MA386 - Introduction to Numerical Analysis
Homework Assignment 8, 100 points
Due In Class Dec. 9th

1. Consider the linear system in Section 7.3, 1a. It might be useful to know how many iterations we need for our methods to converge to a solution. You may use whatever norm you wish in what follows, but you must be consistent throughout the problem.

   (a) Use Corollary 7.20 (i) and an initial guess of $x^{(0)} = [0, 0, 0]^T$ to estimate the number of iterations it will take for the Jacobi method to produce an iterate within 10^{-8} of the actual solution.

   (b) Use Corollary 7.20 (i) and an initial guess of $x^{(0)} = [0, 0, 0]^T$ to estimate the number of iterations it will take for the Gauss-Seidel method to produce an iterate within 10^{-8} of the actual solution.

   (c) Use `MATLAB` to verify your answers.

2. Section 7.3, 17. Does the outcome surprise you? Why or why not?

3. Section 7.3, 18. Does the outcome surprise you? Why or why not?

4. Section 7.3, 19. Do the entire problem, then explain what happens in part (d).

5. Section 7.4: Consider the linear systems in 3d and 4b. Compute the following values:

   (a) The relative error: $\dfrac{\|\mathbf{x} - \widetilde{\mathbf{x}}\|_\infty}{\|\mathbf{x}\|_\infty}$.

   (b) The relative residual error: $\dfrac{\|\mathbf{b} - A\widetilde{\mathbf{x}}\|_\infty}{\|\mathbf{b}\|_\infty}$.

   (c) The condition number: $K_\infty(A)$.

   Now discuss the outcome: How does the relative error compare with the residual error in each case? Is this to be expected? Include Theorem 7.27 in your discussion.

BONUS On page 433 the authors comment that for every $\epsilon > 0$, there is a natural norm $\|\cdot\|_\epsilon$ with the property that $\rho(A) < \|A\|_\epsilon < \rho(A) + \epsilon$. Therefore the spectral radius is "very close" to being a norm, but it turns out that it is *not* a norm. Do number 16 in Section 7.2 (Hint: use triangular matrices). Explain why this shows that $\rho(\cdot)$ is not a norm.