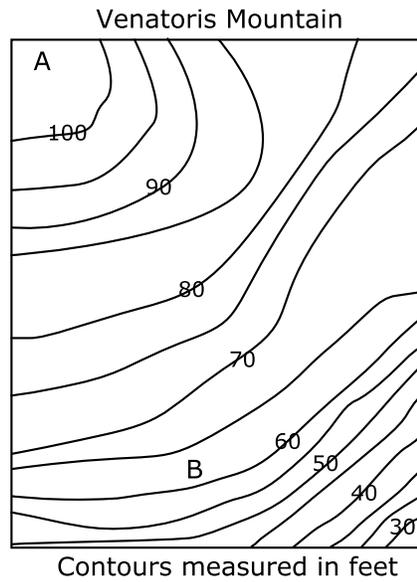
$T \setminus h$	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

(a) What is the value of $f(95, 75)$? What is its meaning?

(b) For what value of h is $f(90, h) = 100$?

(c) For what value of T is $f(T, 50) = 88$?

3. Locate the points A and B on the map of Venatoris Mountain below. How would you describe the terrain near A and B ?



4. A thin metal plate, located in the xy -plane, has temperature $T(x, y)$ at the point (x, y) . The level curves of T are called *isothermals* because at all points on an isothermal the temperature is the same. Sketch some isothermals if the temperature function is given by:

$$T(x, y) = \frac{100}{(1 + x^2 + 2y^2)}$$