Google Earth AND GEOGRAPHIC DISTANCES

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Figure 1: The GPS track of a walk viewed in Google Earth

Topics: Arc-Length, Distance, Dot-Product, Error Analysis, Euclidean Geometry, Geography, Measurement, Modeling, Pythagorean Theorem, Rate of Change, Spherical Geometry, Trigonometry, Visualization (2D & 3D), Vectors.

Figure 1 on the cover page is a Google Earth image of the track of a walk taken by one of the authors. The data that was used to outline the path was collected by a GPS unit. The Pacific Crest Trail, a hiking trail that runs from the southern end of California to the northern end of Washington, is illustrated below in Figure 2[1].

How long was the walk? How long is the trail? Such questions are rich with opportunities to develop mathematical and modeling skills.

Figure 2: Pacific Crest Trail

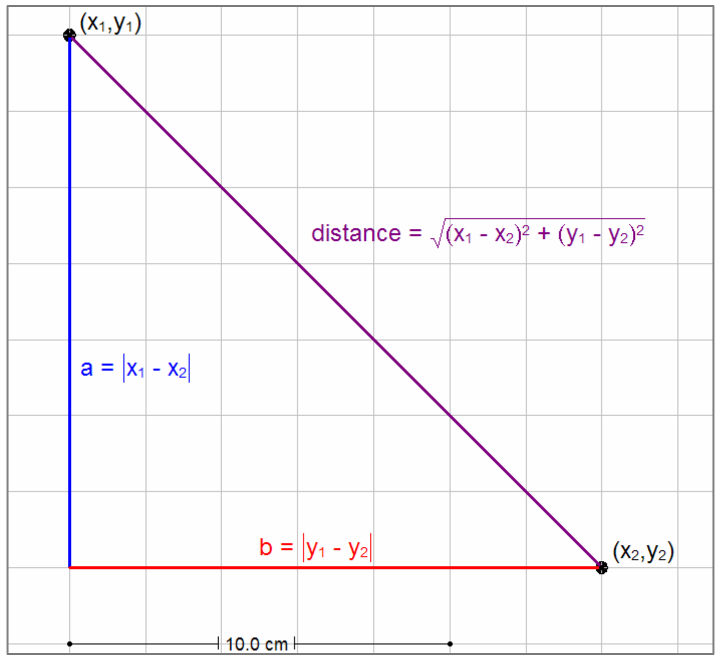


Figure 3: Distance in the Cartesian Plane

As with any modeling problem, let's start with identifying what we know, defining our variables and making any necessary simplifying assumptions.

Finding the distance between two points in the Cartesian plane or Euclidean three-space is a straightforward application of the distance formula based on the Pythagorean Theorem. Using the scale at the bottom of Figure 3, we would say that the distance between the point (x_1, y_1) and the point (x_2, y_2) is roughly 19.8 centimeters.

However, the geographical location data we download from a GPS unit is given in terms of latitude and longitude. Units of latitude and longitude are not the usual units in which we discuss length. A key objective of this document is to examine ways in which we can apply mathematics to find the distance between two locations using their latitudinal and longitudinal coordinates[2].

[1] http://www.fs.fed.us/pct/
[2] For more information, see the article "Latitude and Longitude" provided by NationalAtlas.gov: http://nationalatlas.gov/articles/mapping/a_latlong.html.

Many programs like Google Earth provide the option of expressing latitude and longitude in degrees with decimals rather than degrees, minutes, and seconds. For example, a latitude of 72º 30′ 45′′ (72 degrees, 30 minutes, 45 seconds) can be expressed in decimal degrees as latitude 72.5125. We will work with decimal degrees since they are often more efficient for calculations.

Longitudinal coordinates indicate the east-west coordinate of a location. They vary from 0° (the Prime Meridian) to 180° (the International Date Line). Latitudinal coordinates indicate the north-south coordinate of a location. They vary from $90^{\circ}S$ (the south pole) to 0° (the equator) to $90^{\circ}N$ (the north pole). In order to be consistent with our familiar *xy*-coordinate system we will denote eastern longitudes with positive values and western longitudes with negative values. Similarly, we will denote northern latitudes with positive values and southern latitudes with negative values. We will also denote longitude and latitude coordinates with ordered pairs (θ, φ) where θ represents longitude and φ represents latitude. For example, Sydney, Australia is located at latitude $33.9°S$ and longitude $151.2°E$, so its corresponding coordinates using our notation would be $(151.2, -33.9)$.

We will also need to make some assumptions about our model's "world". The Earth is shaped like a slightly squashed sphere. However, the squashing is not nearly as noticeable as that in Figure 4. The polar radius of the Earth is roughly 6357 kilometers and its equatorial radius is approximately 6378 kilometers. The difference between the two is about 21 kilometers, or 0.3%. The two images in Figure 5 may help to illustrate this difference. Whereas the figure on the left is a representation of the sphere, the figure on the right has a height to width ratio of about 99:100. Our model of the Earth will be a sphere of radius 6368 kilometers (3957 miles). This model will be sufficient for our purposes.

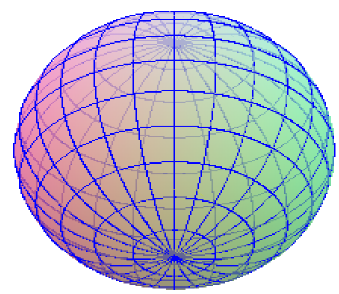
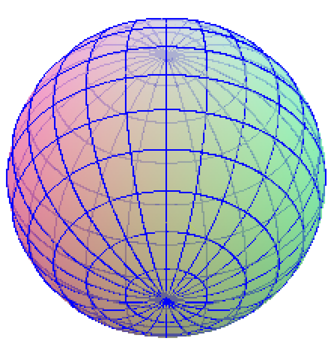


Figure 4: Squashed Sphere



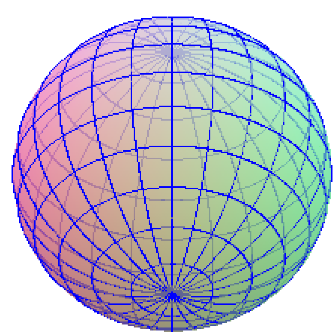


Figure 5: Sphere (left) and slightly squashed sphere (right)

Question: Why did we choose 6,368 kilometers?[3] *What other reasonable choices might we have made?*

Our ultimate goal is to work through an analysis of the data collected by a GPS unit that was used to create Figure 1. A similar analysis can be used with any set of data. Luckily our lives are abundant with opportunities for us and our students to collect interesting data. Data can come from a walk, a run, a cross-country ski trip, a bicycle trip, a car trip, a parachute jump, or even a helicopter flight!

