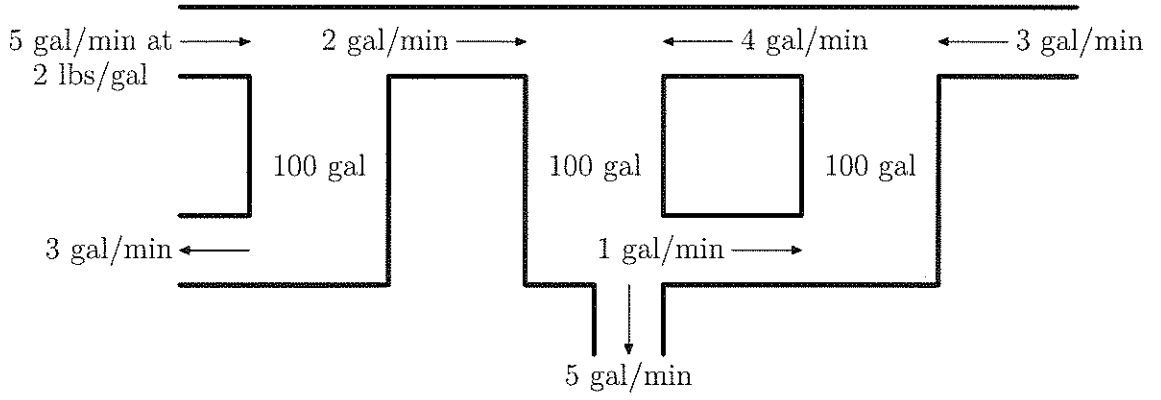
Block III Goal Problems

By the end of this block of instruction, you should be very comfortable analyzing problems similar to these:

1. Using the figure below derive three differential equations that specify the amount of dye in each of the three 100 gallon chambers. Assume all chambers start off with 0 lbs of dye.



$$\frac{dA}{dt} = 10 - \frac{5A(t)}{100}$$

$$\frac{dB}{dt} = \frac{2A(t)}{100} - \frac{6B(t)}{100} + \frac{4C(t)}{100}$$

$$\frac{dC}{dt} = \frac{B(t)}{100} - \frac{4C(t)}{100}$$

4. A 16 lb weight stretches a spring 4 ft. The spring-mass system is in a medium with a damping constant of 4.5 lb-sec/ft. What is the solution function describing the position of the mass at any time if the mass is released from 2 ft below the equilibrium position with an initial velocity of 4 ft/sec downward?

$$x(t) = \frac{20}{7}e^{-t} - \frac{6}{7}e^{-8t}$$

5. A mass of 1 slug is attached to a spring whose constant is 25/4 lb/ft. Initially the mass is released 1 ft above the equilibrium position with a downward velocity of 3 ft/sec, and the subsequent motion takes place in a medium that offers a damping force numerically equal to 3 times the instantaneous velocity. An external force given by $f(t) = 10$ (in pounds) is driving the system. Formulate and solve an initial value problem that models the given system. Interpret your results.

$$x'' + 3x' + \frac{25}{4}x - 10 = 0, \quad x(0) = -1, \quad x'(0) = 3$$

$$x(t) = \frac{1}{20}e^{-3t/2}\left(32e^{3t/2} - 52\cos(2t) - 9\sin(2t)\right)$$