SOLUTION

MA 206 Suggested Problems
Lesson ~~4. Fitting Models to EDFs~~

NOTE: The problems below are a continuation of the Lesson 4 Suggested Problems. You are expected to use the EDF you created for Lesson 4, Suggested Problem # 2 to complete the following exercises.

1. Plot the function $F(x)=1-e^{-ax}$ where $x,a \geq 0$ on the same graph as the EDF. To begin with, set $a = 1$. Note that the function *F(x)* does not appear to match the EDF very well.

   a. Increase the value of *a* in small increments and observe the result.

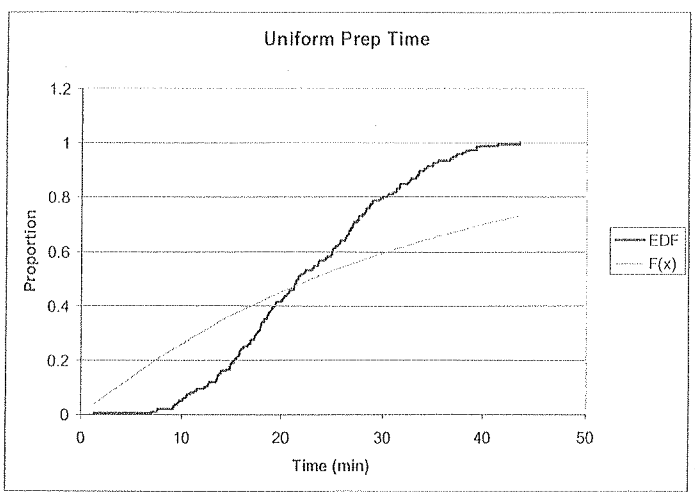
      Increasing the value of *a* moves *F(x)* even farther from the EDF.

   b. Now, decrease the value of *a* in small increments and observe the result.

      Decreasing the value of *a* moves *F(x)* closer to the EDF.

   c. Using your results from above, conduct further experimentation to determine the value of *a* that enables *F(x)* to fit the EDF as closely as possible.

      Using ***a = 0.03*** yields the following graph:



   d. Use Excel's Solver to determine the best value for *a*.

Using Solver, we find that the best value for *a* is 0.033604. This has a corresponding minimized SSE of 10.60.

2. Next, add the function $G(x)=1-e^{-(ax)^b}$ where $x,a,b \geq 0$ to your graph. To begin with, set $a = 1$ and $b = 1$. It appears that the function *G(x)* does not fit the EDF very well either.

SOLUTION

a. Change the value of a to the number you determined in Problem 1d (should be approximately 0.034) and observe the result. Why does the graph of $G(x)$ look exactly like $F(x)$?

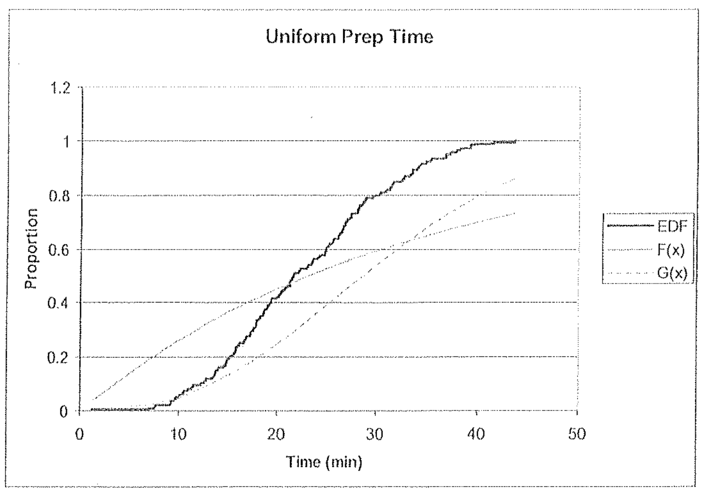
When $b = 1$, $G(x) = 1 - e^{-(ax)^b}$ is mathematically equivalent to the function $F(x) = 1 - e^{-ax}$.

b. Adjust the value of b in small increments (up and down) and observe the results.

Increasing the value of b moves $G(x)$ closer to the EDF. Decreasing the value of b moves $G(x)$ farther from the EDF.

c. Determine the value of b that enables $G(x)$ to fit the EDF as closely as possible.

Using $a = 0.03$ for $F(x)$ and $a = 0.03$, $\boldsymbol{b = 2.5}$ for $G(x)$ yields the following graph:

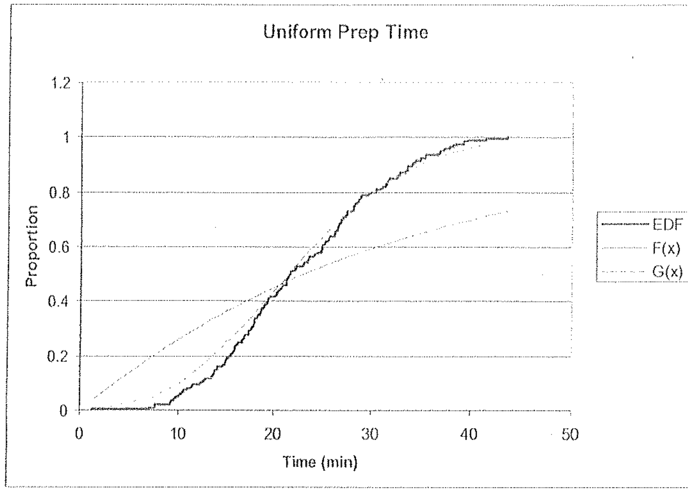


This solution is starting to look better for G(x). In part *e* (below) we will use Solver to find the optimal value of the parameters in G(x).

d. At this point, the function G(x) should provide a relatively good fit to the EDF. As a final step, check to see if any further adjustments to the value of a (up or down) are necessary in order to find a better fit.

SOLUTION

Using $a = 0.03$ for $F(x)$ and $\boldsymbol{a = 0.04}$, $b = 2.5$ for $G(x)$ yields the following graph:



e. Use Excel's *Solver* to determine the best values for both a and b.

Using Solver, the minimized SSE for G(x) occurs when $a = 0.039443$ and $b = 2.850205$.

f. Why does $G(x)$ provide a much better fit to the EDF than the function $F(x)$?

As we determined in part (a) of this problem, $G(x)$ is mathematically equivalent to the function $F(x)$ when $b = 1$. However, for $b > 1$, $G(x)$ changes concavity and therefore more closely models the behavior of our EDF. $F(x)$ is a more simplistic, less flexible model.