Keep on Moving—Energizers in Mathematics Education Reform

Brian Winkel

Introduction

"Thank you for those words of wisdom you offered at graduation years ago. I have used them often," said the returning engineer to the dean at her class reunion. Puzzled as to what he might have said, the Dean asked for clarification. She replied, "When I came across the stage you shook my hand, gave me my degree, and said 'Keep on moving.'" Science, engineering, and mathematics faculty sponsored by the National Science Foundation (NSF) and the good folks at NSF do just that—keep on moving!

With curriculum reform spurred by NSF support of the calculus movement, mathematics PIs were the largest group at the 1994 Project Impact conference. Addressing change, action–reaction, and dialogue, the meeting featured PIs and publishers grappling with issues of disseminating, evaluating, piloting, and upscaling projects. However, there was never an insular atmosphere as exhibit booths and many discussion groups cut across disciplines.

In leading the mathematics discussion groups, Division of Undergraduate Education (DUE) Program Director Bill Haver reported that nearly two-thirds of the nation's colleges and universities are undertaking some type of calculus reform involving at least some of their students. Yet the reform effort is definitely not complete. Only an approximate 15–20 percent of students are enrolled in courses making heavy use of reform approaches. Despite this, I believe an inner clock ticks toward progress in undergraduate curriculum reform. The best programs effectively use cooperative learning; graphical, analytical, and numerical approaches; oral and written communication; and technology. Faculty are responding to these reform efforts.

Reform is being spurred in two major ways: (1) professional meetings devoting large portions of their programs to change and improvements in teaching and (2) local administrators encouraging experimentation—doing something to improve retention. Evidence of reform includes the following:

- Standards produced by the National Council of Teachers of Mathematics (NCTM) making an impact in textbook publishing for K–12 mathematics.
- Technology engaging teachers.

- Cooperative learning becoming an alternative to lecturing.
- Journal literature featuring new approaches to mathematics education.

Published results of reform and peer interaction spread curricular change, for as Haver said, "Some mathematics faculty are feeling pressure to change, hopefully to improve." Stimuli for change abound!

I believe we could do no worse than to continue to lecture on the same topics with the same emphases on analysis and symbol manipulation. We now see before us an expanding menu of options from emerging approaches, such as graphical, numerical, symbolic, analytic, and communicative aspects of mathematics, coupled with significant use of technology and small-group, cooperative learning in which teachers act as coaches and mentors rather than knowledge dispensers. A growing number of faculty believe that their courses should concentrate on a smaller set of materials through which students can "learn how to learn" and use mathematics with confidence. Less is more! Leaner is better! The emphasis shifts from content to process—an appropriate change—for mathematics offers process, not just facts.

The Project Impact conference brought publishers and faculty together to facilitate commercial publication and sustainable dissemination of NSF-funded activities. The commercial market offers the exposure and push to enable faculty to examine, adopt, and deliver new materials. Commercial dollars support continued reform by sustaining the innovators in the process of developing, testing, evaluating, and improving materials. Moreover, in the eyes of some faculty the "proven" value of commercial texts may be just the edge to convince them to change. There are, however, others who believe that placing materials on the Internet permits fast sharing of materials and significant "localization" and customization. This approach can produce dialogue and feedback, but not extensive and sustained reform. Only the commercial market, coupled with a good pedagogical approach and a ready audience of faculty, can ensure real dissemination and systemic change.

Those who are working in mathematics education reform need to be more aware of one another's efforts and should incorporate developed ideas rather than reinvent them. In interviewing PI exhibitors, I heard a number of common themes and approaches, although the PIs were unaware of what the other was doing. PI meetings such as the Project Impact conference help, but the PIs themselves need to spend time and funds publishing materials about their projects, examining other programs, and visiting or hosting colleagues. They all need to read the reports of abstracts of NSF-funded projects published by NSF and other sources, e.g., *UME Trends*, and to establish contact with colleagues doing similar projects. Four years ago the Rose-Hulman Institute of Technology founded a journal, *PRIMUS—Problems, Resources, and Issues in Mathematics Undergraduate Studies*, for the specific purpose of supporting a wider dialogue.

Finally, a renewed emphasis on the value of teaching needs to accompany mathematics teaching reform, and institutions must encourage, support, and reward innovative teaching. We need "at home" recognition of such efforts in the recruitment, mentoring, tenure, and promotion processes.