However, before we tackle the larger problem let's examine the simpler problem of determining the distance between two geographical locations.

PROBLEM 1: Finding the distance between two locations having the same longitude and different latitudes.

Let us suppose that two points have the same longitude, θ, but different latitudes φ_1 and φ_2. Note that θ, φ_1, φ_2 will be given in degrees. The difference between the two latitudes is $|\varphi_2 - \varphi_1|$. This is the measure of the angle between the points where the vertex of the angle is located at the center of the Earth. As illustrated in Figure 6, the two locations lie on the arc[4] of a circle of radius *R*, where *R* represents the Earth's radius.

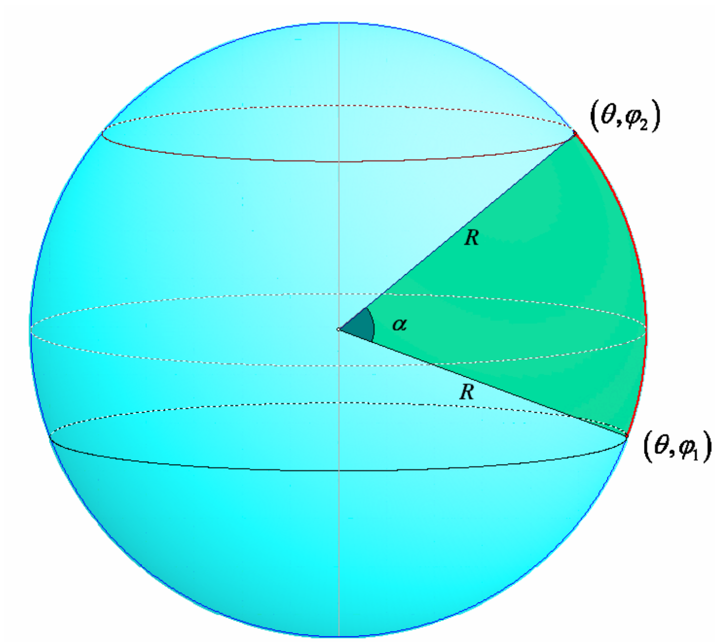


Figure 6: Two points with the same longitude

[3] The average of the polar radius and the equatorial radius is 6367.5 kilometers. Since this is still an approximation, we might as well round-off to 6368 km.

[4] This arc yields the shortest distance between the two points since it is a minor arc on a great circle.

The length of the arc extending from (θ,φ_1) to (θ,φ_2) is given by the product $R\cdot\alpha$ where α is the radian angle measure of $|\varphi_2-\varphi_1|$, i.e.,

$\alpha=|\varphi_2-\varphi_1|\cdot\dfrac{\pi}{180}$ radians.

The distance between (θ,φ_1) and (θ,φ_2) may now be given by the formula

$$R\cdot|\varphi_2-\varphi_1|\cdot\frac{\pi}{180} \qquad \textbf{(1.1)}$$

where R is the radius of the Earth.

EXAMPLE 1: Find the distance in kilometers between Bogotá, Columbia and New York City.

Bogotá is located at $(74°W,4.6°N)$ or $(-74,4.6)$. New York City is located at $(74°W,40.7°N)$ or $(-74,40.7)$.

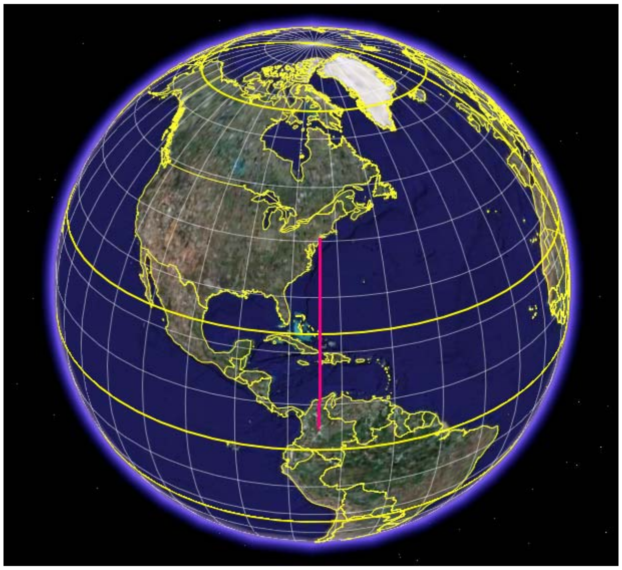


Figure 7: Path from Bogotá to New York City

Since both cities have the same longitude, we can apply Formula (1.1). With $R=6368$ kilometers, we estimate that the distance between Bogotá and New York City is approximately

$6368\cdot|4.6-40.7|\cdot\dfrac{\pi}{180}\cong 4012$ kilometers.

Let's see how our estimate measures up to those given by a couple of online distance calculators. The site http://www.mapcrow.info/ gives a distance of 4013 kilometers and the site http://www.indo.com/distance/ gives a distance of 3992 kilometers. Both are reasonably close to our estimate.

PROBLEM 2: Finding the distance between two locations having the same latitude and different longitudes.

Let us suppose that two points have the same latitude, φ, but different longitudes θ_1 and θ_2.

