

LSN # 9

Mechanics Based Problems

1. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)dt$ represent?

The number of pounds the child gained (or lost) between year 5 and 10.

- * 2. The current, $I(t)$, in a wire is defined as the derivative of the charge, $Q(t)$; in other words: $I(t) = Q'(t)$. What does $\int_a^b I(t)dt$ represent?

Total charge from time a to time b .

3. If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t , what does $\int_0^{120} r(t)dt$ represent?

Total amount of oil that leaked from the tank in the first 120 mins.

4. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

The honeybee population after 15 weeks.

5. In section 4.7 we defined marginal revenue function $R'(x)$ as the derivative of the revenue function $R(x)$, where x is the number of units sold. What does $\int_{1000}^{5000} R'(x)dx$ represent?

Total revenue from selling unit 1000 through unit 5000

6. If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_3^5 f(x)dx$ represent?

The change in vertical distance (Altitude) between mile 3 and mile 5.

7. If x is measured in meters and $f(x)$ is measured in Newtons, what are the units for $\int_0^{100} f(x)dx$?

$$\begin{array}{l} H \times W \\ \text{(Newtons)} \quad \text{(meters)} \end{array} \Rightarrow \text{Newton} \times \text{Meter (N} \cdot \text{m)} \quad \text{or} \quad \text{Joule (J)}$$

* 8. If the units for x are feet and the units for $a(x)$ are pounds per foot, what are the units for da/dx ? What units does $\int_2^8 a(x)dx$ have?

$$\frac{da}{dx} = \frac{\text{lbs}}{\text{ft}^2}$$

ANS

$$\int_2^8 a(x)dx = \frac{\text{lbs}}{\text{ANS}}$$

9. Evaluate the integral: $\int_1^2 \frac{4+u^2}{u^3} du$

$$\ln 2 + \frac{3}{2}$$

ANS

10. Evaluate the integral: $\int_0^5 (2e^x + 4\cos(x))dx$

$$2e^5 - 2 + 4\sin(5)$$

ANS

Problem Solving Problems

1. The velocity function (in meters per second) for a particle moving along a line is $v(t) = t^2 - 2t - 8$ where $1 \leq t \leq 6$. Find (a) the displacement and (b) the distance traveled by the particle during this interval. Use Mathematica to check your answers.

$$a) \text{ Displacement} = \frac{-10}{3} \text{ meters}$$

$$b) \text{ Distance} = \frac{98}{3} \text{ meters}$$

2. The linear density of a rod, denoted $\rho(x)$, of length 4 meters is given by $\rho(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

$$\text{Mass} = \frac{140}{3} \text{ Kilograms}$$

ANS

3. An animal population is increasing at a rate of $200 + 50t$ per year (where t is measured in years). By how much does the animal population increase between the fourth and tenth years?

$$3300 \text{ Animals}$$

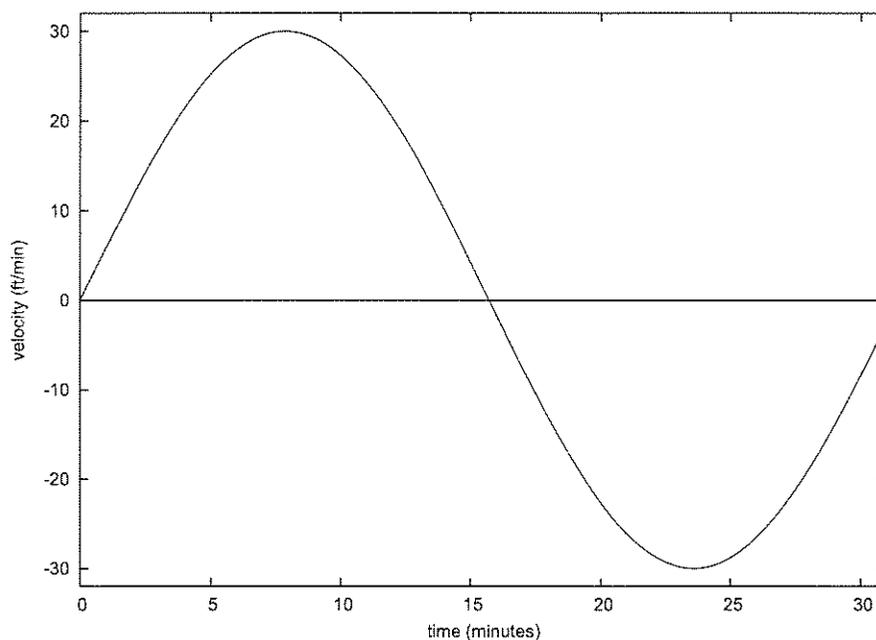
ANS

4. The marginal profits of an electric can opener manufacturer are given by $P(x) = 0.1x\sqrt{x+2}$, where x is measured in thousands of can openers and $P(x)$ is measured in thousands of dollars. Find the total profits generated by increasing the number of can openers from 7000 to 14,000.

$$\text{Total Profit} = \$ 26,306.70$$

ANS

5. The velocity of an object traveling in a straight line over a period of 30 minutes is shown below. Positive velocities indicate movement away from the start point, and negative velocities indicate movement towards the start point.
- Estimate the total distance traveled.
 - Estimate the displacement.
 - Where and when does the object come to rest?



a) ≈ 539 ft

b) ≈ 8 ft. From start point in pos direction

c) ≈ 273.5 ft c) $t = 16$ minutes