One way to realize the forces necessary for reform is to examine specific efforts. Within the main framework of mathematics innovation are lab approaches, resource materials, and totally structured curricula. One finds a broad spectrum of efforts, from the community college level, where projects build on the NCTM Standards approach and make use of appropriate technology, to efforts at large state universities, such as the University of Michigan's efforts to upscale reform calculus instruction, coupled with cooperative learning. Let us look at sample innovations in precalculus, calculus, upscaling of a calculus effort, cognate courses, statistics, linear algebra, upper-level specialty and mainline courses, and integrated courses involving science, engineering, and mathematics.

Precalculus

- "Functioning in the Real World," a project of Sheldon Gordon (Suffolk Community College) demonstrates that a good time to change precalculus occurs when you change the calculus to deemphasize manipulations. The precalculus course can develop more useful skills, e.g., fitting functions to data using linear regression and nonlinear regression by recognizing a pattern, linearizing, fitting a line, and converting back to the nonlinear model.
- "Redesign of College Algebra," by Linda Kime (University of Massachusetts—Boston), fulfills a general university quantitative reasoning requirement. The course, taught in a computer lab, includes linear and nonlinear fits of data collected in the physics lab, e.g., inverse square law using light meter. The course stresses group work, writing, reading, and reporting with more work for students. Performance, however, correlates well with effort and amount of work rather than with innate ability.

Calculus

- At CUNY—Borough Manhattan Community College, Patricia Wilkinson, Chair, added a 2-hour lab to calculus for commuting students. Students meet in small groups and submit portfolios of work using Uri Treisman's model of challenging problems. Students learn word processing on their own and use Mathematica. Exams look the same, but professors include lab grades for as much as they want and influence lab activities, permitting reluctant faculty to join in at their comfort level.
- With regard to Duke University's "Project CALC," Lang Moore says they have a revised pedagogical approach driven by real-world lab problems. Jack Bookman, also involved in the project, just finished a longitudinal study concluding that "Project CALC" students become better problem-solvers, possess better attitudes, and, on average, tend to take one more mathematics course than students who took traditional calculus. It was also revealed that weaker students succeed at a higher rate because of group work. Faculty who have taught this approach will no longer teach the regular approach even though this new approach takes more resources and increases the intellectual load for teachers. D C Heath is publishing the first draft of "Project CALC" material.
- Wiley publishes thorough material for the widely used reform effort of Sheldon Gordon (Suffolk Community College) and Deborah Hughes Hallet's (University of Arizona) "Harvard Calculus Project." Students who learn by algorithm and pattern matching find this approach scary. "You have to be gentle with them at the start and then firm with them else they will hold on to your hand forever," says Hughes Hallett. Gordon says they get a deeper level of questions back, and community college students find a release in thinking, not just grinding (algorithmic approach). Implicit plots are hard to get, but after doing derivatives and slope of tangent line to an

implicit plot, students said why not use the tangent line "envelope" to outline the function, thus discovering an algorithm to construct the implicit plot. Faculty do not present solutions in class. It is tough to make up exams. The key here is the interplay of ideas—students think, faculty think.

- Greg Foley (Sam Houston State University) works with "Calculus for Comprehensive Universities and 2-Year Colleges," a project in which two faculty from each of four local schools meet biweekly to discuss approaches to a calculator-driven calculus course in which students work in groups in lab projects. Write-ups ask students leading questions and expect written responses. Labs lead to healthy discussions, and each group member rates the others in terms of contribution. In addition to the use of labs and technology, there are more extensive, long-range projects. There is one "gateway" exam on skills in differentiation, but major exams are not based on skill, focusing instead on problem-solving and concepts. A higher percentage of students go on to more mathematics courses from this program than from the traditional approach.
- Kenneth Hoffman (Hampshire College) states that Five Colleges, Inc.'s, "Calculus in Context" seeks to show how calculus grew out of attempts to deal with science—starting with a science problem and doing mathematics to address that problem. This technology-based course opens with nonlinear differential equations. Students enter program codes for 1 1/2 semesters, then use Mathematica. Downstream faculty are happy with student strengths in modeling and interpretation, but they say the students' skills in manipulation are weak. The course philosophy is to get a method that works first, in all situations, then show students the few examples in which they may get a closed-form solution, e.g., integration is taught with numerical methods first, then a few special methods. Grading homework becomes more intense because faculty need to read it very carefully.

