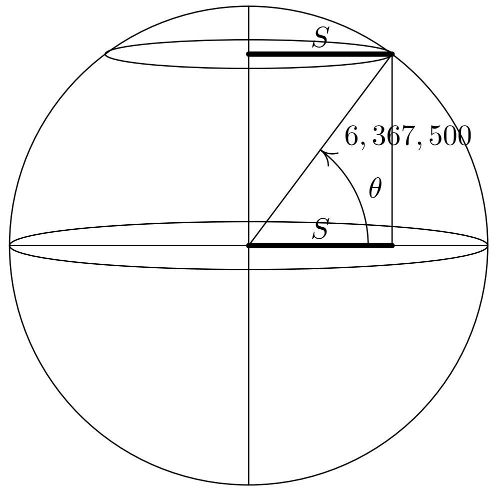


Figure 1: Circles at different latitudes

Estimating Distances in Meters

The location data we download from a GPS unit is given in terms of latitude and longitude. These units are not the usual units in which we discuss length. We want to work with units like feet, miles, meters, or kilometers. This note discusses a rough method for converting distances given in degrees to distances given in meters.

Many programs, like Google Earth, give you the option of expressing latitude and longitude in degrees with decimals rather than degrees, minutes, and seconds. For example latitude 42 degrees 30 minutes can be expressed in decimal degrees as latitude 42.5. We will work with latitude and longitude expressed in degrees with decimals because it is much easier to work with this form than degrees, minutes, and seconds.

The earth is shaped like a slightly squashed sphere. Its polar radius is roughly 6,357 kilometers and its equatorial radius is roughly 6,378 kilometers. The difference between the two is about 21 kilometers, or 0.3%. Our model of the earth will be a sphere of radius 6,367.5 kilometers, or 6,367,500 meters. See Figure 1. This model will be good enough for our purposes.

Suppose that two points have the same longitude but that their latitude is different. Suppose that the latitude of one point is y_1 and the latitude of the second point is y_2. The distance measured in degrees between these two points is just $|y_2 - y_1|$. Since the radius of the earth is 6,367,500 meters we can convert this distance to meters using the formula

$$|y_2 - y_1| \times \frac{2 \times \pi \times 6,367,500}{360}$$

or, roughly,

$$111,134 \times |y_2 - y_1|.$$

Next we look at two points with the same latitude but different longitudes, x_1 and x_2. The same formula would work if the two points were on the equator but most of us do not live on the equator. Circles going around the earth at latitudes close to either of the poles are much smaller than circles going around the earth at latitudes close to the equator. Using Figure 1 we see that the radius, S, of the circle around the earth at latitude θ is

$$S = 6,367,500 \cos\theta.$$

Before using the cosine function on the latitude we must convert latitude expressed in degrees to latitude expressed in radians. This give us the formula

$$|x_2 - x_1| \times \frac{2 \times \pi \times 6,367,500 \times \cos\left(\frac{2\pi y}{360}\right)}{360}$$

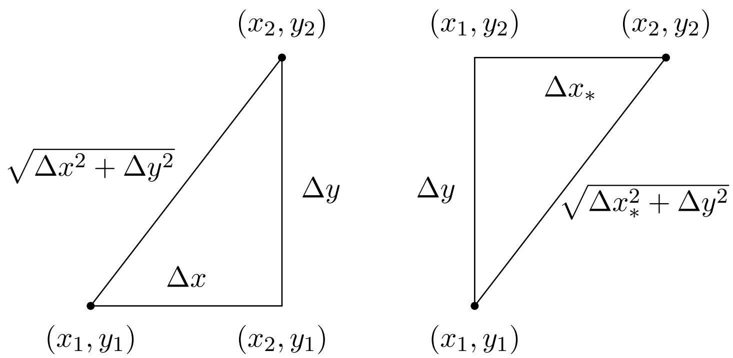
or roughly

$$111,134 \times |x_2 - x_1| \times \cos\left(\frac{2\pi y}{360}\right)$$

where y is the latitude in degrees of the two points. We can summarize our two formulas using the notation Δx for the east-west distance in meters and Δy for the north-south distance in meters by

$$\begin{aligned}\Delta x &= 111,134 \times |x_2 - x_1| \times \cos\left(\frac{2\pi\theta}{360}\right)\\ \Delta y &= 111,134 \times |y_2 - y_1|.\end{aligned}$$

Unless you're a pilot or an astronaut, you probably live your day-to-day life as if the world were flat. For example, if you walk west for 1.53 hours at 4 miles per hour you can be quite confident that you will be 6.12 miles from your starting point and the fastest way back is to walk 6.12 miles east. The International Space Station, however, is flying so fast that we cannot ignore the fact that the earth is not flat. After flying in the same direction for 1.53 hours it is right back where it started.



The first triangle A second triangle

Figure 2: The distance between two points

As long as our data is collected in a relatively small area, we can work with a flat model for our part of the earth. Using the triangle on the left side of Figure 2 and the Pythagorean Theorem we see that the distance between two points (x_1, y_1) and (x_2, y_2), using latitude and longitude coordinates in decimal degrees, is approximately[1]

$$\sqrt{\Delta x^2 + \Delta y^2} = 111,134\sqrt{(x_2 - x_1)^2 \cos^2\left(\frac{2\pi y_1}{360}\right) + (y_2 - y_1)^2}.$$

Note that x_1 and x_2 are the longitude coordinates and that y_1 and y_2 are the latitude coordinates.

We might have used the the second triangle, on the right side of Figure 2, to compute the distance between the two points. If we weren't working with points on a sphere, the two figures would give exactly the same results. Because we are working on a sphere, however, the results are slightly different. Using this second triangle in Figure 2 we compute the distance, using Δx_*, by

$$\sqrt{\Delta x_*^2 + \Delta y^2} = 111,134\sqrt{(x_2 - x_1)^2 \cos^2\left(\frac{2\pi y_2}{360}\right) + (y_2 - y_1)^2}.$$

The only difference between the two computations is in the use of

$$\cos\left(\frac{2\pi y_2}{360}\right) \quad \text{instead of} \quad \cos\left(\frac{2\pi y_1}{360}\right).$$

[1]We must use the qualifying adverb "approximately" because this formula relies on a less than perfect model.

In practice, the difference between these two approximations is very small if the points are close together. In fact, comparing both estimates is a good way to get some feeling for the difference between our flat model and the nearly spherical earth over the distances of interest. One thing that is a bit disturbing about either of the two estimates is that the formulas are not symmetric – that is, the distance from (x_1, y_1) to (x_2, y_2) is slightly different than the distance from (x_2, y_2) to (x_1, y_1). One way to avoid this is to use the formula

$$111,134\sqrt{(x_2 - x_1)^2 \cos^2\left(\frac{\pi(y_1 + y_2)}{360}\right) + (y_2 - y_1)^2}.$$

This formula is the result of using the average

$$\frac{y_1 + y_2}{2}$$

instead of either y_1 or y_2 in the factor

$$\cos\left(\frac{2\pi y_1}{360}\right) \quad \text{or} \quad \cos\left(\frac{2\pi y_2}{360}\right).$$