

Lesson 57 - Solving Homogeneous Linear Systems III - Phase Portraits

Objectives

- Use technology to create a phase portrait of a system of linear differential equations.
- Sketch the trajectory corresponding to the initial conditions of the system.

READ

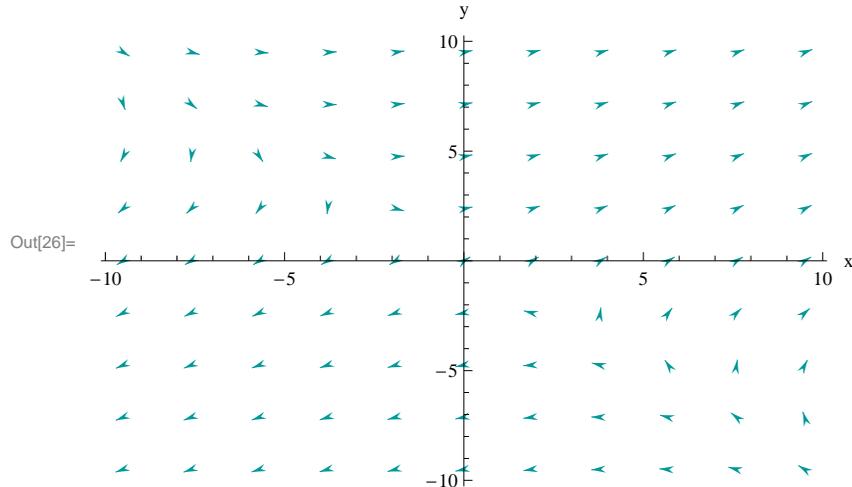
- Differential Equations Supplemental, section 8.2 (Homogeneous Linear Systems), section titled “Phase Portrait”, page 340-341.

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

In order to produce a direction field, or a phase portrait, in Mathematica, you can load the same package that we used to produce slope fields earlier this semester. Instead of the command `DEPlot`, we will use the `PhasePlot` command. The following code draws the direction field for Example 1 in section 8.2.1 (Distinct Real Eigenvalues):

```
In[25]:= << DiffEqs`DEGraphics`
PhasePlot[{2 x + 3 y, 2 x + y}, {t, 0, 100}, {x, -10, 10},
{y, -10, 10}, InitialPoints -> None, AxesLabel -> {"x", "y"}]
```

which produces the direction field shown below:



If you wish to see the trajectory corresponding with specific choices of c_1 and c_2 of the general solution (that is, corresponding with the initial conditions of the system), you can input these initial conditions into the command by altering the option `InitialPoints`. For example, the initial points of $(-1,3)$, $(-8,7)$, and $(-5,5)$ would be entered like the following:

```
In[39]:= << DiffEqs`DEGraphics`
PhasePlot[{2 x + 3 y, 2 x + y}, {t, 0, 100}, {x, -10, 10}, {y, -10, 10},
InitialPoints -> {{-1, 3}, {-8, 7}, {5, -5}}, AxesLabel -> {"x", "y"}]
```

which produces the direction field shown below:

