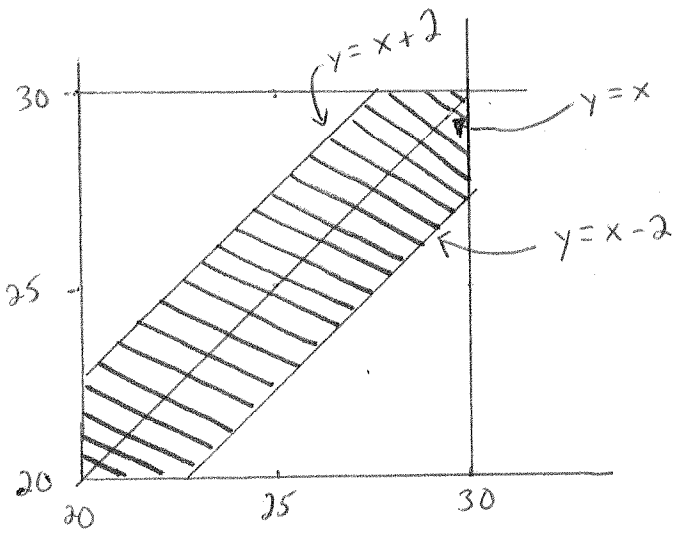
Lesson 15

(9) $f(x,y) = \begin{cases} K(x^2+y^2) & , 20 \le x \le 30 \ , \ 20 \le y \le 30 \\ 0 & , \text{otherwise} \end{cases}$

(a) $\int_{20}^{30}\int_{20}^{30} K(x^2+y^2)\,dy\,dx = 1$, Solve for K , $\underline{\underline{K = \frac{3}{380{,}000}}}$ ANS

(b) $\int_{20}^{26}\int_{20}^{26} \frac{3}{380{,}000}(x^2+y^2)\,dy\,dx = \frac{189}{625} = \underline{\underline{0.3024}}$ ANS

(c) $P(|Y-X| \le 2)$ Draw a picture!



Find volume of region over shaded domain!

$$\int_{20}^{22}\int_{20}^{x+2} \frac{3}{380{,}000}(x^2+y^2)\,dy\,dx + \int_{22}^{28}\int_{x-2}^{x+2} \frac{3}{380{,}000}(x^2+y^2)\,dy\,dx + \int_{28}^{30}\int_{x-2}^{30} \frac{3}{380{,}000}(x^2+y^2)\,dy\,dx$$

$$= \frac{4267}{11875} \Rightarrow \underline{\underline{0.3593}}$$ ANS

(d) X is right tire, Y is left tire

Task: Find marginal distribution for right tire $\Rightarrow f_X(x)$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \Rightarrow \int_{20}^{30} \frac{3}{380{,}000}(x^2+y^2)\,dy$$

$$\boxed{f_X(x) = \frac{1}{20} + \frac{3x^2}{38{,}000}, \quad 20 \le x \le 30}$$

(e) first, find $f_Y(y) = \int_{20}^{30} \frac{3}{380{,}000}(x^2+y^2)\,dx$

$$f_Y(y) = \frac{1}{20} + \frac{3y^2}{38{,}000}$$

If independent, $f(x,y) = f_X(x) \cdot f_Y(y)$

$$\frac{3}{380{,}000}(x^2+y^2) \stackrel{?}{=} \left(\frac{1}{20} + \frac{3x^2}{38{,}000}\right)\left(\frac{1}{20} + \frac{3y^2}{38{,}000}\right)$$

$$\stackrel{?}{=} \left(\frac{1{,}900 + 3x^2}{38{,}000}\right)\left(\frac{1{,}900 + 3y^2}{38{,}000}\right)$$

