

**MA206 Suggested Problems**  
**Unnamed Continuous Distributions**

1. The grade  $X$  that a student receives on a very difficult mathematics exam is a continuous random variable that can be represented by the following probability density function:

$$f(x) = \begin{cases} \frac{4}{3}(1-x^3) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- a. Compute the average grade on a very difficult math exam.

$$E[X] = \mu = \int_{-\infty}^{\infty} x f(x) dx = 0.4 = \underline{\underline{40\%}}$$

- b. Find the standard deviation of the grades on a very difficult math exam.

$$\sqrt{E[X^2]} = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = 0.0622 \Rightarrow \sigma = \sqrt{E[X^2]} = 0.24944 \approx \underline{\underline{25\%}}$$

- c. Determine the probability a student passes a very difficult math exam (his/her score must be above 65%).

$$P(X \geq 0.65) = \int_{0.65}^1 f(x) dx = 0.192835 \approx \underline{\underline{19.3\%}}_{ANS}$$

2. Let  $X$  be the number of gallons of unleaded gas purchased by any customer at the AAFES Service Station. The cost per gallon of gas is \$2.09. Suppose the probability density function for  $X$  is as follows:

$$g(x) = \begin{cases} 0.01x & 0 \leq x \leq 10 \\ 0.2 - 0.01x & 10 \leq x \leq 20 \\ 0 & \text{o.w.} \end{cases}$$

- a. Calculate the expected amount each customer spends on gas.

$$E[X] = \int_0^{10} x f(x) dx + \int_{10}^{20} x f(x) dx = 10 \Rightarrow 10 \cdot \$2.09 = \underline{\underline{\$20.90}}$$

- b. Find the CDF,  $G(x)$ ; ensure that you completely specify the CDF using the appropriate intervals.

$$G(x) = \begin{cases} 0.005x^2 & 0 \leq x \leq 10 \\ 0.2x - 0.005x^2 - 1 & 10 \leq x \leq 20 \\ 0 & x < 0 \\ 1 & x > 20 \end{cases}$$

- c. Compute the probability that a customer spends less than \$20 on unleaded fuel in a single visit.

$$P(\$2.09X < \$20) = P(X \leq 9.57) = \int_0^{9.57} f(x) dx = \underline{\underline{0.458}}_{\text{ANS}}$$

- d. What is the 85<sup>th</sup> percentile of the random variable  $X$ ?

$$G(x) = .85 \Rightarrow x = 14.523 \text{ gallons}$$