MA206

(11) $X \sim$ checkout duration

$$F(x) = \begin{cases} 0 & x<0 \\ \frac{x^2}{4} & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

a. $P(X \le 1) = F(1) = \underline{\underline{\frac{1}{4}}}_{ANS}$

b. $P(.5 \le X \le 1) = F(1) - F(.5) = \frac{1}{4} - \frac{1}{16} = \underline{\underline{\frac{3}{16}}}_{ANS}$

c. $P(X > .5) = 1 - P(X < .5) = 1 - F(.5) = \underline{\underline{\frac{15}{16}}}_{ANS}$

d. $P(X > a) = 0.5 \Rightarrow F(a) = 0.5 = \frac{a^2}{4} \Rightarrow \underline{\underline{a = \sqrt{2}}}_{ANS}$

e. $F'(x) = f(x) = \frac{x}{2}$ for $0 \le x \le 2$

f. $E(X) = \int_{-\infty}^{\infty} f(x)\, x\, dx = \int_0^2 x \frac{x}{2}\, dx = \underline{\underline{\frac{4}{3}}}_{ANS}$

g. $V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx = \int_0^2 (x - \frac{4}{3})^2 f(x)\, dx = \underline{\underline{\frac{2}{9}}}_{ANS}$ $\quad \sigma = \underline{\underline{\sqrt{\frac{2}{9}}}}_{ANS}$

h. $h(x) = x^2 \Rightarrow$ Find $E[h(x)] = E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_0^2 \frac{x^3}{2}\, dx = \underline{\underline{2}}_{ANS}$

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(13) $X \sim$ time between cars

$$f(x) = \begin{cases} \frac{k}{x^4} & x > 1 \\ 0 & x \le 1 \end{cases}$$

a. Find k $\quad k \int_1^{\infty} \frac{1}{x^4}\, dx = 1 \Rightarrow k = \frac{1}{\int_1^{\infty} x^{-4}\, dx} = \underline{\underline{3}}_{ANS}$

b. Find CDF

$$F(x) = \int_1^x f(y)\, dy = \int_1^x \frac{3}{x^4}\, dx = 3\int_1^x x^{-4}\, dx = -\frac{1}{x^3}\Big|_1^x = -\frac{1}{x^3} + 1$$

$$F(x) = \begin{cases} 0 & x \le 1 \\ 1 - \frac{1}{x^3} & x > 1 \end{cases}$$

c. $P(X > 2) = 1 - P(X < 2) = 1 - F(2) = 1 - (1 - \frac{1}{2^3}) = \underline{\underline{\frac{1}{8}}}_{ANS}$

$P(2 < X < 3) = F(3) - F(2) = \underline{\underline{0.087963}}_{ANS}$

d. $E[X] = \int_1^{\infty} \frac{3}{x^3}\, dx = \underline{\underline{1.5}}_{ANS}$ $\quad \sigma = \sqrt{\int_1^{\infty} \frac{3}{x^4}(x-1.5)^2\, dx} = \underline{\underline{\sqrt{\frac{3}{4}}}}_{ANS}$

e. $P(1.5 - \sqrt{\frac{3}{4}} \le X \le 1.5 + \sqrt{\frac{3}{4}}) = P(0.634 \le X \le 2.366) = \int_{.634}^{2.366} f(x)\, dx = \underline{\underline{0.9245}}_{ANS}$

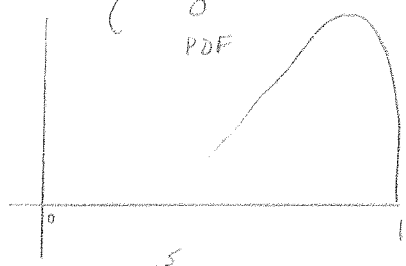
15

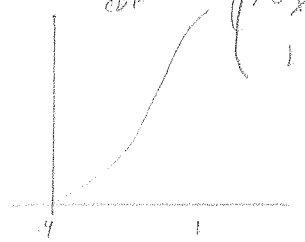
$X \sim$ space occupied

$$f(x) = \begin{cases} 90x^8(1-x) & 0<x<1 \\ 0 & o/w \end{cases} \Rightarrow F(x) = \begin{cases} 0 & x<1 \\ 10x^9 - 9x^{10} & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

PDF CDF

a.





b. $P(x \le .5) = \int_0^{.5} f(x)dx = \underline{0.0107422}$ ANS

c. $P(.25 < x \le .5) = P(.25 \le x \le .5) = \int_{.25}^{.5} f(x)dx = \underline{0.0107126}$ ANS

d $P(x \le a) = 0.75$

CDFinvse$(.75) \Rightarrow \int_0^a f(x)dx = 0.75$ $\quad a = \underline{0.903596}$ ANS

e. $E[x] = \underline{\frac{9}{11}} = \mu$ ANS $\quad \sigma = \underline{0.11134}$ ANS

f. $1 - P(\mu - \sigma < x < \mu + \sigma) = 1 - P(0.706841 \le x \le 0.929522) = 1 - 0.686329$

$= \underline{0.313671}$ ANS