MA205 – LSN 9
Indefinite Integrals and Net Change

It is no use saying, 'We are doing our best.' You have got to succeed in doing what is necessary.

--Winston Churchill

Objectives

- Given initial conditions and a marginal cost function, find the total cost of production.
- Understand the net change theorem as it applies to area, concentration, mass, population, cost and distance.
- Understand the difference between definite and indefinite integrals.
- Compute antiderivatives of polynomials, rational and trigometric functions.
- Understand the differences between total distance vs. total displacement when integrating a velocity function.

Definition

The Fundamental Theorem of Calculus, Part 2

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Where F is an antiderivative of f, that is a function such that $F' = f$.

Stewart, p. 384

Definition

The Net Change Theorem

The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Stewart, p. 394

The Chain Rule

If *f* and *g* are both differentiable and *F* = *f* · *g* is the composite function defined by *F(x)* = *f(g(x))*, then *F* is differentiable and *F'* is given by the product

$$F'(x) = f'(g(x))g'(x)$$

In Leibniz notation, if *y* = *f(u)* and *u* = *g(x)* are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Given F(x) Find F′(x) Using the Chain Rule

$$F(x) = \sqrt[3]{1 + \tan x}$$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

$f'(u) = 1/3(u)^{-2/3}$

$g'(x) = \sec^2 x$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

$f'(u) = 1/3(u)^{-2/3}$

$g'(x) = \sec^2 x$

$$F'(x) = f'(g(x))g'(x)$$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

$f'(u) = 1/3(u)^{-2/3}$

$g'(x) = \sec^2 x$

$$F'(x) = f'(g(x))g'(x)$$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

$f'(u) = 1/3(u)^{-2/3}$

$g'(x) = \sec^2 x$

$$F'(x) = f'(g(x))g'(x)$$

$$F'(x) = f'(1 + \tan x))g'(x)$$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

$f'(u) = 1/3(u)^{-2/3}$

$g'(x) = \sec^2 x$

$$F'(x) = f'(g(x))g'(x)$$

$$F'(x) = f'((1 + \tan x))\sec^2 x$$

First Simplify

$$F(x) = (1 + \tan x)^{\frac{1}{3}}$$

Let $f(u) = (u)^{1/3}$

Let $g(x) = 1 + \tan x$

$f'(u) = 1/3(u)^{-2/3}$

$g'(x) = \sec^2 x$

$$F'(x) = f'(g(x))g'(x)$$

$$F'(x) = 1/3(1 + \tan x)^{-2/3} \sec^2 x$$

Mathematica

Untitled-2 *

In[9]:=

```
Clear[f, g, x, u]
f[x_] := ∛(1 + Tan[x])
f'[x]
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Out[11]=

$$\frac{\mathrm{Sec}[x]^2}{3\,(1+\mathrm{Tan}[x])^{2/3}}$$