MA205 - Integral Calculus and Introduction to Differential Equations

PROJECT I - 150 Points

Instructions: The following assignment is due on **20 SEPT 07 IN CLASS**. In accordance with the Documentation of Written Work (DWW), you must document any and all collaboration with sources other than your partner(s), your text book, and your instructor. Documentation must be IAW Paragraph 1 - "Documentation Standard" of Section VI - "Complying with Documentation Standards" from the DWW. Work will be assessed for your group performance; however, the project final grade may not be shared equally if the work was not equally shared by all group members. Sufficient work is required to indicate clearly the method of reasoning and the operations performed. **SHOW ALL WORK**.

You have just been hired as a consultant to a new company that is expanding their business, and is building a new distribution plant. Current geological surveys have identified the optimal location for the plant on the west side of Lake Euler. The company must now determine a feasible and cost-effective route between the distribution center and the closest airport, located on the east side of the lake. The company can either build a road around the lake to the north, a road around the lake to the south, a bridge across the lake, or any combination of these options. The land north of the lake is marshy and very soft; it will require much preparation prior to placing a road. The land south of the lake is lightly forested and will require less preparation.

Option A

From the most recent land survey, a Computer Algebra System (CAS) modeled the elevation above sea level (in miles) in the area of interest by the following function:

$$T(x,y) = 0.5 - 0.04(e^{-(x^2+y^2)} + 0.5e^{-(2(x-1.5)^2+(y-1.5)^2)} + e^{-((x-1)^2+2(y+1)^2)} + 0.25e^{-(2(x-1.2)^2+(y-0.9)^2)})$$

for

$$-3 \leq x \leq 3$$
$$-3 \leq y \leq 3$$

where x and y are represented in miles, and $T(x,y)$ represents elevation above sea level in miles. (To verify your accuracy when inputting $T(x,y)$ in Mathematica, $T(0,0) = 0.457735$.) The normal water table is at an elevation of 2613.6 ft (0.495 miles) above sea level. The distribution center is located at the coordinates (-1.0, 1.5) and the airport is located at the coordinates (3.0, 0.5). See Figure 1 for a graphical representation of the area of interest.

Geologists, civil engineers, and environmental engineers have already looked at placing roads (drawn in Figure 1), one to the north of the lake, and one to the south. The roads they planned are reported to minimize the impact on the ecology and existing town area, and are modeled by the functions below.

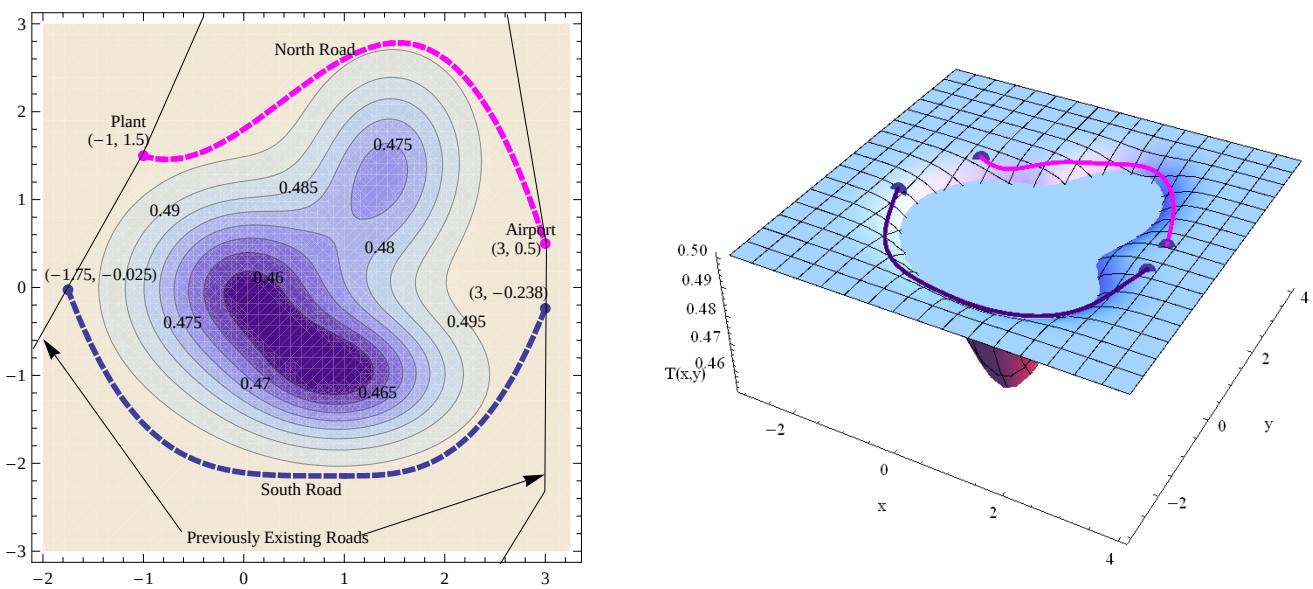
North Road

$$n(x) = \frac{9}{5} + \frac{23}{30}x + \frac{1}{4}x^2 - \frac{13}{60}x^3, \text{ for } x \text{ measured in miles}$$

