

**MA206 Suggested Problems**  
**Exponential Distribution**

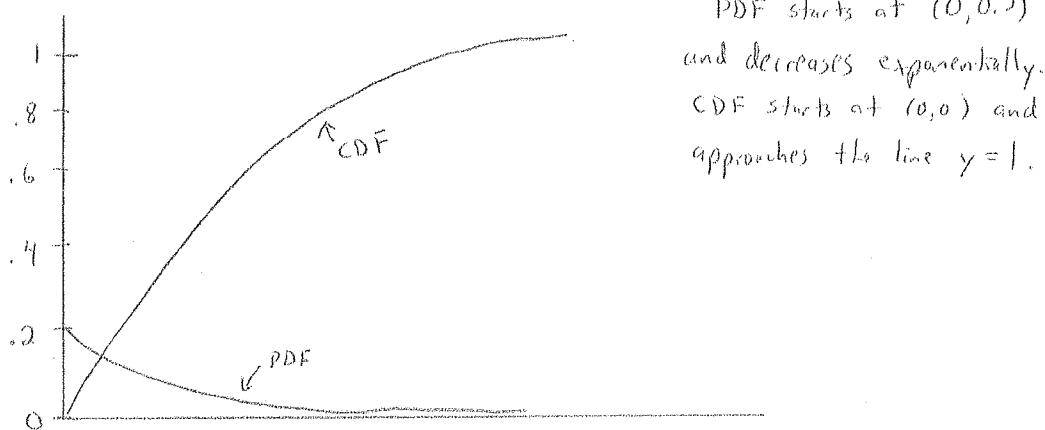
Let  $X$  = the time between crashes in days of the corps of cadets network server. The cumulative distribution function of  $X$  is as follows:

$$F(x) = \begin{cases} 1 - e^{-0.2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

- a. Take the derivative of  $F(x)$  to find the PDF of  $X$ ; ensure that you completely specify the PDF using the appropriate intervals.

$$f(x) = \begin{cases} 0.2 e^{-0.2x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

- b. Sketch the PDF and the CDF of  $X$ . Comment on the relationship between the PDF and the CDF.



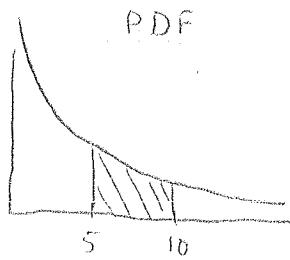
- c. Calculate the average time between crashes.

$$E(X) = \mu = \int_0^{\infty} x \cdot f(x) dx = \frac{1}{\lambda} = 5 \text{ days}$$

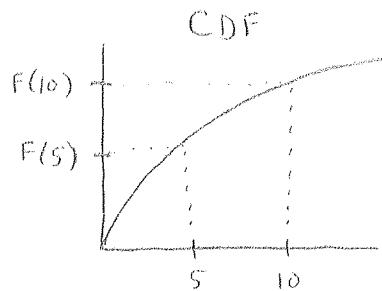
- d. Compute the variance of the time between crashes.

$$V(X) = \sigma^2 = \int_0^{\infty} (x - \mu)^2 \cdot f(x) dx = \frac{1}{\lambda^2} = 25 \text{ days}$$

- c. Using both the PDF and the CDF, find the probability that there are between 5 and 10 days before the next crash. Illustrate your answers on the respective graphs.



$$\int_5^{10} f(x) dx = \underline{0.2325}$$



$$F(10) - F(5) = \underline{0.2325}$$

- f. Determine the median time between crashes.

$$F(x) = 0.5, \text{ Solve for } x \Rightarrow x = 3.4657$$

- g. Find the 90<sup>th</sup> percentile of the random variable  $X$ .

$$F(x) = 0.9, \text{ Solve for } x \Rightarrow x = 11.5129$$

- h. Given that the cadet network has not crashed for the past 6 days, what is the probability that the network will work for at least 3 more days before crashing?

Memoryless Property

$$\int_3^{\infty} f(x) dx = \underline{0.5488}$$

$$CH_2 \Rightarrow 45, 50, 53$$

(45)

$$P(A) = .106 + .141 + .200 = 0.447$$

a)  $P(C) = .215 + .200 + .065 + .020 = 0.500$

$$P(A \cap C) = 0.200 \text{ (from table)}$$

b)  $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0.2}{0.5} = 40\%$

Given that a person is of ethnic group 3, there is a 40% probability that he/she has type A blood

$$P(C|A) = \frac{P(C \cap A)}{P(A)} = \frac{0.2}{0.447} = 44.74\%$$

Given that a person has type A blood, there is a 44.74% probability that he/she is of ethnic group 3.

c)  $P(\text{ethnic group 1} \mid \text{not } B \text{ blood})$

$$P(B) = 0.008 + 0.018 + 0.065 = 0.091$$

Therefore,  $P(\text{not } B) = 0.909$

$$P(\text{group 1} \mid \text{not } B) = \frac{P(\text{group 1} \cap \text{not } B)}{P(\text{not } B)} = \frac{0.082 + 0.106 + 0.004}{0.909} = \underline{\underline{0.2112}}$$

(50)

a.) 0.05  $\Rightarrow$  from table

b.)  $(0.07) + (0.05) = 0.12$

c.) short sleeve  $\Rightarrow$  (sum 9 probabilities in short sleeve table)  $= 0.56$

long sleeve  $\Rightarrow 0.44$

d.) medium?  $\Rightarrow 0.08 + 0.07 + 0.12 + 0.10 + 0.05 + 0.07 = 0.49$

print?  $\Rightarrow 0.02 + 0.07 + 0.07 + 0.02 + 0.05 + 0.02 = 0.25$

e.)  $P(\text{medium} \mid \text{SS \& plaid})$

$$P(\text{SS \& plaid}) = 0.04 + 0.08 + 0.03 = 0.15$$

$$P(\text{medium} \mid \text{SS \& plaid}) = \frac{P(\text{medium} \cap \text{SS \& plaid})}{P(\text{SS \& plaid})} = \frac{0.08}{0.15}$$

$$= \underline{\underline{0.5333}}$$

f.)  $P(\text{med. plaid}) = (0.08) + (0.10) = 0.18$

$$P(\text{SS} \mid \text{med plaid}) = \frac{P(\text{SS} \cap \text{med. plaid})}{P(\text{med. plaid})} = \frac{0.08}{0.18} = \underline{\underline{0.4444}}$$

$$P(\text{LS} \mid \text{med plaid}) = \frac{P(\text{LS} \cap \text{med plaid})}{P(\text{med. plaid})} = \frac{0.1}{0.18} = \underline{\underline{0.5556}}$$

(53)

A - Audio

B - CD Player (contained in A)

$$P(A) = 0.6$$

$$P(B) = 0.05$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

(since B is contained in A,  $P(B \cap A) \Rightarrow P(B)$ )

$$= \frac{P(B)}{P(A)} = \frac{0.05}{0.6} = \underline{\underline{0.0833}}$$