Lesson 45 - Analytic Solutions II: Second Order Differential Equations

Objectives

- Given a second order, linear, homogeneous differential equation, determine the characteristic or auxiliary equation.
- Determine a general solution to a given second order, linear, homogeneous differential equation.
- Find the particular solution to a second order, linear, homogeneous initial-value problem.

READ

- Stewart, Chapter 17.1, pages 1111-1116.

THINK ABOUT

- What is an oscillation?
- What is the relationship between position velocity and acceleration?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

No new commands.

Mechanics Based Problems

For the next three problems find the general solution to the given second order differential equation.

1. $2y'' + 5y' + 3y = 0$

$$y = C_1 e^{-\frac{3}{2}x} + C_2 e^{-x}$$

2. $y'' + 8y' + 16y = 0$

$$y = C_1 e^{-4x} + C_2 x e^{-4x}$$

3. $y'' + 8y' + 41y = 0$

$$y = e^{-4x}\left(C_1 \cos(5x) + C_2 \sin(5x)\right)$$

For the next two problems find the particular solution(by hand) to the given second order differential equation by hand.

4. $2y'' + 5y' + 3y = 0, \quad y(0) = 3, \quad y'(0) = -4$

$y = 2e^{-\frac{3}{2}x} + e^{-x}$

5. $y'' + 16y = 0, \quad y(\pi/4) = -3 \quad , y'(\pi/4) = 4$

$y = 3\cos(4x) - \sin(4x)$

6. $y'' + 12y' + 36y = 0, \quad y(1) = 0, \quad y'(1) = 1$

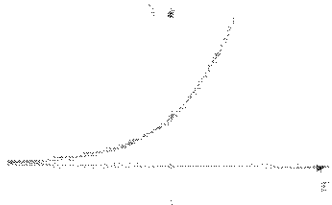
$y = e^{6}e^{-6x} - e^{6}xe^{-6x}$

Problem Solving Problems

1. Match the second order differential equations below to a solution curve graph:

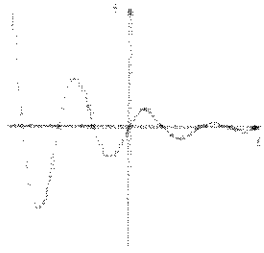
$a)$ $y'' - 3y' - 4y = 0$ $b)$ $y'' + 4y = 0$
$c)$ $y'' + 2y' + y = 0$ $d)$ $y'' + y = 0$
$e)$ $y'' + 2y' + 2y = 0$ $f)$ $y'' - 3y' + 2y = 0$

f



a

e



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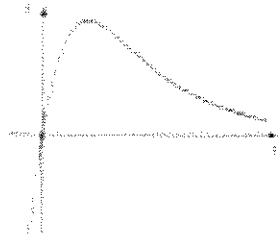

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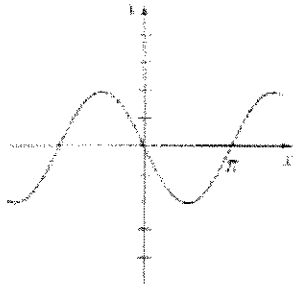
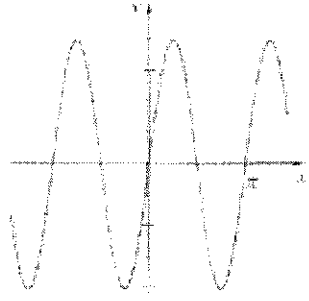


b



2. Find the necessary value of β so that $2y'' + \beta y' + 6y = 0$

(a) has a solution function that oscillates with constant amplitude.

$\beta = 0$

(b) has a solution function that oscillates with a decaying amplitude.

$-\sqrt{48} < \beta < \sqrt{48}$

(c) has a solution function with real and distinct roots.

$\beta < -\sqrt{48}$ OR $\beta > \sqrt{48}$

(d) has a solution function with repeated roots.

$\beta = \pm\sqrt{48}$

3. Given that $y(t) = e^{-5t} - te^{-5t}$ is a particular solution to a second order linear differential equation, what is the differential equation? What are the initial conditions?

$y = e^{-5t} - te^{-5t}$

$y(0) = 1$

$y'(0) = -6$

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