MA 371 WPR 2 **26 October 2003**

Name: ______________________________

1. Use of notes, text, and TI-89 **are** authorized.
2. Use pencil.
3. You **need not** show the details of computations obtained via TI-89. Show all other work.
4. All problems are weighted as indicated.

1. (15 pts.) Find the orthogonal complement of the span of the vectors $v_1 = (1, 2, 0, -1)$, $v_2 = (0, 1, 1, 0)$ in $\mathbf{R}^4$.

2. (20 pts.) For each of the following, determine whether the given set of vectors is linearly independent. Briefly explain your reasoning.

   (a) $v_1 = (-1, 1, -2, 1)$, $v_2 = (-1, 1, 0, 1)$, $v_3 = (-2, 2, 2, 2)$.

   (b) $v_1 = (1, 2, 1, 2)$, $v_2 = (0, 1, 0, 1)$, $v_3 = (1, 1, 1, 0)$.

3. Suppose that $A = (v_1, v_2, v_3, v_4)$ where the v_i are column vectors in $\mathbf{R}^4$, and suppose that $RREF(A) = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

(a) (5 pts.) Are the rows of A linearly independent? Explain.

(b) (5 pts.) Are the columns of A linearly independent? Explain.

(c) (10 pts.) Find the solution to $Ax = 0$ and use it to express v_1 as a linear combination of v_2, v_3, and v_4.

4. (10 pts.)

   Find the 4×4 elementary matrix E such that the product EA has the given effect on a $4 \times n$ matrix A.

   (a) Interchanges rows 3 and 4.

   (b) Multiplies row 3 by 4 and adds it to row 1.

5. (10 pts.) Suppose that A and B are $n \times n$ matrices such that $AB = \mathbf{0_{n\times n}}$.

   (a) What relationship holds between the rows of A and the columns of B? Briefly explain your reasoning.

   (b) Show, using the determinant, that either A or B, (or both) must be singular. **Hint:** Use Theorem 4.2.5.

6. (5 pts.) Suppose C, D, and E are $n \times n$ matrices with $CDE = I_{n\times n}$. Find D^{-1}.

7. (10 pts.) Find all values of k for which $A = \begin{pmatrix} k & k & -1 \\ k & 0 & 0 \\ -1 & 1 & 1 \end{pmatrix}$ non-singular.

8. (10 pts.) The matrix determinant $\begin{vmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & 1 & -1 \\ 1 & x & x^2 & x^3 \end{vmatrix}$ is a polynomial in x. What is the coefficient of x^2?

EXTRA: (5 pts.) Show that the set of vectors of the form (x_1, x_2, x_3) where x_1, x_2, and x_3 are real numbers such that $x_1 + 2x_2 - 3x_3 = 0$ is a subspace of $\mathbf{R}^3$ either by: 1. Showing directly that the closure properties hold, **or**, 2. Finding a set of spanning vectors.