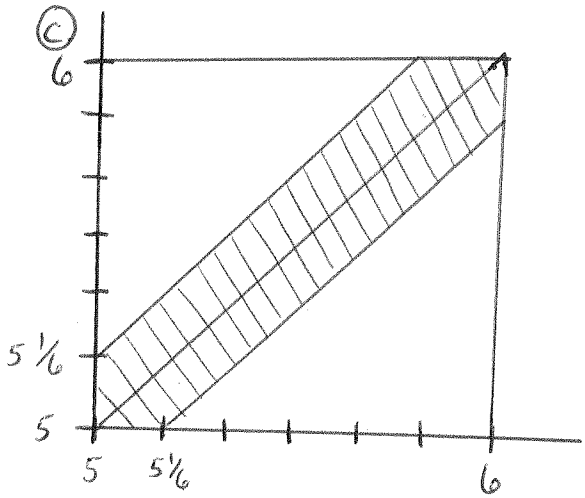
$$\stackrel{?}{=} \frac{3{,}610{,}000 + 5700x^2 + 5700y^2 + 9x^2y^2}{(38{,}000)^2}$$

No! $\therefore$ <u>NOT</u> independent!

(10) (a) $\int_5^6 \int_5^6 1 \, dy\, dx$

(b) $\int_{5.25}^{5.75} \int_{5.25}^{5.75} 1 \, dy\, dx = \underline{\underline{0.25}}$

(c)



Find volume over shaded domain

1 − volume over other regions

By symmetry & since $f(x,y) = 1$

$\hookrightarrow$

$1 - \left[2\left(\frac{1}{2} \cdot \text{length} \cdot \text{height}\right)\right]$

$1 - \left[2\left(\frac{1}{2}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\right] = \underline{\underline{0.3056}}$

ANS

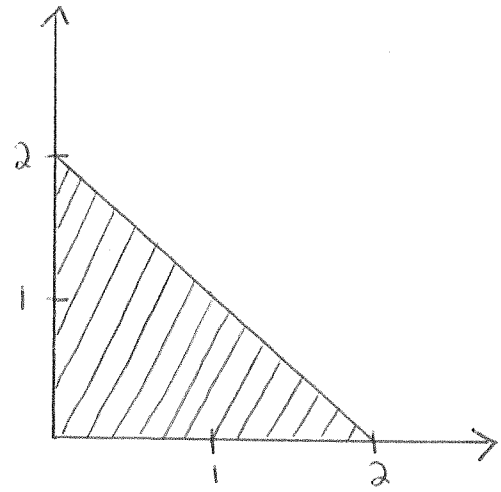
(13) (a) by independence, $f(x,y) = f_X(x) \cdot f_Y(y)$

$f(x,y) = (1e^{-1x})(1e^{-1y}) = (e^{-x})(e^{-y})$

$$f(x,y) = \begin{cases} e^{-x-y}, & x \geq 0,\ y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

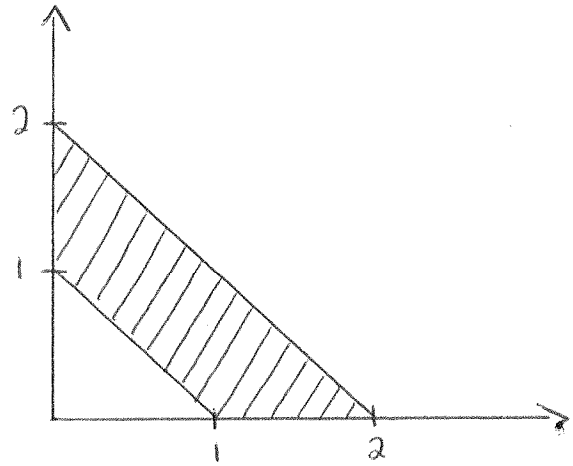
(b) $\int_0^1 \int_0^1 e^{-x-y} \, dy\, dx = \underline{\underline{0.3996}}$

ⓒ $P(x+y \le 2)$



$$\int_0^2 \int_0^{-x+2} e^{-x-y}\, dy\, dx = \underline{\underline{0.5940}}$$

ⓓ $P(1 \le x+y \le 2)$



$$\int_0^1 \int_{-x+1}^{-x+2} e^{-x-y}\, dy\, dx + \int_1^2 \int_0^{-x+2} e^{-x-y}\, dy\, dx = \underline{\underline{0.3298}}$$