MA 371 WPR 3 selected practice problems.

1. (From 4.4 #8) Find the characteristic equation for the matrix $\begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{pmatrix}$, find the eigenvalues and eigenvectors, and find the geometric and algebraic multiplicities of the eigenvalues.

2. Show, using matrix multiplication, that $(1, -1, 2)$ is an eigenvector of $\begin{pmatrix} 3 & 4 & -1 \\ -1 & -2 & 1 \\ 3 & 9 & 0 \end{pmatrix}$.

3. A country is divided into two demographic regions. Each year, 20% of the residents of Region 1 move to Region 2. Of the residents of Region 2, 30 % move to Region 1. Initially, 20% of the population resides in Region 1 and 80% in Region 2.

   After a long time, what percentage of the population resides in each region?

4. Let $T : \mathcal{R}^3 \to \mathcal{R}^2$ be defined by $T_A(x_1, x_2, x_3) = (2x_2 - 3x_3, x_1 - 2x_2 + x_3)$

   (a) Find the standard matrix representation, A, of T_A, and use it to find $T(3, 4, 5)$.

   (b) Find $ker(T_A)$

   (c) Determine whether T is

      i. one-to-one,

      ii. onto,

      iii. invertible.

5. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ -1 & 2 & -1 \end{pmatrix}$

   (a) Find a basis for $null(A)$.

   (b) Find a basis for $row(A)$.

   (c) Find a basis for $col(A)$.

   (d) Is T_A 1-1?

6. Let A be as defined in the last problem. Is the operator T_A onto $\mathcal{R}^3$. Justify your answer.

7. Show that $v_1 = (1, 0, 1), v_2 = (0, 2, 1)$, and $v_3 = (2, 2, 2)$ form a basis for $\mathcal{R}^3$, or show that they don't.

8. Suppose that $RREF(A) = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}$.

   (a) Find rank(A).

   (b) Find nullity(A)

   (c) Is AA^T invertible? Why or why not?

   (d) Is A^TA invertible? Why or why not?

9. Let A be defined in problem 5.

   (a) Show that $(3, 1, 2) \in col(A)$.

   (b) Show that $(4, 1, 1) \in row(A)$.

10. #20 Sec 7.4.

11. #5 Sec. 7.5.

12. Suppose that A is 4×5 with rank(A)=3

    (a) Find nullity(A^TA).

    (b) Find nullity(AA^T).

13. Find the projection of $(1, -2, 2)$ onto the subspace W spanned by the vectors $v_1 = (1, 1, 1)$, and $v_2 = (2, 0, 1)$. Show that the error vector is orthogonal to W.

14. Let A be defined in problem 5. Find the projection of $(1, -2, 2)$ onto

    (a) $row(A)$.

    (b) $col(A)$.

15. # 12 Sec. 7.8.