MA205 – LSN 38

Modeling with Differential Eqns

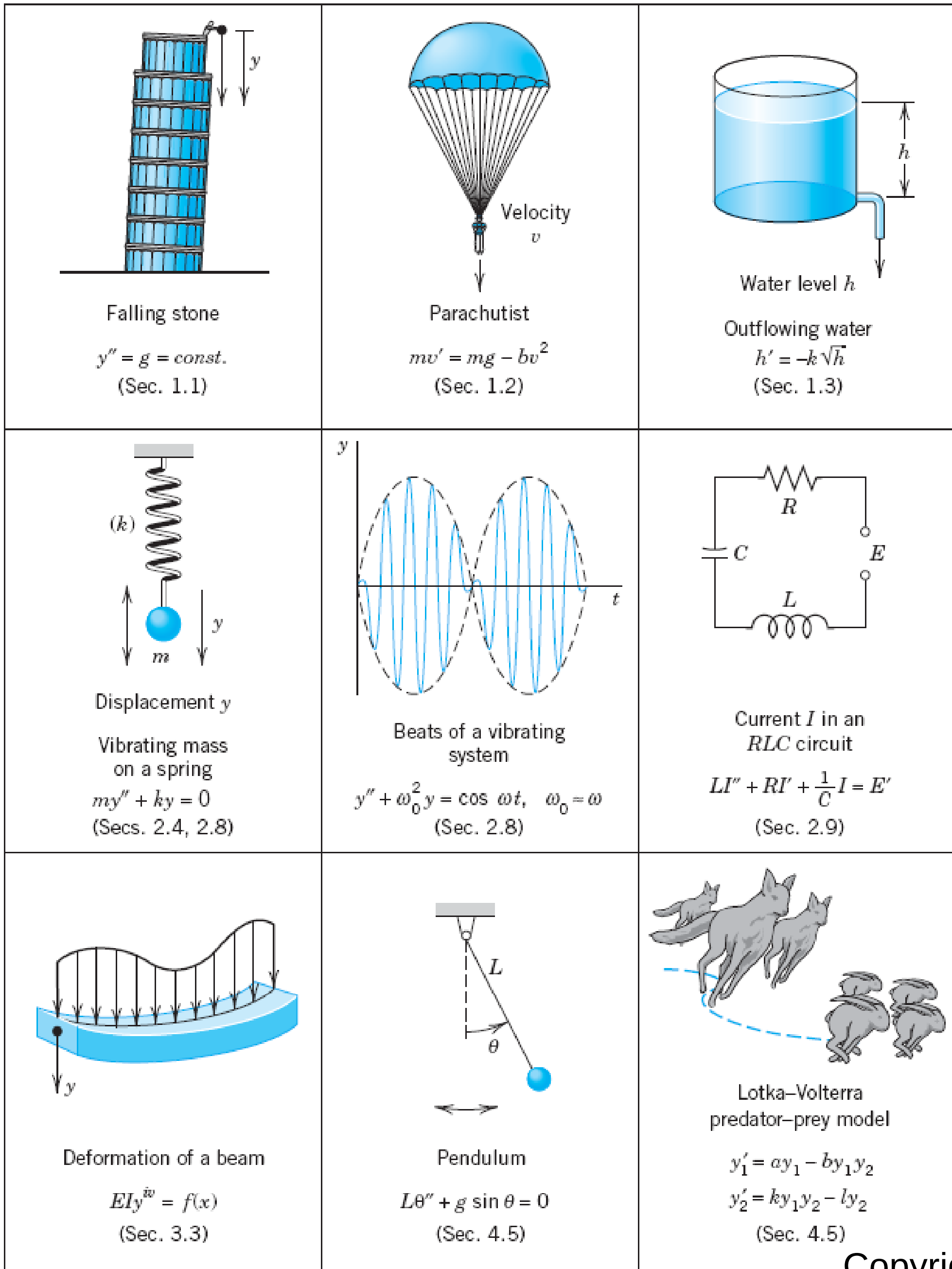
Reverend Thomas Robert Malthus
Pioneer in Classic Economics
(1766 – 1834)

