MA 371 11/6/03 Quiz 6. **NAME:**

You may use a calculator, text and notes. Show all other work.

1. A country is divided into two demographic regions. Each year, 15% of the residents of Region 1 move to Region 2. Of the residents of Region 2, 20 % move to Region 1. Initially, 40% of the population resides in Region 1 and 60% in Region 2.

   (a) Model this situation with a system of 2 DDS.

   (b) Express the system in matrix form.

   (c) Write the solution using the eigenvector expansion (that is, in terms of the eigenvectors and eigenvalues).

   (d) After a long time, what percentage of the population resides in each region?

2. Let $A = \begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{pmatrix}$.

(a) Find the characteristic equation of A.

(b) Find the eigenvectors and eigenvalues of A and state their geometric and algebraic multiplicities.

(c) Use your answer to (b) to determine the eigenvalues and eigenvectors of A^{10}.

3. Suppose that A is a matrix of the form $\begin{pmatrix} 5 & d & e & f \\ 0 & -1 & b & c \\ 0 & 0 & -2 & a \\ 0 & 0 & 0 & 1 \end{pmatrix}$ where a, b, c, d, e, and f are constants. Find the eigenvalues of A, if possible, and decide whether A is singular or not, if possible.