Upscaling of a Calculus Reform Effort

- In 1990–91, Morton Brown (University of Michigan) taught one section of Calculus I using graphing calculators with standard text. In 1991–92, several sections used graphing calculators with supportive texts, while others used calculators with traditional texts. In 1992–93, the first year of Michigan's NSF support, cooperative learning and reform materials, such as "Harvard Calculus," were used in 10 pilot sections. Brown thought they were doing cooperative learning, but in the second week of the term he found the excellent book, *Active Learning: Cooperation in the College Classroom*, by David Johnson, Roger Johnson, and Karl Smith (Interaction Book Company, 7208 Cornelia Drive, Edina, MN 55435 USA). Brown read the book, modified the teaching style, and began to assign roles to students and do true cooperative learning. In 1993–94, Michigan used calculators and the "Harvard Calculus" materials in all Calculus I and II classes. Some classes were still in traditional rooms with 35 students in each class; however, a number of sections were in smaller rooms (24 per class) using full cooperative learning.

  The goal is to limit class size to 24 students with groups of 4 executing cooperative classwork and group homework. All faculty and teaching assistants in this program attend a summer workshop on sensitivity training, cooperative learning, technology utilization, and the "Harvard Calculus" materials.

Cognate Courses

- An example of this genre is the "Quantitative Science Curriculum for Life Science Students" developed by Louis Gross (University of Tennessee, Knoxville). Few biology departments require statistics or linear algebra. Gross put together a 1-year course driven by data and hypothesis testing to teach necessary calculus, statistics, probability, and linear algebra (with some dynamical systems). Gross conducts workshops to encourage educators to offer this approach.

Statistics

- Two examples of enrichment in statistics are Richard Schaeffer's efforts to create an "Activity-Based Introductory Statistics Course" at the University of Florida and an electronic encyclopedia of examples and exercises at the University of Ohio. Elizabeth Stasny explains that the Florida project adds a lab component to the "Introductory Statistics" course. Students see statistical principles unfold with materials they use in small demonstration groups. Faculty are working toward a lab manual of modules. This approach changed the tests from "plug and chug" to "what would happen if . . . " There is more work such as thinking about how to use materials and grading assignments. The Ohio State project produces HyperCard materials based on interesting activities found in the media: "Are baseball players worth their salary?" and "What is the real story on doctor-to-patient HIV spread issue?" Projects, indexed by topic and by statistical concepts, are on-line, and a user can specify the level.

Linear Algebra

- Steven Leon (University of Massachusetts, Dartmouth) heads up "Project ATLAST" to encourage and facilitate the use of software in teaching linear algebra. Summer workshops using MATLAB are offered for faculty to develop projects. Participants develop modules, try them in courses, and return the finished product.

Examples of Upper-Level Courses

- Through "Geometry at Cornell," David Henderson (Cornell University), as a member of a geometry group, offers a sequence of geometry courses particularly for prospective high school teachers. Efforts stress writing, relate geometry to the real world, and offer classical geometry concepts.
- By implementing an "Upper-Division Mathematics Computer Classroom/Laboratory," Gary Sherman (Rose–Hulman Institute of Technology) offers students discovery-based algebra and discrete mathematics in a Magma/Cayley software environment where algebraic structures are real, substructures and relationships are computed, and theorems are founded on tactile sense and experience. Sherman has motivated undergraduates at Rose–Hulman, and his Research Experience for Undergraduates summer efforts have produced published research results based on this approach.

Integrated Approaches

Integrated approaches in which mathematics is taught as a part of, and in relation to, elements of a larger picture do exist and are increasing in number. During the summer of 1994 at a conference sponsored by NSF, GE Foundations, and Rose–Hulman Institute of Technology, faculty from 30 schools gathered to discuss the merits of an integrated approach in first-year science, engineering, and mathematics curricula. A number of schools will pilot such an approach in the fall of 1994, among them Texas A&M University, University of Alabama, and Arizona State University.