"The power of population is so superior to the power of the earth to produce subsistence for man, that premature death must in some shape or other visit the human race. The vices of mankind are active and able ministers of depopulation. They are the precursors in the great army of destruction, and often finish the dreadful work themselves. But should they fail in this war of extermination, sickly seasons, epidemics, pestilence, and plague advance in terrific array, and sweep off their thousands and tens of thousands. Should success be still incomplete, gigantic inevitable famine stalks in the rear, and with one mighty blow levels the population with the food of the world."

To give a mathematical perspective to his observations, Malthus proposed the idea that population, if unchecked, increases at a geometric rate (i.e. 1, 2, 4, 8, 16, etc.), whereas the food-supply grows at an arithmetic rate (i.e. 1, 2, 3, 4, 5 etc.).

Objectives

- Understand the following vocabulary:
  - Independent vs Dependent Variable
  - Order of a Differential Equation
  - Linearity of a Differential Equation
  - Homogeneous or Non Homogeneous
  - Analytical Solution, Graphical Solution, Numerical Solutions
  - General vs Particular Solutions
  - Equilibrium
- Classify a Diff Eq with respect to Order, Linearity, and Homogeneity
- Become familiar with Diff Eq that model exponential growth and the growth given some carrying capacity
- Verify that a given function is a solution to a differential equation and/or initial value problem



Differential Equations are equations that contains an unknown function and some of its derivatives (rates of change).

Examples

Key Characteristics

Number of Independent variables
Order
Linearity
Homogeneity

Types of Solutions

Analytical
Graphical
Numerical

Key Concepts

General vs Particular Solutions
Equilibrium

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ODE vs PDE

Ordinary Differential Equations is an equation that contains one or several derivatives of an unknown function, which we usually call *y*(*x*) or *y*(*t*). The equation may contain itself, known functions of *x* or *t* and constants.

$$y' = \cos x$$

$$y'' + 9y = 5$$

$$x^2 y''' y' + 2e^x y'' = (x^2 + 2)y^2$$

Partial Differential Equations is an equation which involve partial derivatives of an unknown function of two or more variables.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Classification/Characteristics

Order – based on the highest derivative in the equation.

Linearity – If a differential equation has no products, powers or functions of the dependent variable or any of its derivatives, then it is ***linear***. Otherwise it is ***non-linear*** (linearity does not apply to the independent variable).

Homogeneity – If all terms involve the dependent variable or one of its derivatives. Otherwise it is ***non homogeneous***.

$y' = \cos x$	First Order, Linear, Non Homogeneous
$y'' + 9y = 5$	Second Order, Linear, Non Homogeneous
$x^2 y''' y' + 2e^x y'' = (x^2 + 2)y^2$	Third Order, Non Linear, Homogeneous

Solutions

Analytical - Calculate exact or “closed form” solution which satisfies the Diff Eq as well as any given initial condition.

Numerical – Solution is approximated using a table of values of the dependent variable for preselected values of the independent variables.

Graphical – Solution is approximated by a graphical representation of a continuous function. The curve whose slope at any point is the value of the derivative.

Exponential Growth, Exponential Decay

From calculus we know that $y = ce^{3t}$ (c any constant) has the derivative (chain rule!)

$$y' = \frac{dy}{dt} = 3ce^{3t} = 3y.$$

This shows that y is a solution of $y' = 3y$. Hence this ODE can model **exponential growth,** for instance, of animal populations or colonies of bacteria. It also applies to humans for small populations in a large country (e.g., the United States in early times) and is then known as ***Malthus's law.***[1] We shall say more about this topic in Sec. 1.5.

