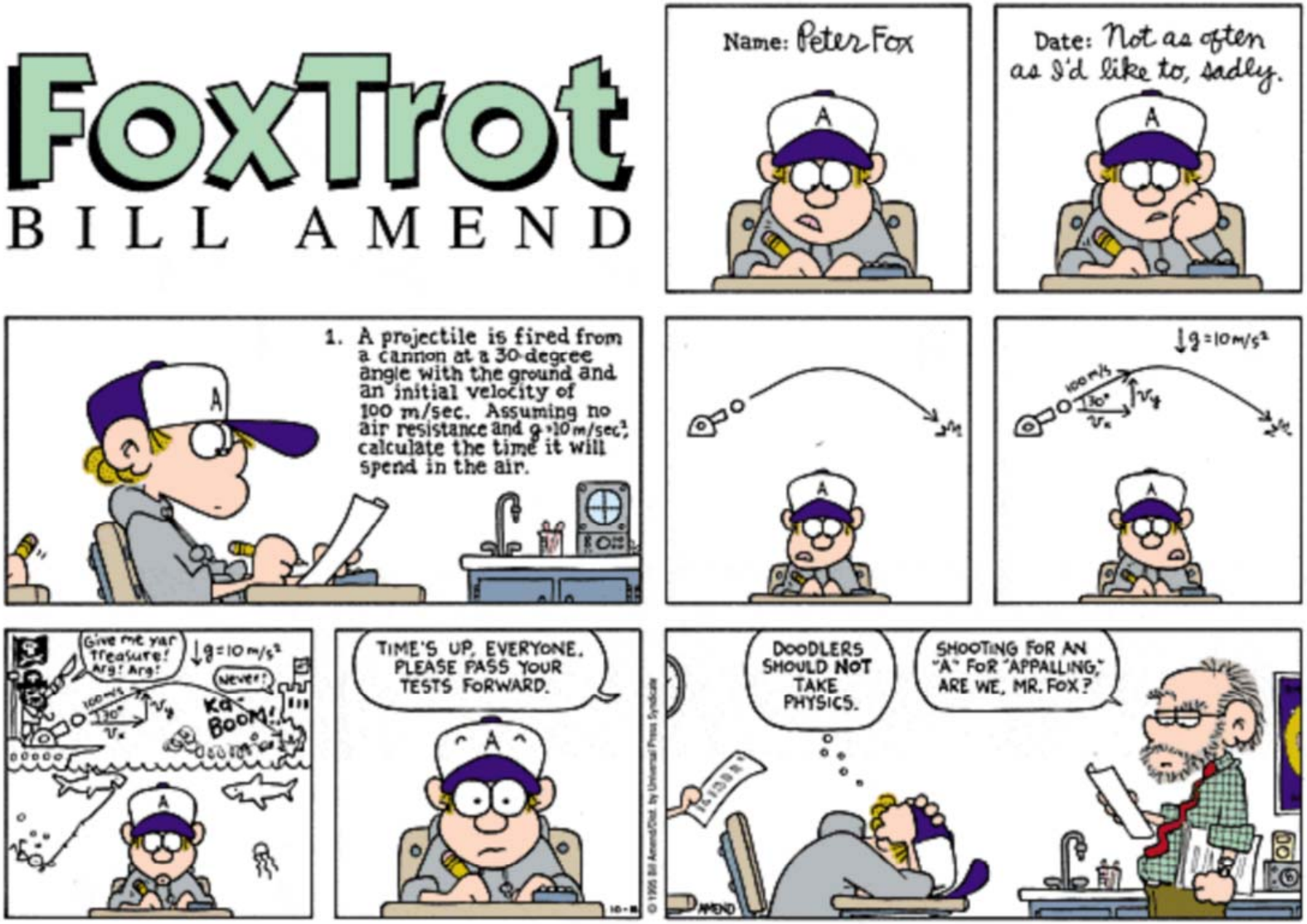
# MA205 – LSN 21, Motion in Space I



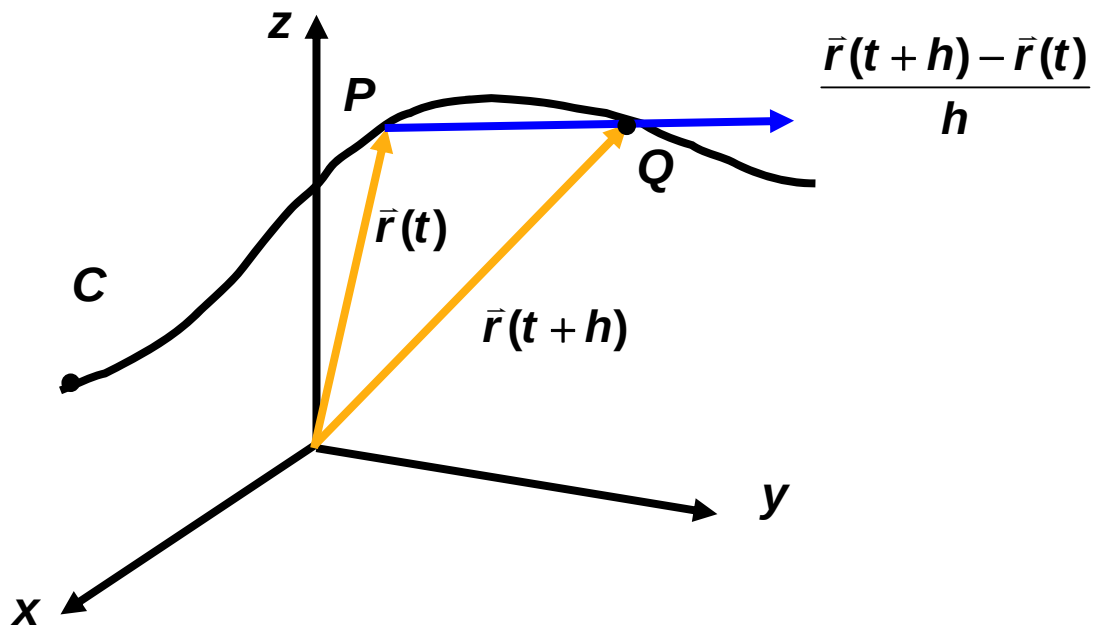
# Objectives

- **Find a position function when acceleration, initial velocity and initial position are all known.**
