Research Statement

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My research falls into several categories. One category is in continuing research based on coordinate percolation, the topic of my disertation, and another is research to expand my knowledge of graph theory. Because many of the problems in percolation have eluded solution methods so far, I find myself drifting more towards the graph theory research and applications of graph theory. Recently, this has led to an interest in the oriented line graph, although I am continuing to seek new graph theory problems.

- **Oriented Line Graph**

  The oriented line graph (OLG) construction follows the construction of a line graph, but the goal is to study the cycle structure of a graph rather than the edge adjacency structure. To construct the OLG L^oX from a graph X, we let

  $$\begin{aligned} V(L^oX) &= \{(u,v) \in V(X) \times V(X) | \{u,v\} \in E(X)\} \\ E(L^oX) &= \{((u,v),(v,w)) \in V(L^oX) \times V(L^oX) | u \neq w\}. \end{aligned}$$

  Notice that every edge in X gives rise to two vertices in the digraph L^oX.

  My interest in the OLG is in the structural characterization of the OLG in terms of forbidden subdigraphs. I have found a set of 11 subdigraphs which are forbidden by the form that the oriented line graph must obtain. Together with a degree condition, this appears to fully characterize which graphs may be oriented line graphs.

- **Percolation**

  In percolation on the coordinate lattice $\mathbb{Z}^2$ each vertex is declared to be open or closed with a certain probability. A path in $\mathbb{Z}^2$ is open if all the vertices in the path are open. An open cluster is a maximal set of vertices connected by open paths. A typical question is whether or not the origin is in an infinite open cluster.

  In coordinate percolation, the decision as to whether a vertex is open or closed is based on events associated with the lines or planes through that vertex. Some versions of coordinate percolation are notoriously difficult; my objective is to develop a version which has parallels to independent percolation and yields to methods from combinatorial probability. Recently I have been working with the following model: each line $x = a$ or $y = b$ in $\mathbb{Z}^2$ is given a uniformly random value from the interval $[0, 1]$. A vertex (a, b) is declared open if the sum of the values of the lines $x = a$ and $y = b$ is less than a given threshold value, t.

  In this form of percolation, there is a critical threshold, t_c. In other words, for $t < t_c$, the probability that the origin is in an infinite open cluster, $\theta(t)$, is 0. For $t > t_c$, $\theta(t) > 0$. For coordinate percolation where the lines are assigned values

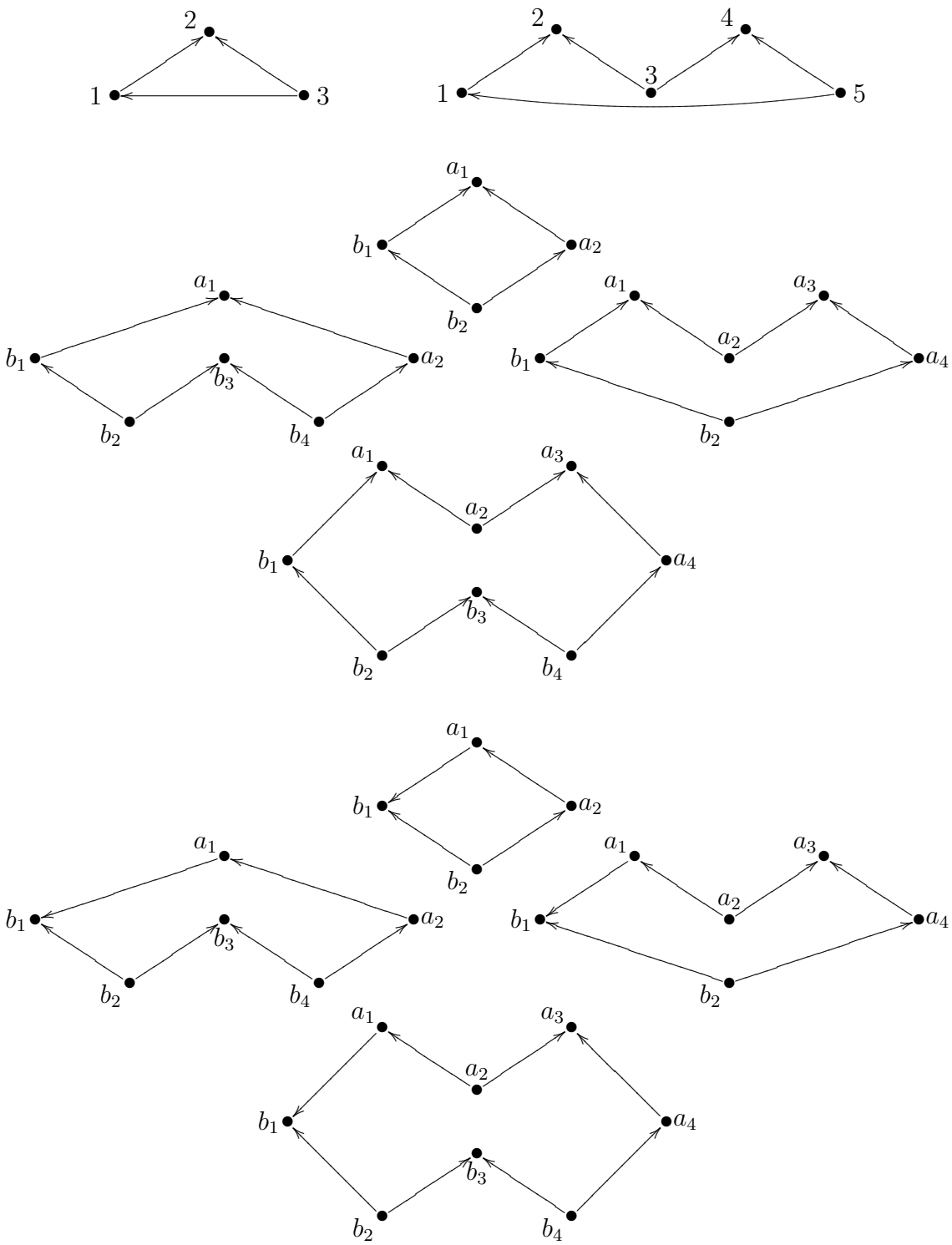

Figure 1: The forbidden subdigraphs of an OLG. One more, an alternating path of length more than 5, is only forbidden as an induced subdigraph.

uniformly from $[0,1]$, $t_c = 1$. By replacing the independent uniform values with a dynamically generated order of the integers 1 through n, I have shown that

$$\Theta(t) = \begin{cases} 0 & \text{if } t \leq 1 \\ \frac{5+3\sqrt{5}}{10}(t-1)^{\frac{3-\sqrt{5}}{2}} - \frac{3\sqrt{5}-5}{10}(t-1)^{\frac{3+\sqrt{5}}{2}} & \text{if } 1 \leq t \leq 2 \\ 1 & \text{if } t \geq 2. \end{cases}$$

Although the probability of an infinite open cluster is known, there are still more questions to ask. The size of the cluster containing the origin is also of interest. In addition, future work may focus on higher dimensional lattices. In $\mathbb{L}^n$, values may be assigned to each line (one-dimensional subspace) or each plane ($n-1$-dimensional subspace). The former seems to be less tractable than the latter but has interesting parallels to the coordinate percolation model in the triangular lattice. The triangular lattice is also interesting; most questions are still open in this lattice, including the value of the critical threshold. I also hope to explore more connections to independent percolation and find some direct applications of coordinate percolation.