South Road

$$s(x) = \begin{cases} -\frac{398}{189} - \frac{4}{27}x + \frac{2}{9}x^2 - \frac{4}{27}x^3 + \frac{1}{27}x^4 & \text{for } -1.75 \leq x < 1 \\ \frac{-50}{21} + \frac{5}{7}x - \frac{5}{7}x^2 + \frac{5}{21}x^3 & \text{for } 1 \leq x \leq 3 \end{cases}, \text{for } x \text{ measured in miles}$$

Previously existing roads already connect the west ends of the proposed northern and southern road, and the east ends of the proposed northern and southern road, as shown in Figure 1.



(a) Contour Plot of Terrain

(b) 3D Plot of Terrain

Figure 1: Graphical Representations of Terrain

Below is a chart containing information about the available options for construction of the route. The distribution center will be completely operational in 6 months. Once the plant becomes operational, based on the construction status of the route, the company calculates it will lose about $C a day in transportation costs until construction is complete.

Table 1
Route Construction Cost Data

Option	Cost per Mile ($Millions of Dollars)	Days Required Preparation	Average Days per mile of construction
Build Surveyed Road (North)	A	45	25
Build Surveyed Road (South)	.87(A)	45	21
Bridge - Deepest point less than 50 feet	0.01(D)+A	35	B
Bridge – Deepest point between 50 and 100 feet	0.015(D)+A	35	1.2(B)
Bridge – Deepest point between 100 and 150 feet	0.02(D)+A	40	1.35(B)

D = Average depth of water covered by bridge in feet

where

A = _____

B = _____

C = _____

You, a recent graduate of the United States Military Academy, skilled in logical reasoning and armed with an unprecedented ability to work effectively in a crisis situation, are tasked to prepare and possibly present a decision brief to the company CEO. The decision brief will detail the options you analyzed and propose a recommendation. Which bridge or road should the company invest in? You must analyze the costs for both roads and at least one bridge in your report. Your report is due 20 SEPT 07.

Option B

The company, in addition to hiring **your** services as a consultant, hired a third party to research the bridge/road problem. This third party recommended that building a bridge was the appropriate option when all construction and time delay costs were considered, and reported that the optimal bridge would cost \$5.8 million; however, they did not provide an estimate for the cost of the road.

An issue of concern that the CEO wants you to study is that the flat terrain north of the lake has a high average annual rainfall. Its annual rainfall (in inches, R) follows the probability density function below:

$$g(R) = \begin{cases} \frac{1}{\sqrt{2\pi}R} e^{-\frac{(\ln R - \sqrt{5})^2}{2}} & \text{for } R \geq 0 \\ 0 & \text{for } R < 0 \end{cases}$$

From the terrain graph, it can be seen that all the rain's runoff flows into the lake which sometimes results in the northern road flooding. Based on the number of inches of rain during the year, the road floods a certain number of days with the probability density function below, where R is the number of inches of rain per annum, which can be determined from the above distribution ($g(R)$), and x is the number of days the road is flooded:

$$h(x, R) = \frac{1}{\sqrt{6\pi}} e^{-\frac{(x - 20e^{\frac{R-20}{10}})^2}{6}}, \text{ for } x \in \Re$$

Using this information, you need to determine if the third party research is worth considering by examining what occurs during a typical year. More specifically, the CEO wants you to illustrate what the range of costs would be for plus or minus two standard deviations of the average number of days the road is flooded if the option of building the northern road is considered. Your cost analysis should include both road construction costs and inclement weather costs. See Option A for North Road Construction data. The company calculates it will lose about \$C a day in transportation costs if the northern road is flooded.

You, a recent graduate of the United States Military Academy, skilled in logical reasoning and armed with an unprecedented ability to work effectively in a crisis situation, are tasked to prepare and possibly present a decision brief to the company CEO. The decision brief will detail the options you analyzed and propose a recommendation. Should the company invest in the bridge the third party suggests, or in the northern road? Assume that the reported cost of the bridge is correct in your analysis. Additional readings that may be useful in your analysis are available on the USMA MA205 Webpage (http://www.dean.usma.edu/departments/math/courses/ma205/). Your report is due 20 SEPT 07.