- **Understand the derivation of the physics equations of motion (the so-called kinematics equations.**
- **Use parametric equations to describe the motion of a projectile.**

# Motion in Space

**(From Lesson 16)**

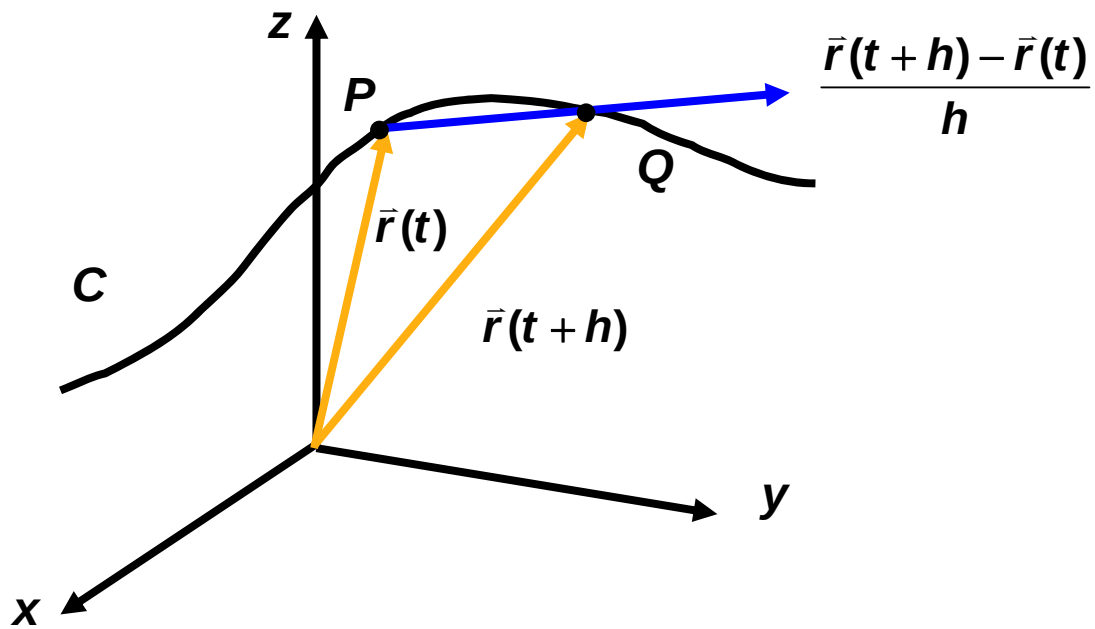
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

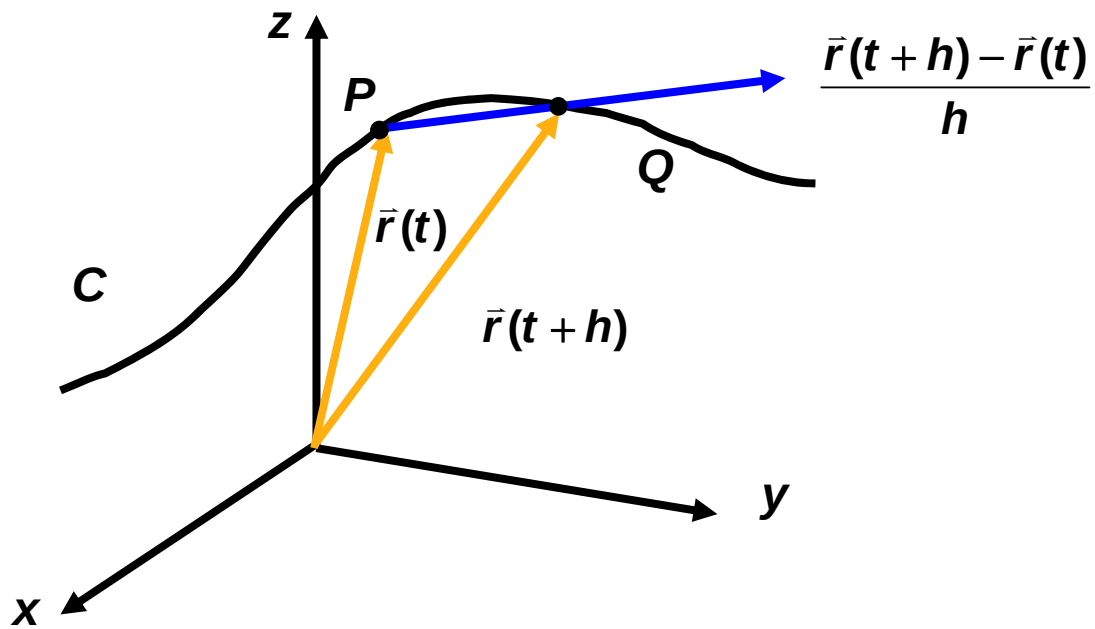
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

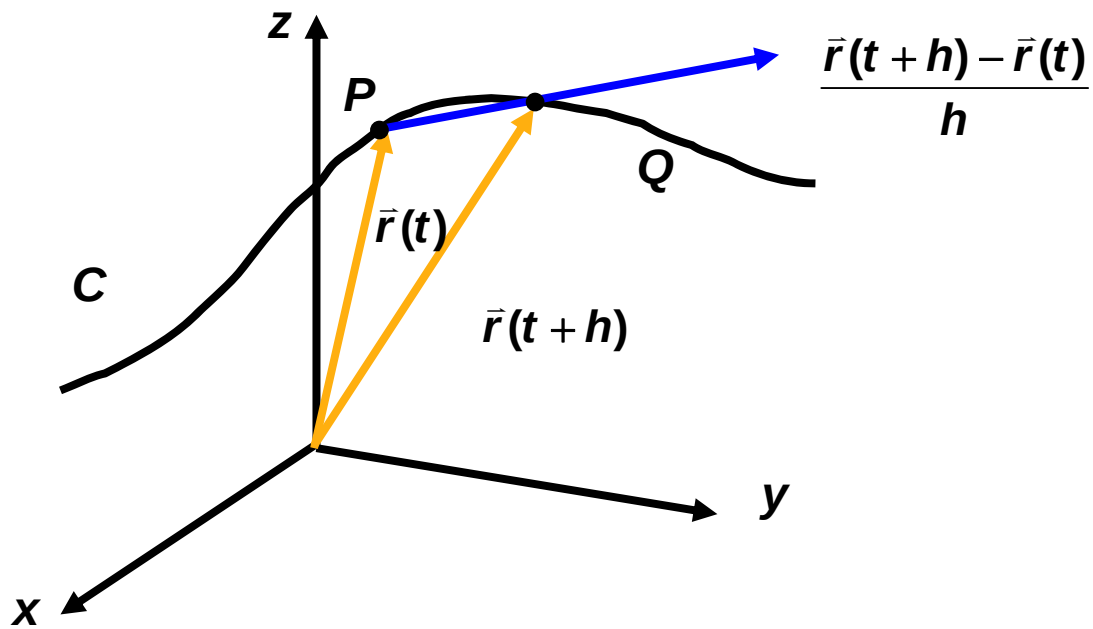
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

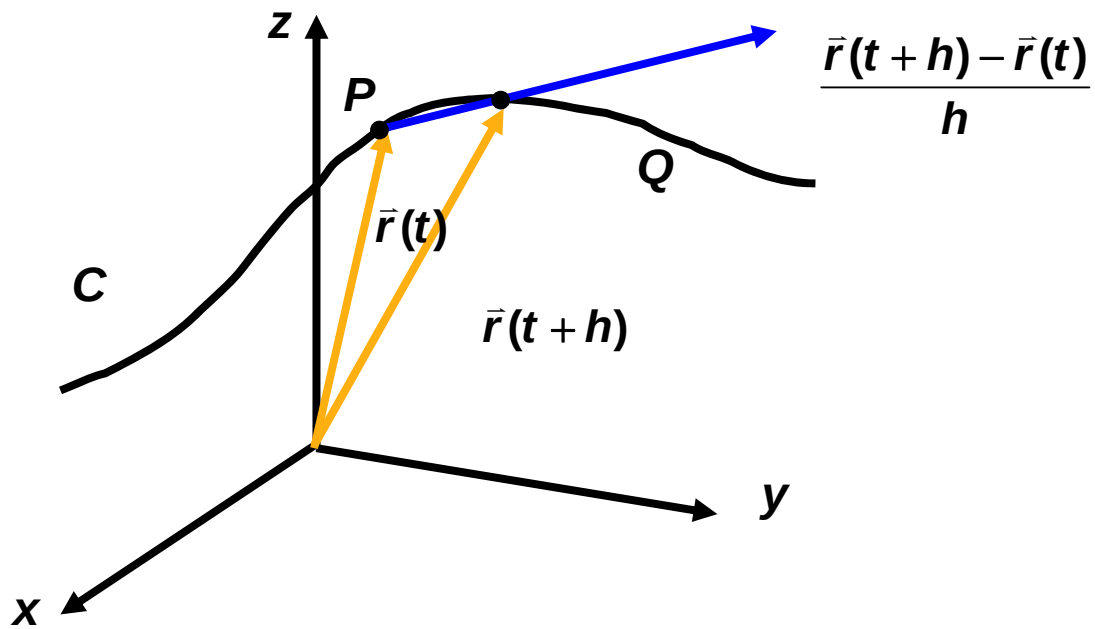
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

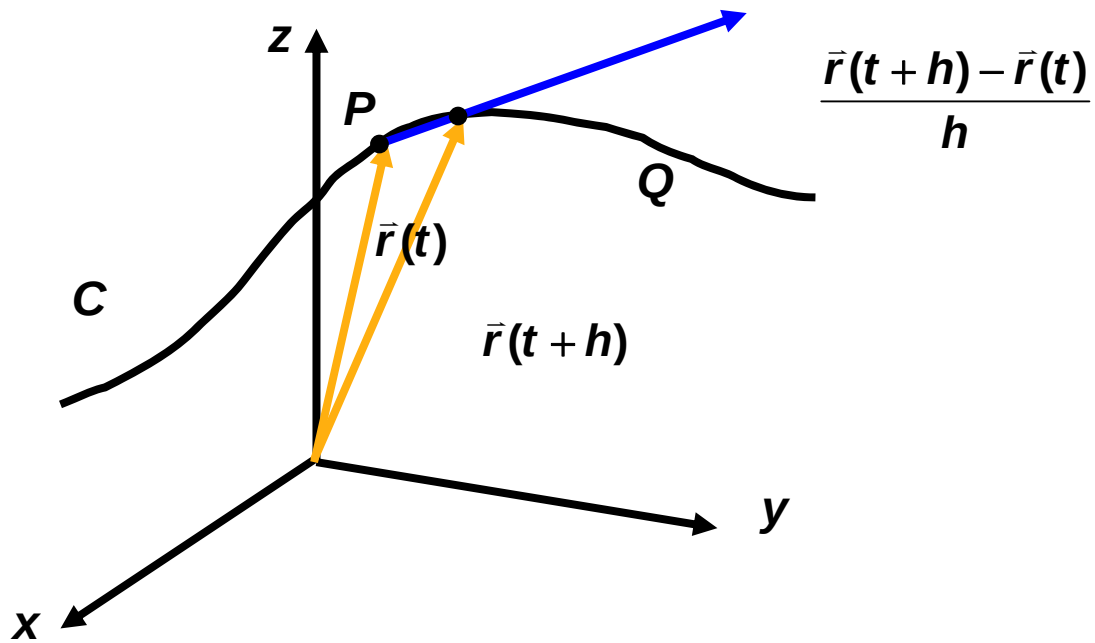
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

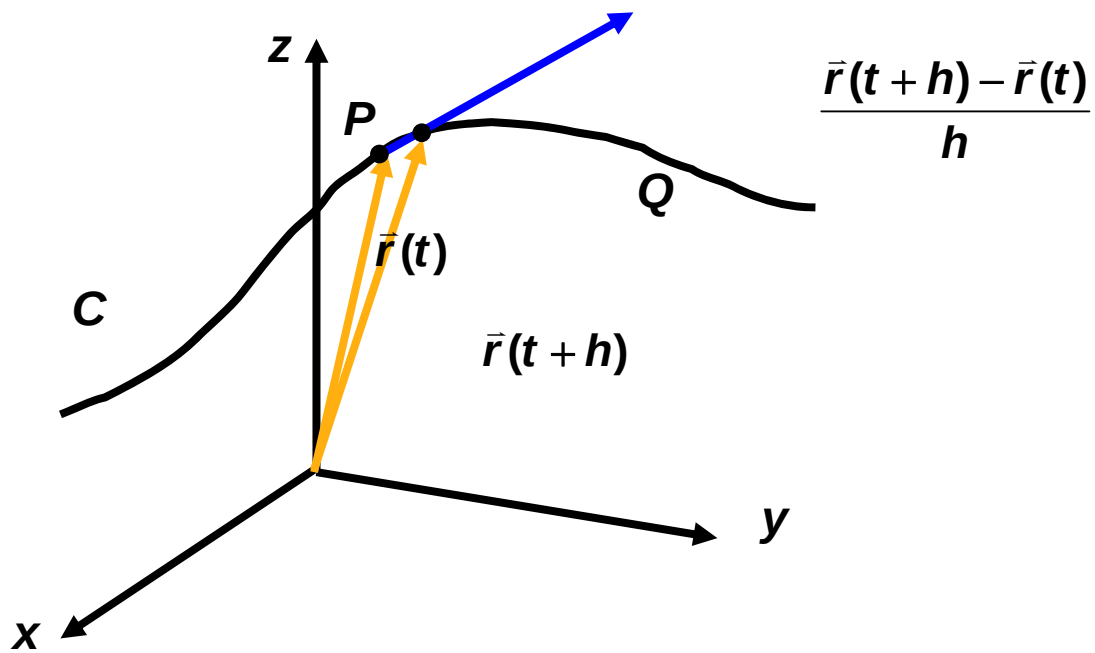
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

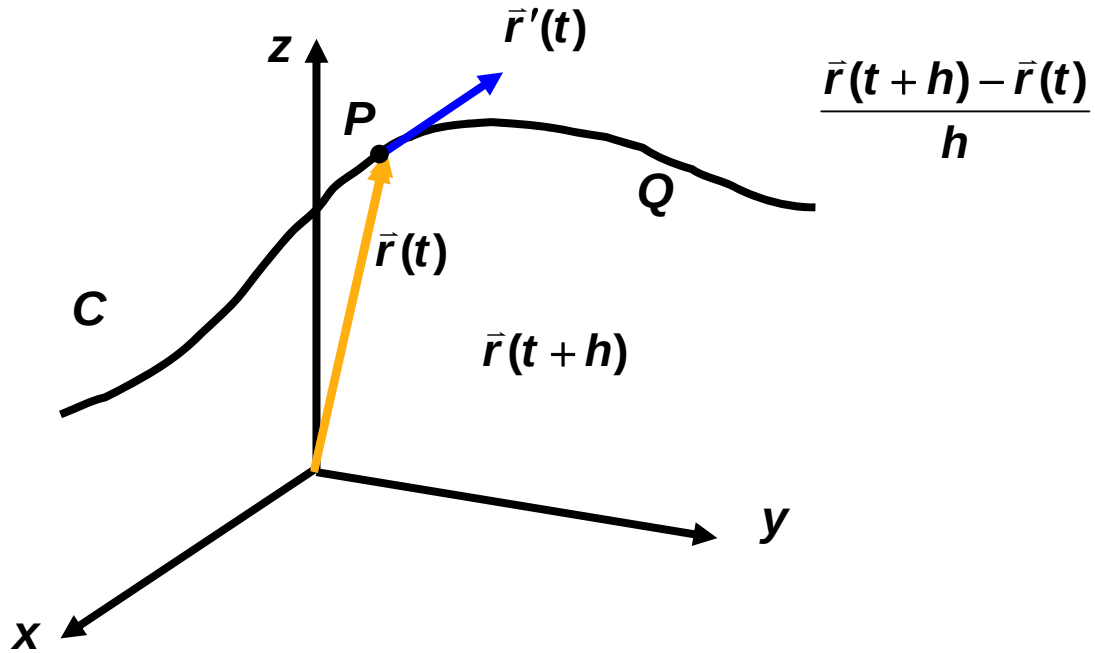
**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



# Motion in Space

**(From Lesson 16)**

**Suppose a particle moves through space, so that its position vector at time *t* is $\vec{r}(t)$. For small values of h, the vector, $\frac{\vec{r}(t+h)-\vec{r}(t)}{h}$, approximates the direction of the particle moving along the curve $\vec{r}(t)$. Its magnitude measures the size of the displacement vector per unit time. The vector gives the average velocity over a time interval of length *h* and its limit is the velocity vector $\vec{v}(t)$ at time *t***



$$\boxed{\vec{v}(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \vec{r}\,'(t)}$$

**The velocity vector is also the tangent vector and points in the direction of the tangent line.**

**The speed of the particle at time t is the magnitude of the velocity vector.**

$$\left|\vec{v}(t)\right| = \left|\vec{r}\,'(t)\right| = \frac{ds}{dt} = \text{ rate of change of distance with respect to time}$$

**Just as you saw in previous math courses, the following relationship will hold true.**

$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$$