- Robert Quinn's (Drexel University) "Enhanced Learning Experience for Engineering Students" (E4) attempts to integrate calculus with physics in the first-year engineering curriculum. Robin Carr has developed an excellent text of applications-driven calculus using Maple technology. E4 is now the standard curriculum in engineering at Drexel.
- "Integrated, First-Year Curriculum in Science, Engineering, and Mathematics" (IFYCSEM) by Brian Winkel *et al.* (Rose–Hulman Institute of Technology) is in its fifth year of teaching one-quarter of entering students in a 3-term, 12-credit/term, one-grade, team-taught (8 faculty members from science, engineering, and mathematics) course. All technical material in the first-year curriculum is included in this one course and is available for mathematics integration on the spot! Mesa Community College, as a part of the NSF-funded Foundation Coalition, is teaching an IFYCSEM project modeled after the one at Rose–Hulman.
- Dennis DeTurck (University of Pennsylvania) team teaches an integrated course with physics and chemistry colleagues. They found it very exciting and dared to be different—students can only ask questions of faculty not from the field in question.

Learning from and with Colleagues Outside Mathematics

There is a rich collection of experiences, ideas, and successes outside mathematics. David Hestenes (Arizona State University), a leader in the physics community, has discovered misconceptions that students bring to physics courses (and they leave with these same misconceptions). Physics teachers are examining what they do and are introducing many active, tactile learning opportunities. The new NSF initiative in chemistry education will produce an effort comparable with calculus reform. Mathematics educators do not have a monopoly on major reform efforts. Ask around your campus community to find others interested in reform. Perhaps you can form a team interested in integrated curriculum efforts, or you might learn something from these reformers that works for them and could work for you.

The NSF initiative, Mathematical Sciences and Their Applications Throughout the Curriculum, will stimulate dialogue among reformers across the disciplines and provide excellent opportunities to work with colleagues outside the mathematics community. In addition to the support of upper-division course development, this initiative will enable dialogue across disciplines concerning introductory courses.

Examples of Sources

- "Calculus Reform in Liberal Arts College," represented by Wayne Roberts (Macalester College), delivered modules and readings from five books from the Mathematical Association of America and Roberts' book *Resources for*

Calculus. These books serve faculty who do not want to invest in an entire reform program or who wish to supplement a reform program with other projects.

- "Snapshots of Applications in Mathematics" by Dennis Callas (SUNY College of Technology, Delhi) is creating *Real World, Contrived, and Charming Applications*. The material comes from faculty, popular press, and scientific journals. Two mathematics journals have agreed to feature these snapshots in regular columns: *The AMATYC Review* and *NY State Mathematics Teacher Journal*.

General Observations

Commitment and involvement, both individual as well as institutional and consortial, are essential. Consider the efforts of Lang Moore and David Smith (Duke University) on "Project CALC," who speak at conferences; visit projects; host visitors; write, publicize, work and rework materials; and provide source materials for review and scrutiny in the development stage. The same applies to the team involved with the "Harvard Calculus" project. They field tested many versions of their material before approaching a publisher.

Evaluation of materials is generally not done in small projects, but often there is at least one other site at which the material is being developed and tested. Some of the bigger projects have extensive evaluation. Formal evaluation is scarce.

Students learn better when they have to formulate their ideas and communicate them. People talk about cooperative learning and small groups, and some do it in labs that accompany lectures. There is a movement to do more small-group work in class and use writing, e.g., logs, journals, papers, homework write-ups, and portfolios, both for internal assessment and external evaluation.

Students also benefit when they are given more attention. It may well be economical to do so, for attrition rates are reduced, the need for repeat courses is lowered, and students are more likely to pursue a subject area that makes them happy.

Final Words

Several years ago, at a national engineering education meeting, Richard Boyce (Rensselaer Polytechnic Institute) said, "It is the most exciting time to be teaching calculus." Indeed, it is the most exciting time to be teaching mathematics! More and more faculty are willing to do what it takes to bring about positive changes in their professional lives, in the learning activities of their students, and in the broader mathematics education community. Calculus reform, sponsored by NSF, has been a jump start for mathematics education improvement, and those in the field just "keep on moving!"

Brian Winkel, Professor of Mathematics at Rose-Hulman Institute of Technology, having taught mathematics in liberal arts settings (Albion College) and now engineering settings (Rose-Hulman), is committed to teaching with cooperative learning, to using technology, and to teaching using an integrated approach. He has been a team member in innovating science, engineering, and mathematics education in Rose-Hulman's "Integrated First-Year Curriculum in Science, Engineering, and Mathematics" (a nationally recognized curriculum originally funded by NSF); is part of a larger team of educators who form the Foundation Curriculum, funded by NSF to effect systemic change in engineering education; heads an NSF-funded project to create a development site for complex, technology-based problems in calculus with applications in science and engineering at Rose-Hulman; and edits two journals, PRIMUS *and* Cryptologia.