MA205 - Integral Calculus
Lesson 31: Rectangular Regions II

Problem Solving Problems

1. Electric charge is distributed over a rectangle $1 \leq x \leq 3, 0 \leq y \leq 2$ so that the charge density at (x, y) is $\sigma(x, y) = 2xy + y^2$ (measured in coulombs per square meter). Find the total charge on the disk.

2. The population density of a certain city is described by $f(x, y) = 10,000e^{-0.2|x|-0.1|y|}$ where the origin $(0, 0)$ is the location of city hall. What is the total population in the area 10 miles east and west of the city hall and 5 miles north and south of city hall, where x and y are measured in miles?

   What is the average population density in the same area?

3. The County Workshop's total weekly profit (in dollars) realized in manufacturing and sales of its roll top desks is given by the profit function $P(f, u) = -0.2f^2 - 0.25u^2 - 0.2fu + 100f + 90u - 4000$ where f stands for the number of finished units and u stands for the number of unfinished units manufactured and sold each week. Find the average weekly profit if the number of finished units manufactured and sold varies between 180 and 200 and the number of unfinished unit varies between 100 and 120 per week?

4. An industrial plant is located at the precise center of a square town with each side of length 4 miles. If the plant is placed at the point $(0, 0)$, and certain pollutants are dispersed in such a manner that the concentration at any point (x, y) in town is given by $C(x, y) = 1000(24 - 3x^2 - 3y^2)$, where $C(x, y)$ is the number of particles of pollutants per square mile of surface per day at a point (x, y) in town, then what is the average concentration of these pollutants each day in town?