Similarly, $y' = -0.2y$ (with a minus on the right!) has the solution $y = ce^{-0.2t}$. Hence this ODE models **exponential decay,** for instance, of a radioactive substance (see Example 5). Figure 3 shows solutions for some positive c. Can you find what the solutions look like for negative c? ■

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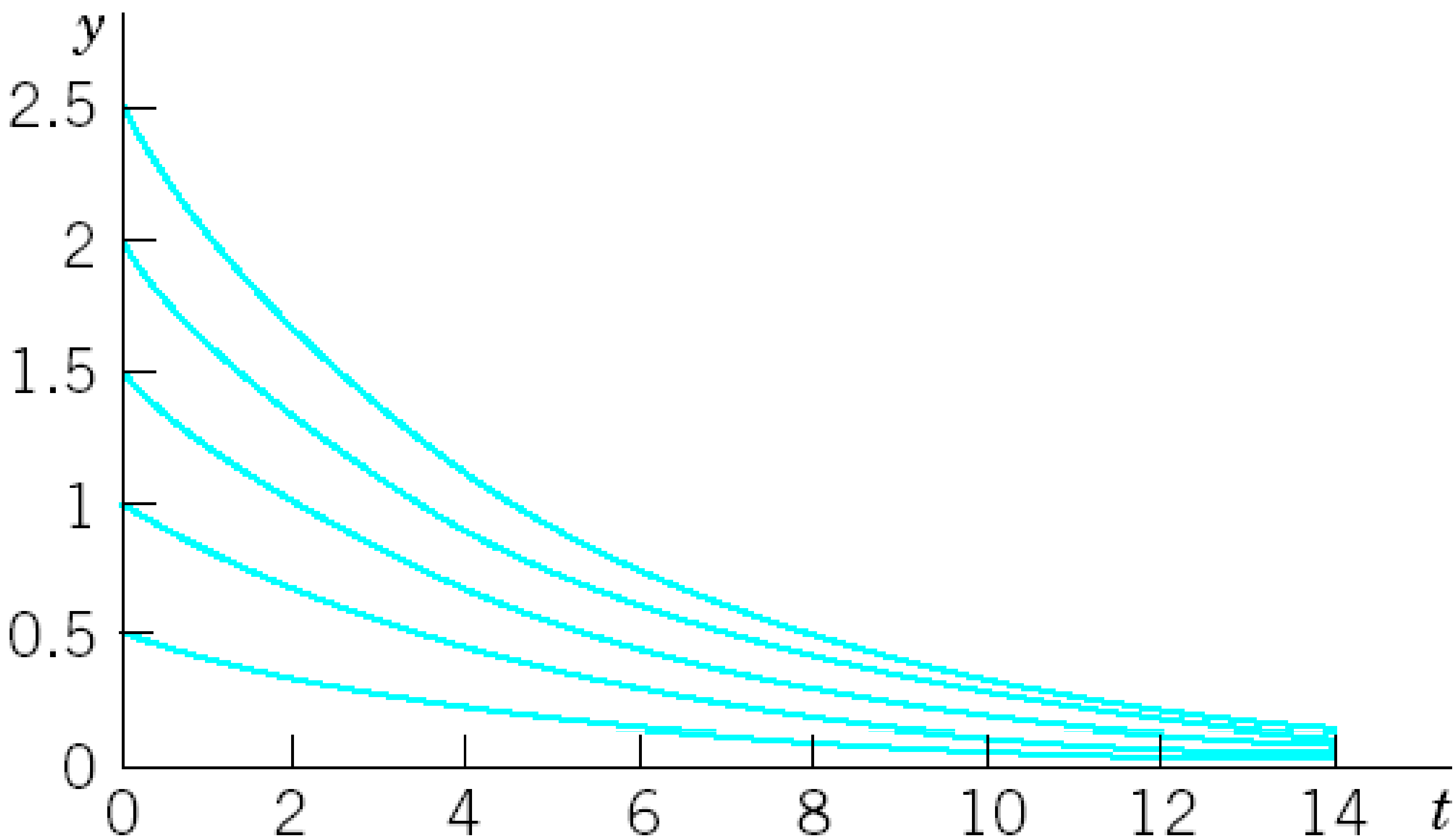

Fig. 3. Solutions of $y' = -0.2y$ in Example 3

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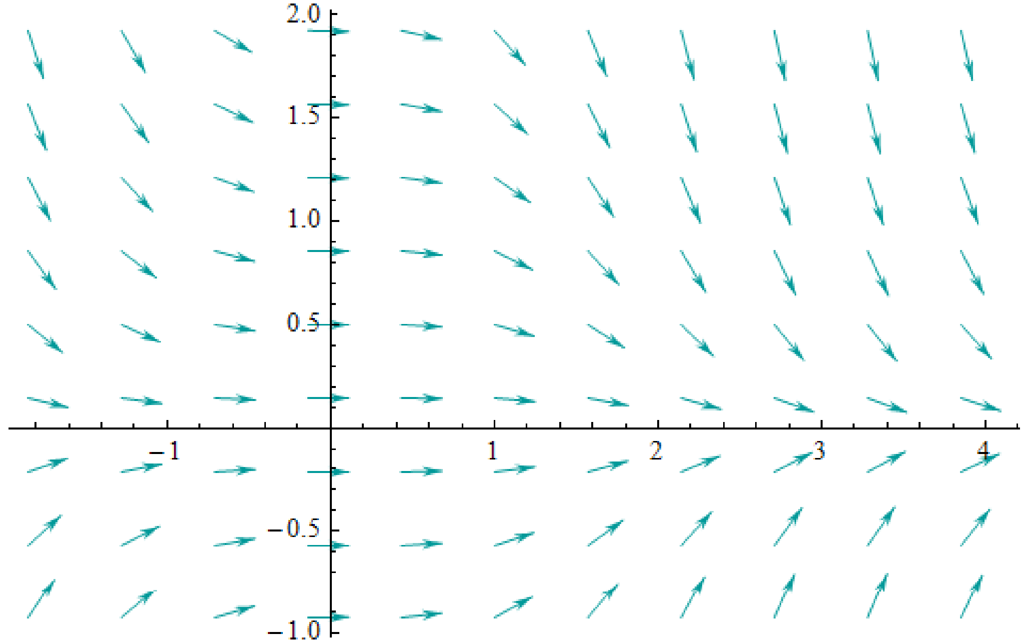
Graphical Solutions

Graphical – An approximate solution in the form of Direction or Slope Fields. For a 1st Order D.E. it is the curve representing the slope or derivative at any point.

The slope field for $$\frac{dy}{dt} = y\cos(t) - y$$

y	*t*	*dy/dt* = *y* Cos(*t*) - *y*
1.50	-2	-2.12
0.00	-2	0.00
-1.00	-2	1.42
1.50	0	0.00
0.00	0	0.00
-1.00	0	0.00
1.50	1	-0.69
0.00	1	0.00
-1.00	1	0.46
1.50	4	-2.48
0.00	4	0.00
-1.00	4	1.65



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We are going to use a package that you will need to install in order to create slope fields. Follow these directions exactly.

1. Go to the MA205 WebSite and download the DETools.zip file. Save it to your desktop.
2. Right Click the icon, then left click on "Extract All", and left click on "Next".
3. When the window pops up left click on "Browse".
4. Left click "c:". Left click "program files".
5. Left click "Wolfram Research" then Left click "Mathematica".
6. Left click "6.0" then Left click "addons"
7. Now, Left click "Applications", Left click "OK", Left click "Next", and Left click "Finish".
8. A window should pop up that shows the applications folder. You should now see a DiffEqs folder listed.

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```
<< DiffEqs`DEGraphics`
```

Note: The marks used in the above syntax are not single quotation marks next to the "ENTER" key. These marks are the accents located on the key above the "TAB" key and below the "ESC" key on the left side of the keyboard.

In order to plot the slope field for the differential equation $\frac{dy}{dt} = y \cos t - y$, enter in the following:

```
DEPlot[y Cos[t] - y, {t, -2, 4}, {y, -1, 2}, InitialPoints → None]
```