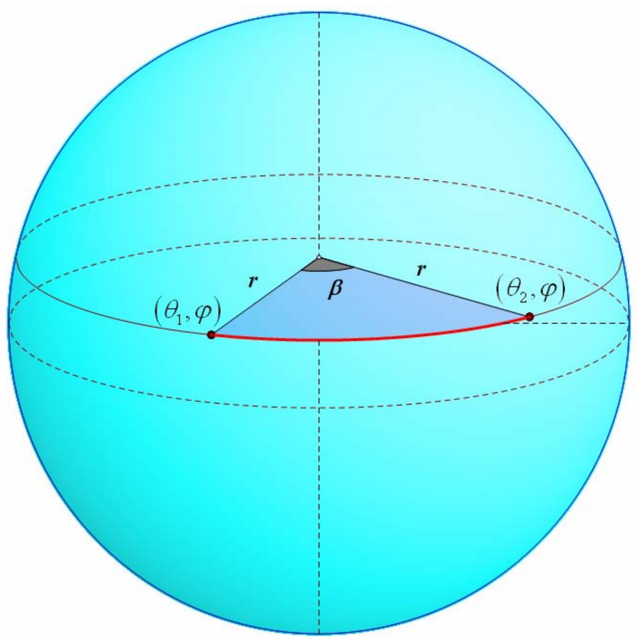


Figure 8: Two points at the same latitude

The difference between the two longitudes is $|\theta_2 - \theta_1|$. This is the measure of the angle with vertex on the polar axis of the Earth at latitude φ that extends to these two points. As illustrated in Figure 8, the two locations lie on the arc of a circle of radius *r*, where *r* represents the radius of the circle at latitude φ. Using some fundamental right angle trigonometry (see Figure 9) we observe that $r = R\cos\left(\dfrac{\pi \cdot \varphi}{180}\right)$, where *R* represents the Earth's radius and $\dfrac{\pi \cdot \varphi}{180}$ is the radian measure of φ.

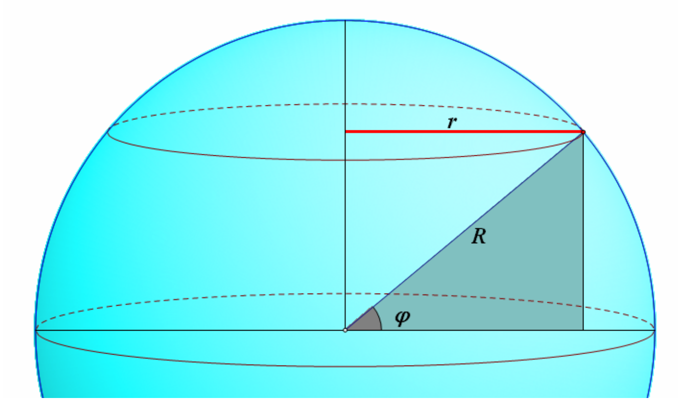


Figure 9: Illustration of $r = R\cos(\varphi \cdot \pi / 180)$

The length of the arc is given by the product $r \cdot \beta$ where β is the radian angle measure of $|\theta_2 - \theta_1|$, i.e., $\beta = |\theta_2 - \theta_1| \cdot \frac{\pi}{180}$ radians.

If we travel from location (θ_1, φ) to location (θ_2, φ) while remaining at latitude φ, this "latitudinal" distance is given with the formula

$$R \cos\left(\frac{\pi \cdot \varphi}{180}\right) \cdot |\theta_2 - \theta_1| \cdot \frac{\pi}{180} , \tag{1.2}$$

where R is the radius of the Earth.

EXAMPLE 2: Use Formula (1.2) to find the "latitudinal" distance between Rome, Italy and Plymouth, Massachusetts.

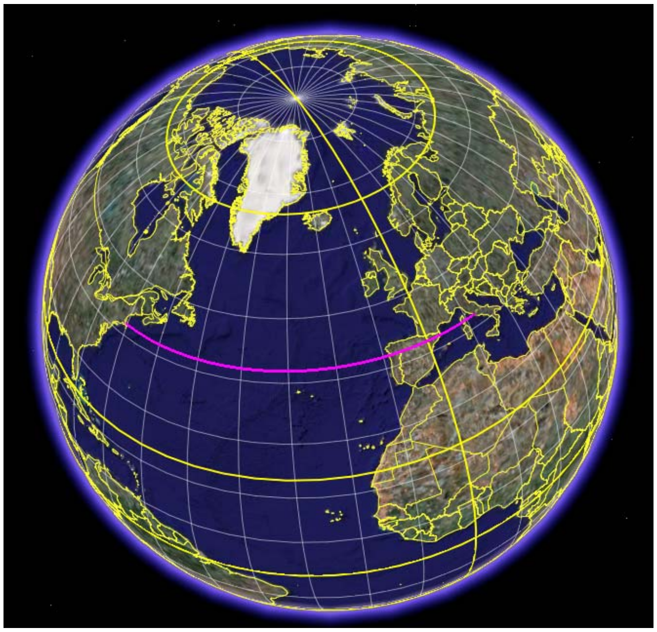


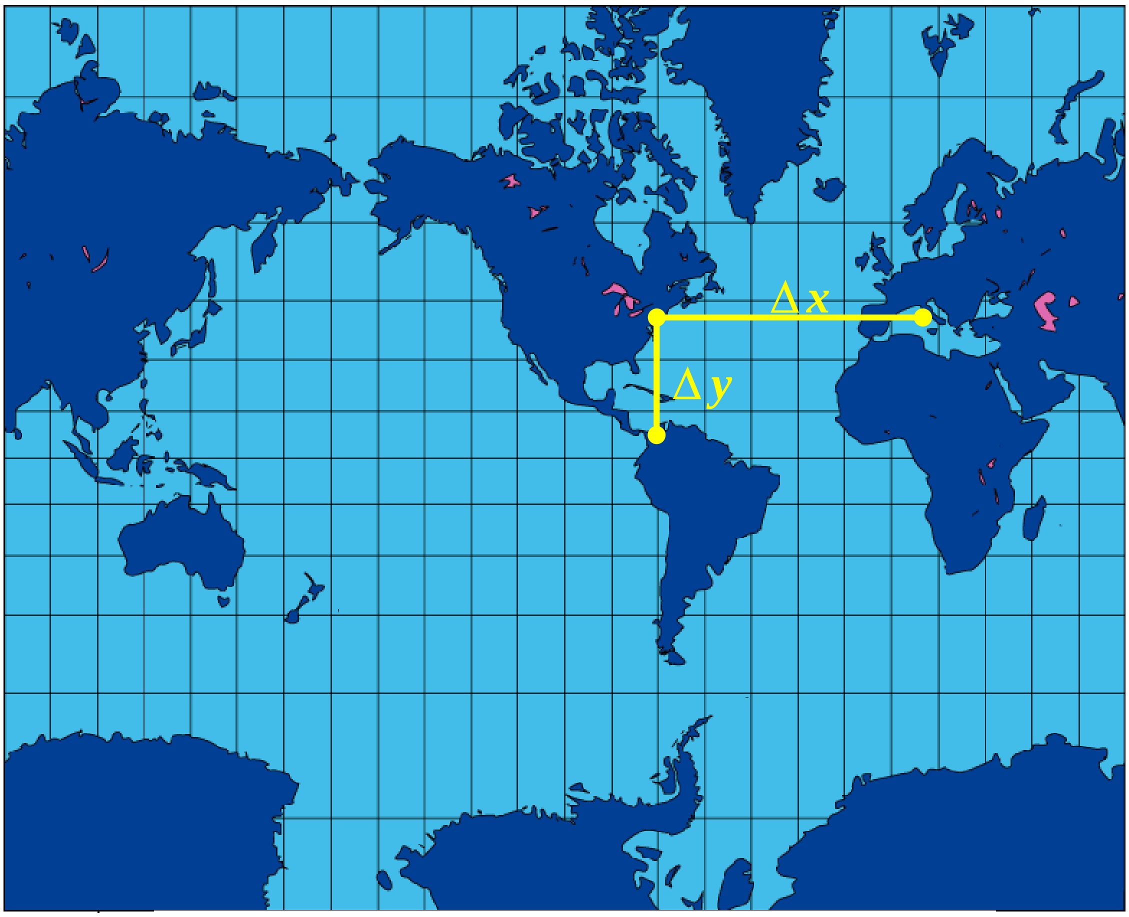
Figure 10: Path from Rome to Plymouth

Rome is located at $(12.5, 42)$ and Plymouth is located at $(-70.7, 42)$. Since both cities have the same latitude, we can apply Equation (1.2). Using $R = 6368$ kilometers for the Earth's radius, we find that the "latitudinal" distance between Rome and Plymouth is roughly $6368 \cdot \cos\left(\frac{42 \cdot \pi}{180}\right) \cdot |12.5 + 70.7| \cdot \frac{\pi}{180} \cong 6872$ kilometers.

Note: Example 2 provides a nice opportunity for a connection to history. Some history sources suggest that the fact that Rome and Plymouth have the same latitude affected the pilgrims' expectations of climate conditions in America. How might this fact affect climate expectations? Is there any real evidence to suggest that it did or did not?

PROBLEM 3: Finding the distance between two locations having different latitudes and different longitudes.

In examples two and three we found distances as we might with a Mercator map, or a flat projection of the world where latitude lines are perpendicular to longitude lines. Figure 11 illustrates such a map with the approximate paths used in Examples 1 and 2.



http://serc.carleton.edu/usingdata/nasaimages/index4.html

Figure 11: Mercator map of the world

For two points (θ_1, φ) and (θ_2, φ) at the same latitude, let Δx represent the east-west "latitudinal" distance between them. Then by Formula (1.2) we have

$$\Delta x = R\cos\left(\frac{\pi \cdot \varphi}{180}\right) \cdot \left|\theta_2 - \theta_1\right| \cdot \frac{\pi}{180} \quad . \tag{1.3}$$

For two points (θ, φ_1) and (θ, φ_2) at the same longitude, let Δy represent the north-south distance between them. Then by Formula (1.1) we have

$$\Delta y = R \cdot \left|\varphi_2 - \varphi_1\right| \cdot \frac{\pi}{180} \quad . \tag{1.4}$$

Those of us who are not pilots or astronauts probably view our immediate world as flat. For example, if you walk due west for 1.53 hours at 4 miles per hour you will have traveled 6.12 miles from your starting point. Moreover, you probably feel quite confident that the shortest distance back will be 6.12 miles due east. The International Space Station, however, cannot ignore the fact that the Earth is not flat. It is flying so fast that if it flew due west for 1.53 hours it would be pretty much back where it started! The shortest distance to its starting point is certainly not the way it came!

As long as our data is collected in a relatively small area, we can work with a flat model for our part of the Earth and apply the Pythagorean Theorem for finding distances between points.

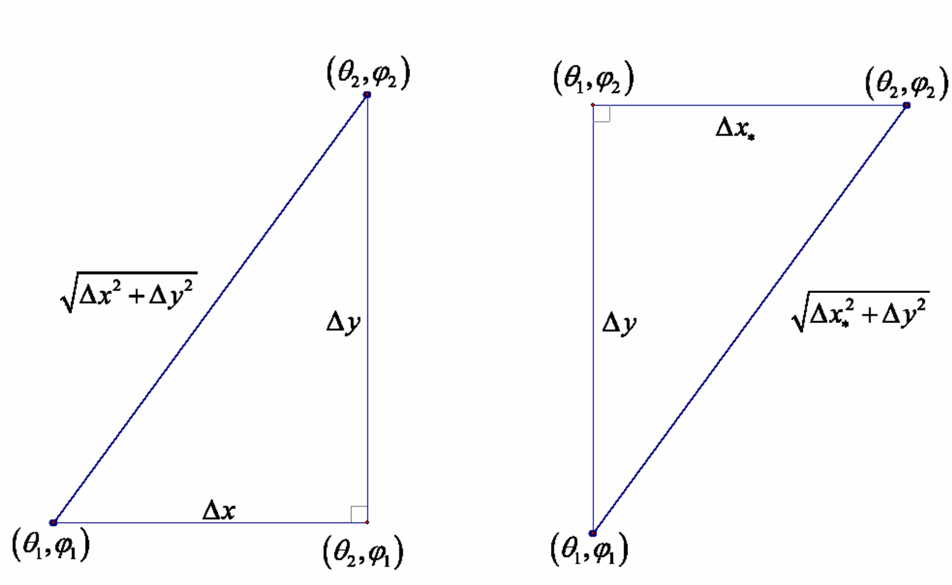


Figure 12: The distance between two points

From the triangle on the left side of Figure 12 we see that the distance between two locations (θ_1,φ_1) and (θ_2,φ_2) is approximately[5]

$$\sqrt{\Delta x^2+\Delta y^2}=R\cdot\frac{\pi}{180}\cdot\sqrt{(\theta_2-\theta_1)^2\cos^2\left(\frac{\pi\cdot\varphi_1}{180}\right)+(\varphi_2-\varphi_1)^2}\quad . \qquad \textbf{(1.5)}$$

If we use the triangle on the right side of Figure 12 the approximate distance between the points (θ_1,φ_1) and (θ_2,φ_2) is given by the formula

$$\sqrt{\Delta x_*^2+\Delta y^2}=R\cdot\frac{\pi}{180}\cdot\sqrt{(\theta_2-\theta_1)^2\cos^2\left(\frac{\pi\cdot\varphi_2}{180}\right)+(\varphi_2-\varphi_1)^2}\quad . \qquad \textbf{(1.6)}$$

Question: How do the two formulas differ?

[5] We must use the qualifying adverb “approximately” because this formula relies on a less than perfect model.

EXAMPLE 3: Estimate the distance between Stonehenge and the London Eye.

Figure 13: Path between Stonehenge and the London Eye

Stonehenge is located at $\left(-1.829,\ 51.179\right)$ and the London Eye is located at $\left(-0.119,\ 51.504\right)$.

According to Formula (1.5), the distance between the two landmarks is

$$\sqrt{\Delta x^2 + \Delta y^2} = 6368 \cdot \frac{\pi}{180} \cdot \sqrt{\left(1.710\right)^2 \cos^2\left(\frac{\pi \cdot 51.179}{180}\right) + \left(0.325\right)^2} \text{ kilometers.}$$

This is approximately 124.50 kilometers.

According to Formula (1.6), the distance between the two landmarks is approximately

$$\sqrt{\Delta x_*^2 + \Delta y^2} = 6368 \cdot \frac{\pi}{180} \cdot \sqrt{\left(1.710\right)^2 \cos^2\left(\frac{\pi \cdot 51.504}{180}\right) + \left(0.325\right)^2} \text{ kilometers.}$$

This is approximately 123.69 kilometers.

Notice that the two answers are different, but by less than a kilometer. This happens because we are using a flat model for a spherical Earth. Since Stonehenge is slightly south of London, and both are in the northern hemisphere, the east-west distance Δx between Stonehenge's coordinates and the London Eye's longitude line is slightly more than the distance between the London Eye's coordinates and Stonehenge's longitude line.

One way to improve our estimate might be to average the two distances:

$$\frac{124.50+123.69}{2}=124.095 \text{ kilometers.}$$

Or perhaps we could average the longitudinal coordinates and obtain a single formula to estimate the distance between (θ_1,φ_1) and (θ_2,φ_2):

$$R\cdot\frac{\pi}{180}\cdot\sqrt{(\theta_2-\theta_1)^2\cos^2\left(\frac{\pi\cdot(\varphi_2+\varphi_1)}{360}\right)+(\varphi_2-\varphi_1)^2}\quad. \tag{1.7}$$

This would yield a distance of about 124.10 kilometers.

Our analysis begs the question, “When is a flat model good enough?” The answer lies at the heart of problem solving – it depends entirely on the problem we are trying to solve!

Recall that in Example 2 we calculated the latitudinal distance between Rome and Plymouth to be roughly 6872 kilometers. However, if we go to the online distance calculators used for Example 1, the site http://www.mapcrow.info/ gives a distance of 6580 kilometers, while the site http://www.indo.com/distance/ gives a distance of 6608 kilometers.

The difference between these and our estimate is nearly 300 kilometers! What happened? Did we make a mistake? Did they? The truth is that neither of us erred, but we were calculating different distances. Although we estimated the “latitudinal” distance correctly, this distance is not the *shortest* surface distance between the two points! This concept is one of the cornerstones of spherical geometry. On a sphere, the shortest path between two points lies on the arc of the great circle[6] passing through those two points. The “latitudinal” arc between two points as described in Problem 2 and illustrated in Figure 8 does not lie on a great circle unless we are at 0$^{\mathrm{o}}$ latitude, i.e., on the equator. Therefore we did not find the shortest distance between the points. See Figure 14 for an illustration of the latitudinal path (red) and the path on a great circle (blue).

Clearly a flat model is not a good choice for the Rome to Plymouth scenario. So, let's investigate how we can better estimate the length of the shortest path between two points when a flat model is clearly out of the question.

[6] A great circle is formed by the intersection of the surface of a sphere with a plane passing through the sphere’s center. Great circles form the “lines” in spherical geometry.

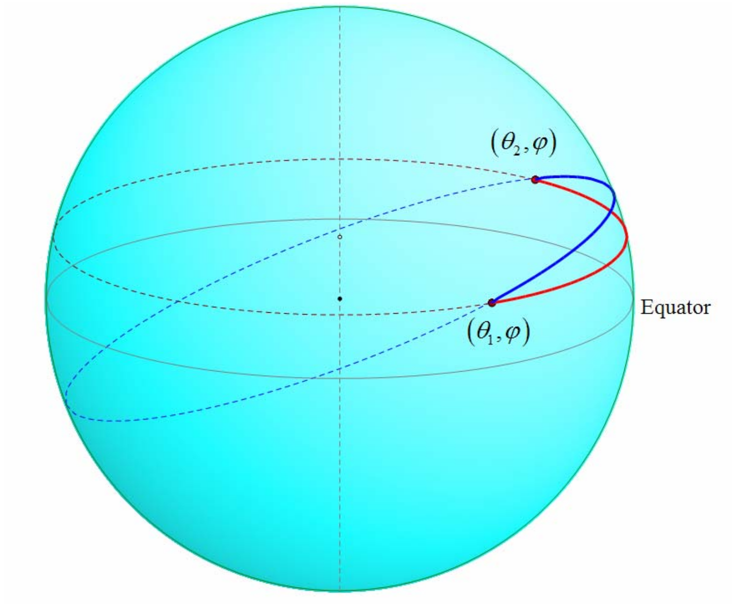


Figure 14: Arc on a great circle vs. arc on a latitude line

PROBLEM 4: Estimate the shortest distance between any two geographic locations P and Q using a spherical model.

Suppose P and Q are two geographical locations on the Earth's surface. The shortest distance between P and Q will lie on an arc of the great circle that passes through them. Let O be Earth's center, R its radius, and $\lambda = m\angle POQ$ in radians. The length of the minor arc passing through P and Q is equal to $R \cdot \lambda$. (See Figure 15.) With a little vector math and spherical geometry, we have all the tools we need to determine λ.

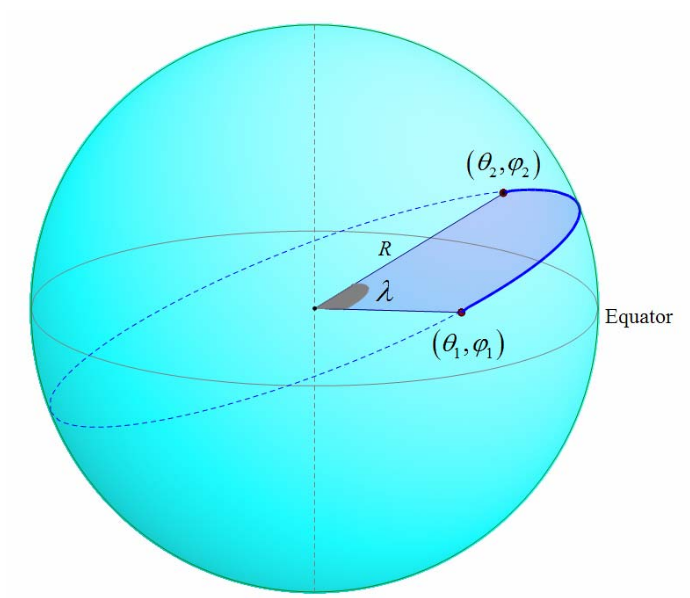


Figure 15: Shortest path between two locations

Let the center of the Earth correspond to the origin in $\mathbb{R}^3$. The positive *x*-axis will extend from the origin through the location with latitude and longitude both equal to $0°$. The positive *y*-axis will extend from the origin through the location with latitude $0°$ and longitude $90°E$. The polar axis will be designated as the *z*-axis.

Figure 16 illustrates how a location $P = \left(\theta_{\text{longitude}}, \varphi_{\text{latitude}}\right)$ on the Earth's surface will be assigned to the unique position vector

$$P\left(x, y, z\right) = \left(R\cos\theta\cos\varphi, R\sin\theta\cos\varphi, R\sin\varphi\right) \; . \qquad \textbf{(1.8)}$$

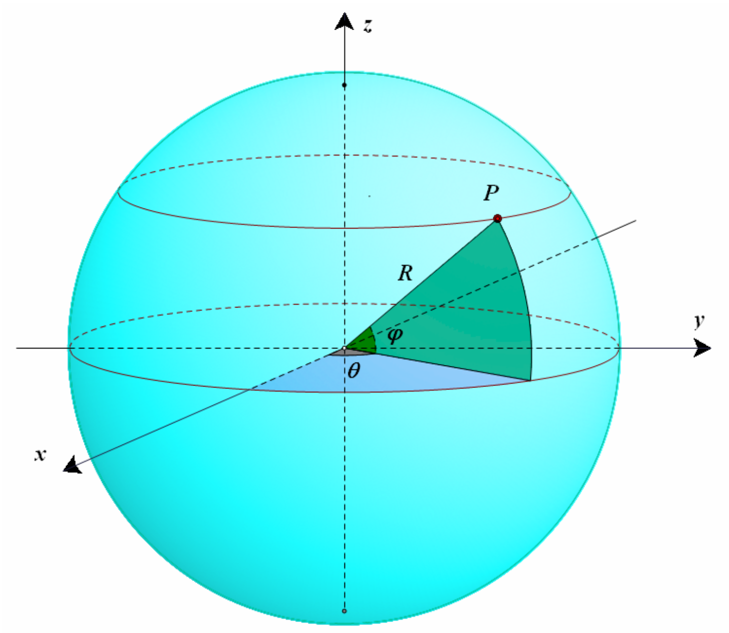


Figure 16: The position vector *P* of a geographic location

If two locations *P* and *Q* are converted to their corresponding position vectors **u** and **v**, respectively, then determining the angle λ is a straightforward application of the dot product. Using the relationship $\mathbf{u} \cdot \mathbf{v} = R^2 \cos\lambda$, we have $\lambda = \arccos\left(\mathbf{u} \cdot \mathbf{v} / R^2\right)$. Therefore, the distance between *P* and *Q* may be approximated with the formula

$$R \cdot \lambda = R \cdot \arccos\left(\frac{\mathbf{u} \cdot \mathbf{v}}{R^2}\right) \; . \qquad \textbf{(1.9)}$$

Question: We may know from previous experience that the inverse cosine function has a restricted range. Will this be a problem for us? Why not?[7]

[7] The central angle, λ, of the shortest arc between two distinct locations on the sphere will have a measure greater than 0° and less than or equal to 180°. These values lie in the range of the arccosine function.

EXAMPLE 3 (REVISITED): Find the distance between Stonehenge and the London Eye using Formula (1.9).

In Example 3 the coordinates of Stonehenge were given by $(-1.829,\ 51.179)$. Using $R = 6368$ kilometers and Formula (1.8) to convert this to Cartesian coordinates we obtain

$$\mathbf{u} = (3990.00,\ -127.41,\ 4961.36)\ .$$

Similarly, the London Eye's coordinates, $(-0.119,\ 51.504)$, correspond to the vector

$$\mathbf{v} = (3963.82,\ -8.23,\ 4983.93)\ .$$

Applying formula (1.9) we have:

$$\text{Distance} = 6368 \cdot \arccos\left[\frac{(3990.00,\ -127.41,\ 4961.36)\bullet(3963.82,\ -8.23,\ 4983.93)}{6368^2}\right].$$

This yields a distance of about 124.09 kilometers, approximately the same as our "flat earth" estimation. For this example a "flat earth" model is reasonable.

EXAMPLE 4: The highest recorded temperature in the world is 136ºF (57.7ºC). It was recorded in 1922 at Al'Aziziyah, Libya. The coldest recorded temperature is –128.6 ºF (–89.2ºC). It was recorded in 1983 at Vostok, Antarctica.[8] Estimate the shortest distance between Al'Aziziyah and Vostok. Compare the "flat earth" estimate to the "spherical earth" estimate.

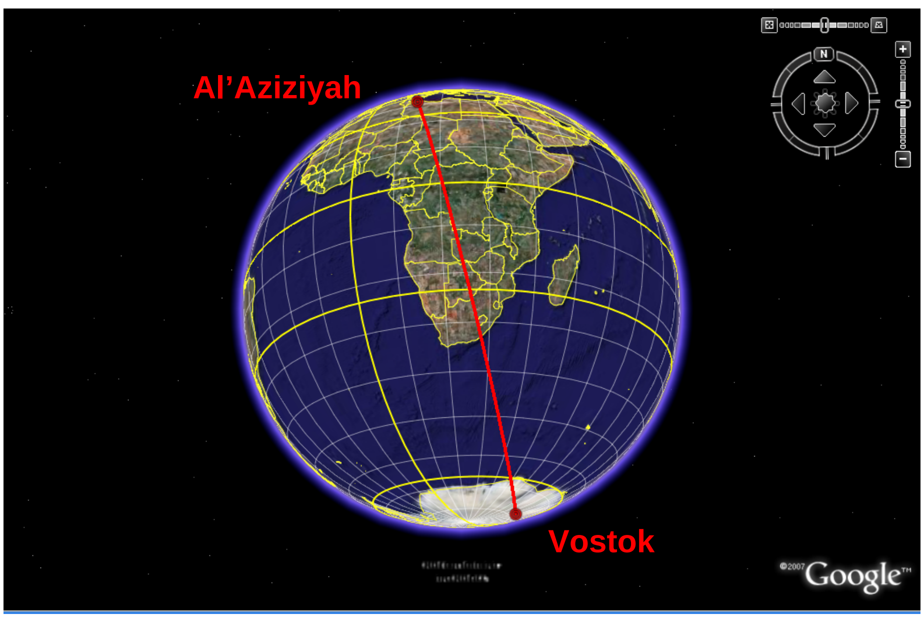


Figure 17: Path from Al'Aziziyah to Vostok

[8] Temperature data obtained from http://www.metoffice.gov.uk/corporate/library/factsheets/factsheet09.pdf .

The longitude-latitude coordinates of Al'Aziziyah are $(13.02, 35.53)$ and the coordinates of Vostok are $(106.87, -78.47)$. If we estimate the distance with the "flat earth" model using Formula (1.7) we obtain a distance of about 15,961 kilometers. Using the "spherical earth" model with Formula (1.9) we obtain a distance of 13,945 kilometers – just over 2000 kilometers less than the "flat earth" estimation!

Let us now return to our opening problem – estimating the length of the walk in Figure 1. This walk may be viewed dynamically in Google Earth by downloading its .kmz file located **here**[9]. Looking at Figure 1 we can get a rough idea of the distance covered by the walk. Using the scale provided and a ruler, we estimate that the walk is about 4 kilometers long.

Now look at the walk as viewed on a topographic map in Figure 18.

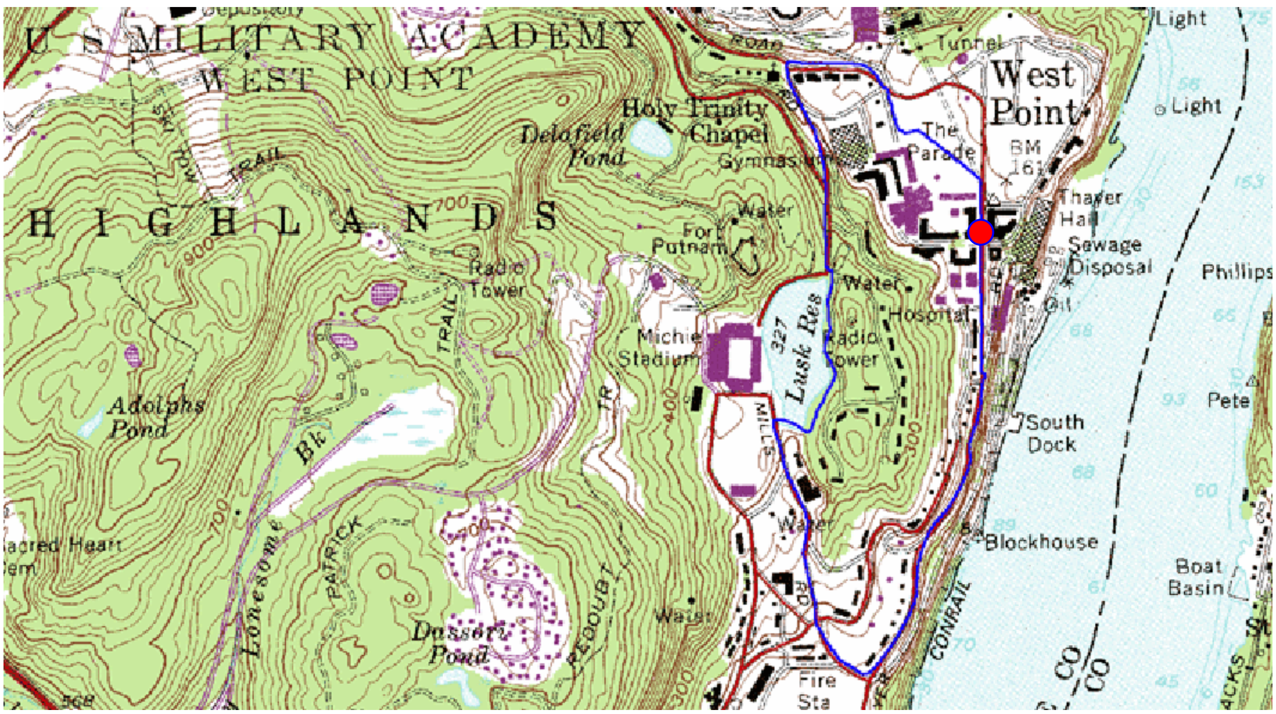


Figure 18: The GPS track of the walk on a topographic map

The walk followed a counterclockwise loop that started and ended at the point marked by the red dot in Figure 18. The first part of the walk was on flat terrain, but then the walk ascended a hill toward Lusk Reservoir. Later it descended and then continued on fairly level terrain back to the starting point. Figure 19 shows the elevation in meters above sea level measured by the GPS during the walk. Notice the erratic nature of the curve. It looks as if the walker was doing some pretty dramatic jumping – he wasn't. Some of the bumps are due to noise in the data. One of the difficulties of working with real data is that the noise that can come from measurement error or

[9] http://www.dean.usma.edu/math/people/Wattenberg/JOMA-GPS/Class-Materials/Intro-modeling/walk.kmz

from unusual events may obscure broad trends. For example, GPS units may be adversely affected by large buildings. However, in this case we can match the broad variations in elevation shown in Figure 19 to the terrain shown in Figure 1 and Figure 18. Table 1 shows a few lines of the raw data. The full data set can be downloaded **here**[10].

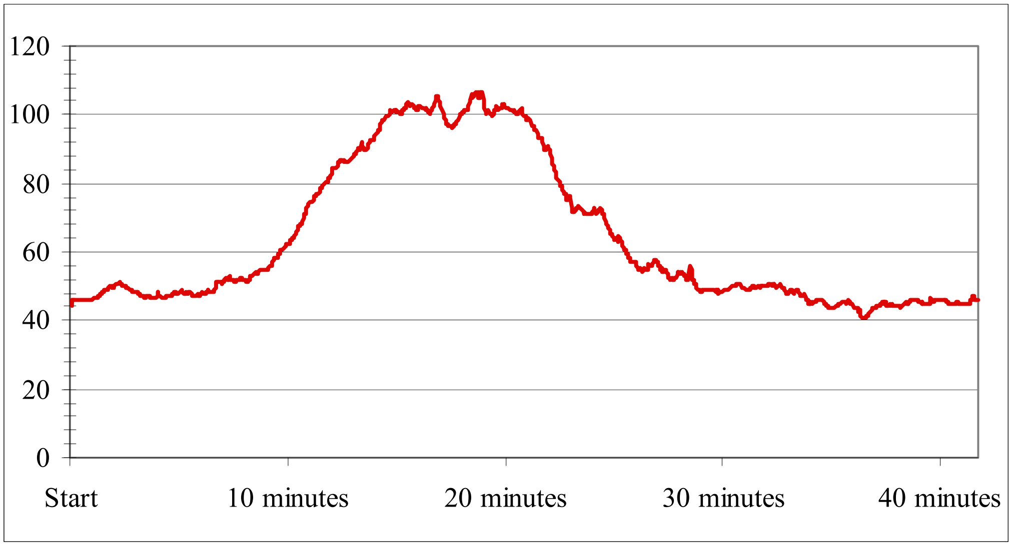


Figure 19: Elevation in meters

Date	Time	Latitude	Longitude	Elevation (meters)
12/15/2006	10:01:22	41.39035	-73.95621	44.4
12/15/2006	10:01:24	41.39035	-73.95623	45.3
12/15/2006	10:01:26	41.39035	-73.95623	46.3
12/15/2006	10:01:28	41.39035	-73.95623	46.3
12/15/2006	10:01:30	41.39039	-73.95623	46.3
12/15/2006	10:01:32	41.39041	-73.95623	45.8
12/15/2006	10:01:34	41.39043	-73.95623	45.8
12/15/2006	10:01:36	41.39048	-73.95623	45.8
12/15/2006	10:01:38	41.39050	-73.95623	45.8

Table 1: A few lines of GPS data from the walk

[10] http://www.dean.usma.edu/math/people/Wattenberg/JOMA-GPS/Class-Materials/Intro-modeling/walk.xls .

How can we apply the mathematics presented in this paper to estimate the total length of the walk? Since the area we are covering is small, we can use our flat earth model. We will estimate the *arc-length* of our walk by using Formula (1.7) to calculate the average distance traveled over each 2 second interval. We can then sum these distances to estimate the total length of our walk. By letting a CAS or *Excel* perform our calculations we arrive at an estimate of 4159.33 meters. This seems to agree with our very rough visual estimate of 4 kilometers. But wait! This is only the length of a two dimensional projection of our walk onto a flat surface. What about that change in altitude? Wouldn't it contribute to the total distance traveled as well?

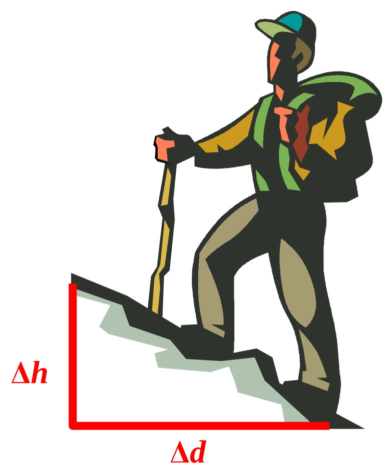


Figure 20: Elevation and horizontal distance

It's time to dust off the Pythagorean Theorem one more time. Over each two second interval we will let the average horizontal change in distance be given by Δd and the average gain in elevation be given by Δh. Then the average distance traveled during those two seconds would be approximately $\sqrt{\Delta d^2 + \Delta h^2}$.

When we sum up all of these distances over the course of the walk we have an estimate of 4203.68 meters. Taking the elevation changes into account added about 44 meters to our previous estimate.

Conclusion

The problem of estimating the distance between two geographic locations – or deciding when the Earth can be flat, and when it can't – is a rich mathematical modeling challenge. Google Earth and mathematical software offer us a wonderful opportunity to apply mathematics while exploring our planet.

Let the journey begin...

EXERCISES

1. **Example 2 (revisited)** Use formula (1.9) to estimate the shortest distance between Rome and Plymouth, Massachusetts. How does this compare to the website estimate of 6580 kilometers?

2. Discover what interesting landmarks are located at the following longitude-latitude coordinates: (147.7, -18.2861) (2.294260, 48.858052) (31.13747, 29.975734) (-113.240014, 36.114526)

3. Using Google Earth the latitude and longitude of four Washington, D.C. landmarks are:

   - The Washington Monument: 38.889448 N 77.035204 W
   - The United States Capitol: 38.889420 N 77.008913 W
   - The Jefferson Memorial: 38.881347 N 77.036587 W
   - The Lincoln Memorial: 38.889237 N 77.050118 W

   Find the distance between each of the buildings. If you were to plan a walk that would take you to all four landmarks, how long would your walk be?

Figure 21: Four landmarks in Washington, DC viewed in Google Earth

4. Discuss the accuracy of the estimate for the sample walk described in the text. What potential sources of inaccuracy might we need to consider?

5. Investigate the "noise" phenomena that can occur when gathering GPS data. Place your GPS in a place where it won't be disturbed and collect measurements for about a half an hour. What do you observe? How can you use these observations to provide error estimates on your distance measurements?

6. Using Google Earth the latitude and longitude of three major US cities are:

- San Francisco: 37.775000 N 122.418333 W
- New York City: 40.714167 N 74.006389 W
- Washington DC: 38.895000 N 77.036667 W

A flight plan makes a loop through the three cities. What is the distance traveled during the total flight?

Figure 22: Three US cities viewed in Google Earth

7. Collect your own GPS data for a walk, car trip, bike ride, or some other excursion. Estimate the length of your trip. Find the best estimate you can and discuss how accurate you think your estimate is. What steps did you take to make it as accurate possible? What potential sources of inaccuracy did you consider?